Basic Electrical, Electronics and Measurement Engineering

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Preface

Basic Electrical, Electronics and Measurement Engineering is designed specifically to cater to the needs of second semester B.E/B.Tech (Mechanical, CSE and IT) students. The book has a perfect blend of focused content and complete syllabus coverage. Solved University question papers, tagged with specific topics, are provided in the book which will help students from the examination point of view. Simple and easy-to-understand text elucidates the fundamentals of electrical, electronics and measurement. Several solved examples, schematic diagrams and adequate questions further help students to understand and apply the concepts.

Salient Features

- Crisp content strictly as per the latest syllabus of Basic Electrical, Electronics and Measurement Engineering
- Comprehensive coverage with lucid presentation style
- Solutions to examination papers present appropriately within the chapters
  - Solved questions as examples incorporated appropriately within each chapter
  - Theory questions tagged within each chapter
- Rich exam-oriented pedagogy
  - Solved Numerical Examples within chapters
  - Theory Questions tagged within chapters
  - Two Mark Questions and Answers at the end of each chapter
  - Unsolved Review Questions

Chapter Organisation

Chapter 1 discusses electrical circuit analysis, which includes Ohm's law, Kirchhoff's law, series and parallel circuit analysis with resistive, capacitive and inductive networks, nodal analysis and mesh analysis of electrical circuits.

Chapter 2 deals with network theorems, three-phase power system, single phase, three-phase balanced circuits and electrical wiring.

Chapter 3 is devoted to electrical machines, construction, principle, emf and torque equation, applications, speed control, basics of stepper motor – brushless DC motors – transformers, working principle of ideal transformer and all day efficiency calculation.

Chapter 4 focuses on utilisation of electrical power, renewable energy sources, illumination by lamps, domestic refrigerator and air conditioner, batteries, charge and discharge characteristics, protection and energy tariff calculation for domestic loads.

Chapter 5 explains electronic devices and circuits, PN junction, VI characteristics of diode, Zener diode, transistor configurations, Op-amps, ADC, DAC, multi-vibrator using 555 timer IC and voltage regulator IC.

Chapter 6 concentrates on electrical measurements and instruments, characteristics of measurement, energy meter and wattmeter, transducers, oscilloscope and CRO.

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Any constructive criticism, suggestions and corrections for further improvement of the book will be most appreciated.

S. Salivahanan
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CHAPTER 1

Electrical Circuit Analysis

1.1 Introduction

The electric circuit design and analysis find its significance in different fields of science and engineering. Depending on the type of components and sources present in the circuit, it is classified as either Direct Current (DC) circuit or Alternating Current (AC) circuit. If a circuit comprises of only resistors, direct voltage or direct current sources, then the circuit is called DC circuit; and if a circuit comprises of elements such as inductors, capacitors, alternating voltage or alternating current sources, then it is called AC circuit. The circuit analysis is a process of determining the values of unknown quantities in a DC or an AC circuit. In a DC or AC circuit, the elements can be either be connected in series, parallel, or combination of both.

The circuit laws that provide the basis for circuit analysis are: Ohm's law, KVL and KCL. Ohm's law provides the basic relationship between the voltage and current. Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) are used to derive the voltage and current equations of a circuit. Fundamentally, there are two forms of interconnection between circuit components, i.e., series and parallel connections. The KVL and KCL are used to derive the equations for series and parallel circuits respectively.

According to KVL and KCL, the algebraic sum of voltages or currents in a circuit is always zero. This leads to division of voltage in the series circuit and division of current in the parallel circuit. The mesh and nodal analysis methods are systematic approaches of applying Kirchhoff's laws in simplifying the process of both DC and AC circuit analysis.

1.2 Basic Circuit Components

Charge

One coulomb of charge is the total charge associated with 6.242×10^{18} electrons, i.e., 1 coulomb = 6.242×10^{18} electrons. A positive charge of 1 coulomb means a deficit of 6.242×10^{18} electrons. A coulomb of positive charge will have the same magnitude but opposite polarity. The unit for charge is coulomb, represented by the symbol Q or q.

Electric Potential

Certain amount of work is required to charge a body. This work done is stored in the form of potential energy. The charged body can move the other charges by either attraction or repulsion and thus capable of performing work. This capacity of the charged body to perform the work is called electric potential, which has the unit of volt or symbol *V*.

If a total of 1 joule (*J*) of energy is used to move the negative charge of 1 coulomb (*q*), then there exist a difference of 1 volt (*V*) between the two points. In this case, joule is the unit used to measure the amount of work done or energy.

$$\text{Electric potential, } V = \frac{\text{Work done}}{\text{Charge}} = \frac{J}{q}$$

For example, if the amount of work done is 2 J to move the charge between two points, then there is an electric potential of 2 V between these points.

Electric Current

The flow of electrons in a conductor is known as electric current, which is represented by the symbol *I*. According to modern electron theory, the actual direction of current in a circuit is from negative terminal to positive terminal. However, conventionally it is considered thought that the direction of current is from the positive terminal to negative terminal. This conventional direction of current is still in use. Clockwise and anticlockwise directions are also used to represent the current direction.

The unit of current is ampere (A) or coulomb per second. If 6.242×10^{18} electrons (1 coulomb) pass through the conductor in 1 second, the flow of charge or current is 1 ampere. It is expressed as

$$I = \frac{dq}{dt}\,(or)\,\frac{q}{t}$$

where q is the charge in coulomb and t is the time in second.

The number of electrons crossing a fixed point per second is calculated as

$$\frac{\text{One coulomb/second}}{1.602 \times 10^{-19}\text{ coulomb/electron}} = 6.242 \times 10^{18}\text{ electrons/second}$$

Electric Circuit

The closed path in which the electrons (or current) flow from a voltage or current source is called an electric circuit.

Circuit Element

Any individual circuit component with two terminals by which it may be connected to other electric components is called circuit element. As shown in Figure 1.1(a) and (b), the circuit element can be a resistor, inductor, capacitor, voltage or current source, etc.

Voltage and Current Sources

The voltage and current sources are classified as independent sources and dependent sources. The independent voltage and current sources and symbols are represented as shown in Figure 1.1(a).

Independent Voltage Source

If the voltage source is independent of the current flowing through the circuit is called independent voltage source.

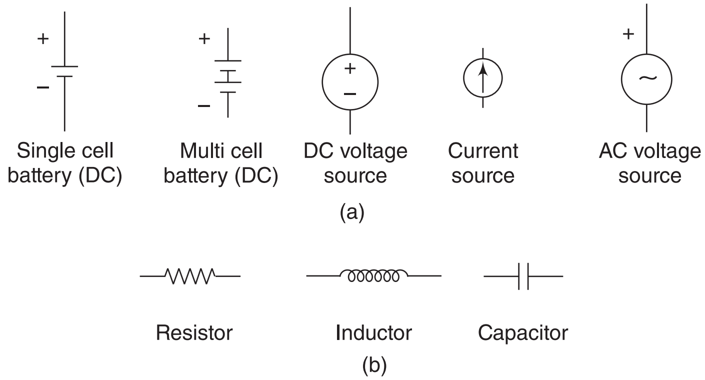


Figure 1.1 Circuit elements and symbols

Independent Current Source

The current supplied by the source is independent of the voltage across it in an independent current source. The arrow mark in its symbol indicates the direction of flow of current.

Dependent Sources

The dependent voltage sources are further classified as voltage-dependent and current-dependent voltage sources. Similarly, the dependent current sources are classified as voltage-dependent and current-dependent current sources as shown in Figure 1.2.

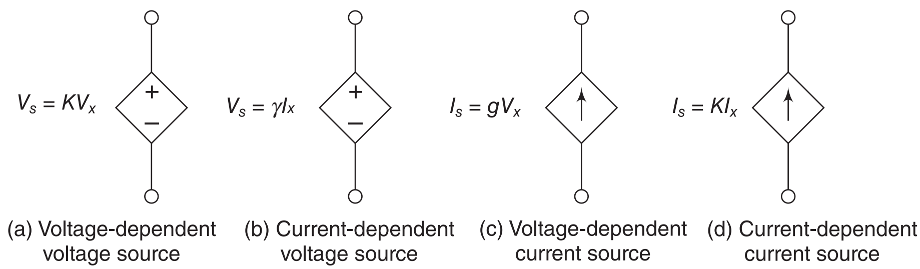


Figure 1.2 Dependent Sources

Voltage-Dependent Voltage Source

The Voltage-Dependent Voltage Source is also called Voltage Controlled Voltage Source (VCVS), in which the source voltage is dependent on voltage at some other location in the circuit. It is represented in Figure 1.2(a), where K is a dimensionless quantity.

Current-Dependent Voltage Source

The Current-Dependent Voltage Source is also called Current Controlled Voltage Source (CCVS), in which the source voltage is dependent on current flowing through any element in the circuit. It is represented in Figure 1.2(b), where γ is a scaling factor with the unit V/A.

Voltage-Dependent Current Source

The Voltage-Dependent Current Source is also called Voltage Controlled Current-Source (VCCS), in which the source current is dependent on voltage at some other location in the system. It is represented in Figure 1.2(c), where g is a scaling factor with the unit A/V.

Current-Dependent Current Source

The Current-Dependent Current Source is also called Current Controlled Current Source (CCCS), in which the source current is dependent on current at some other location in the system. It is represented in Figure 1.2(d), where K is a dimensionless quantity.

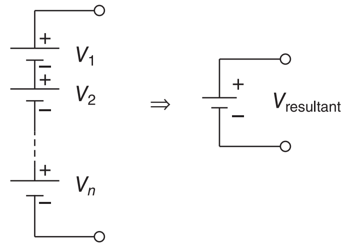


Figure 1.3 Voltage sources in series

Voltage Sources in Series

If the voltage sources, V_1, V_2 … V_n are connected in series as shown in Figure 1.3, then the resultant voltage is given by $V_{resultant} = V_1 + V_2 + \cdots + V_n$.

Current Sources in Parallel

If the current sources, I_1, I_2, … I_n are connected in parallel as shown in Figure 1.4, then the resultant current is given by $I_{resultant} = I_1 + I_2 + \cdots + I_n$.

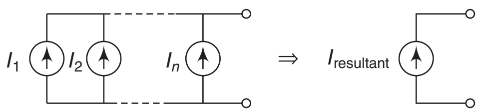


Figure 1.4 Current sources in parallel

Branch

One or more components connected in series between any two nodes are usually called a branch. There will be no other branches connected in between the two nodes of one branch. Figure 1.5 shows an example of a branch connected between nodes 1 and 2. The current flow in a particular branch is called branch current.

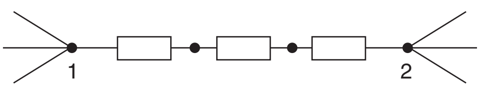


Figure 1.5 A branch

Node (or Junction)

A terminal of any branch of a network, or a terminal common to two or more branches is called a node as shown in Figure 1.6.

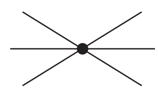


Figure 1.6 A node

Networks

Any interconnection of electric circuit elements or branches is called an electric network.

Lumped Networks

A lumped network is the network in which the individual circuit components cannot be electrically separated or individually isolated and represented as a whole. Example of the distributed network is the transmission line.

Distributed Network

A distributed network is the network containing physically separate and individually isolate circuit components such as resistors, inductors, and capacitors.

Passive Network

A network consisting of passive circuit elements only such as resistors, capacitors and inductors is known as a passive network.

Active Network

A network consisting of active elements such as op-amps, transistors, etc., along with other elements is called an active network.

Linear Element and Network

A circuit element is called linear element if the relationship between the current and the voltage involves a constant coefficient as shown in the following equations.

For a resistor, $V = RI$ · For an inductor, $v = L\dfrac{di}{dt}$. For a capacitor, $v = \dfrac{1}{C}\int idt$.

A network consisting of linear elements is called a linear network.

Bilateral Network

A network is called bilateral if the relationship between the voltage and current does not change in both the directions in that network.

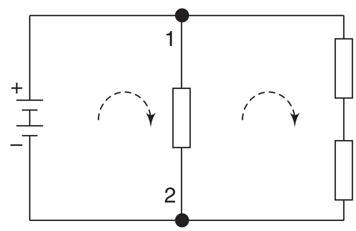


Figure 1.7 Example of circuit showing two possible meshes

Mesh

In a circuit, any closed path or loop starting and ending with the same node, not passing any node or branch twice is called a mesh. Figure 1.7 shows an example circuit with four circuit elements and two nodes. Two different possible meshes can be identified with this circuit as shown by the dotted lines. The arrow mark indicates the assumed clockwise current direction.

Loop

A loop is a closed path in a circuit where two nodes are not traversed twice except the initial point, which is also the final one. But in a loop other paths can be included inside. Figure 1.8 shows an example circuit with four circuit elements and two nodes. Three different possible loops can be identified with this circuit as shown by the dotted lines. The arrow mark indicates the assumed clockwise current direction.

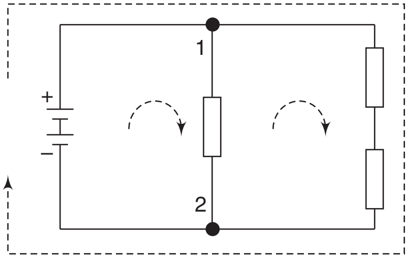


Figure 1.8 Example of circuit showing three possible loops

1.3 Ohm's Law

Ohm's law states that, in a linear network, the voltage across the conducting material is directly proportional to the current flowing through the material, at constant temperature, i.e.,

$$V \propto I$$

i.e., $V = RI$

where R is the ability of material to resist the flow of current and its unit is Ohm.

When the temperature changes, the resistivity (ρ) and the physical dimension (length l or area A) of the resistance material also changes. As $R = \rho \dfrac{l}{A}$, any change in the value of resistivity or physical dimension would affect the resistance value R. Therefore, Ohm's law is only valid at constant temperature.

For an example, assume that a resistor R is connected between the nodes 1 and 2, with voltages, V_1 and V_2 as shown in Figure 1.9.

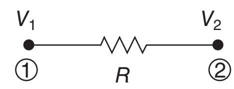


Figure 1.9 An illustration for Ohm's Law

If $V_1 > V_2$, then the potential difference between these nodes 1 and 2 is given by

$$V_1 - V_2$$

If the current I flows because of this potential difference, then according to Ohm's law

$$V_1 - V_2 = IR$$

or simply, $V = IR$, where $V = V_1 - V_2$.

The *V-I* relationship for a linear resistor is shown in Figure 1.10.

From this relationship, the unknown voltage across a resistor can be determined by knowing the current value and resistance value. Similarly, if the values of R and V are provided, the value of I can be calculated using the relationship as

$$I = \frac{V}{R}$$

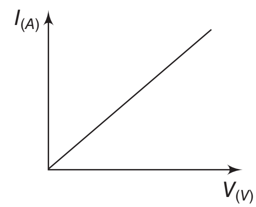


Figure 1.10 *V-I* relationship for a linear resistor

If the values of I and V are known, R can be calculated by using the relationship as

$$R = \frac{V}{I}$$

Limitations of Ohm's Law

The following are the limitations of Ohm's law:

(i) Ohm's law is applicable only for the metallic conductors maintained at a constant temperature. The law is not applicable if the temperature varies.

(ii) Ohm's law is not applicable to all non-metallic conductors.

(iii) It is also not applicable to the nonlinear devices such as diodes, transistors and other semiconductor devices.

1.4 Kirchhoff's Laws

The Kirchhoff's current and voltage laws are used in systematic analysis of the relationships between the voltages and currents in any given electric circuit.

1.4.1 Kirchhoff's Current Law (KCL)

Kirchhoff's Current Law states that algebraic sum of currents at any node is zero, i.e., the total current entering a node is equal to the total current leaving that node.

In the above statement,

(i) The term 'node' represents a junction in a circuit, which is a common point between two or more branches. For example, Figure 1.11 shows a node-x with six branches.

(ii) The term 'algebraic sum' stresses that when applying Kirchhoff's Current Law in a node, one has to consider both the magnitude of the current and its direction. In Figure 1.11, the notations I_1, I_2, I_3, I_4, I_5 and I_6 represent magnitude of branch currents and the arrow mark indicates their directions of flow. The individual branch current can flow towards or away from the node. From Figure 1.11, it can be observed that three currents I_1, I_2 and I_6 flow towards the node-x, while the other three currents I_3, I_4 and I_5 flow away from 'x'.

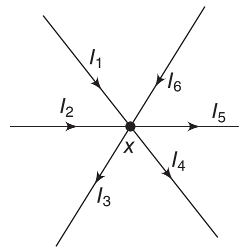


Figure 1.11 A node-x with six branches

The first step in applying Kirchhoff's Current Law is to assign arbitrary sign conventions according to the direction of current. Traditionally, positive (+) sign is assigned to the currents flowing towards the node as they are treated as positive contributions to the algebraic sum, whereas the currents flowing away from the node are assigned with negative (–) sign as they are treated as negative contributions.

From Figure 1.11, it is observed that there are three positive currents I_1, I_2 and I_3 three negative currents I_3, I_4 and I_5. Therefore, according to Kirchhoff's Current Law, the algebraic sum of currents should be equal to zero.

$$\Sigma i = 0$$

or $$I_1 + I_2 + I_6 - I_3 - I_4 - I_5 = 0$$

Hence, $$I_1 + I_2 + I_6 = I_3 + I_4 + I_5 \tag{1.1}$$

If an assumption is made that current entering the node has negative polarity and current leaving the node has positive polarity, then KCL can be written as

$$-I_1 - I_2 - I_6 + I_3 + I_4 + I_5 = 0$$

Hence, $$I_1 + I_2 + I_6 = I_3 + I_4 + I_5 \tag{1.2}$$

Therefore, from KCL Equations (1.1) and (1.2), it can be inferred that the total amount of current entering any node is equal to the amount of current leaving that node.

1.4.2 Kirchhoff's Voltage Law (KVL)

Kirchhoff's Voltage Law (KVL) states that the algebraic sum of the voltages around any closed loop or circuit is zero.

i.e., in a closed path, mesh or loop, $\Sigma V = 0$.

Therefore, the algebraic sum of the voltage drop across the circuit elements of any closed path in a circuit is equal to the sum of the *emf*s or sources of voltages in that path.

i.e., $$\Sigma IR = \Sigma V$$

When applying KVL in a circuit, apart from the magnitudes of voltage drops, their directions or polarities should also be taken into account.

To demonstrate the application of KVL, let us consider a single loop circuit as shown in Figure 1.12. In this circuit, traversal has started from node-a and then nodes 'b', 'c' and 'd' along clockwise direction and ended at the same node 'a'.

If 'I' is the current flowing in the circuit, it creates a voltage drop across each resistor. Then, according to Ohm's law,

$$V_1 = IR_1,\ V_2 = IR_2 \text{ and } V_3 = IR_3$$

Here, the current direction is arbitrarily assumed and the voltage drop it creates across each resistor will also have its sign convention '+' and '–'.

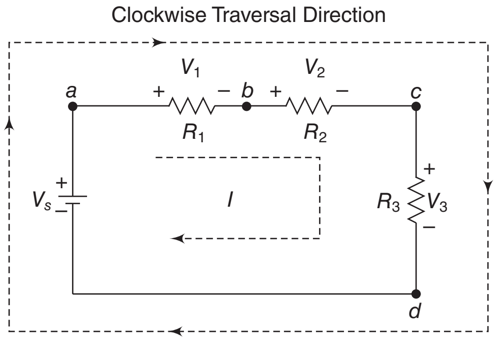


Figure 1.12 An illustration for KVL

Figure 1.12 shows the current traversing the circuit in the clockwise direction is depicted by dashed lines. The elements V_1, V_2 and V_3 are assigned with '+' sign as its positive polarity is met first in the assumed traverse direction. Similarly V_s is assigned with a '–' sign as its negative terminal is met first in the assumed traverse direction. Even though the starting point of loop is entirely arbitrary, it must start and end at the same point. Therefore, by applying KVL along the traverse direction through '$a \to b \to c \to d \to a$', we get

$$V_1 + V_2 + V_3 - V_s = 0$$

and $$\Sigma V = 0$$

or $$IR_1 + IR_2 + IR_3 - V_s = 0$$

$$IR_1 + IR_2 + IR_3 = V_s$$

From the above KVL equation, the current in the circuit is calculated as

$$I(R_1 + R_2 + R_3) = V_s$$

i.e., $$I = \frac{V_s}{(R_1 + R_2 + R_3)}$$

Note: The voltages in the loop may be summed in the anticlockwise direction also. This will cause no difference except to change in all the signs in the resulting equation. Mathematically, this will end up with the same KVL equation but multiplied by –1. The result in both cases is that the algebraic sum of voltages in any loop will be zero, i.e., $\Sigma V = 0$.

Example 1.1

For the circuit shown in Figure E1.1(a), find the values of I and R.

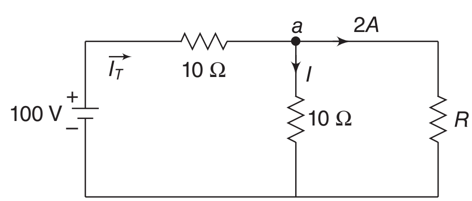


Figure E1.1(a)

Solution

Assuming clockwise traversal current direction for I_T in loop I as shown in Figure E1.1(b), and applying KCL at node-a, we get

$$I_T = I + 2$$

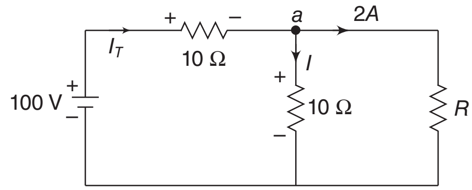


Figure E1.1(b)

Applying KVL for loop 1, we get

$$100 = 10I_T + 10I$$

Substituting the equation of I_T in the above equation, we get

$$100 = 10(I + 2) + 10I$$

i.e., $20I = 80$

Therefore, $I = 4A$

Since 10 Ω and R are in parallel,

$$V_{10\Omega} = V_R$$

i.e., $10 \times 4 = 2 \times R$

Therefore, $R = 20\ \Omega$

Example 1.2

Find the value of V_s in the network shown in Figure E1.2 (a).

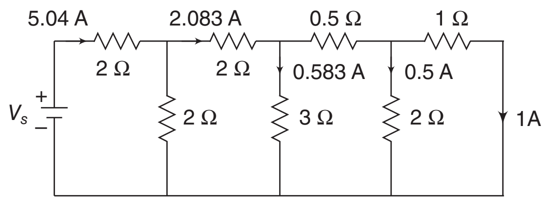


Figure E1.2(a)

Solution

The given circuit can be redrawn as shown in Figure E1.2(b).

Applying KCL at node-b we get

$$I_x = 5.04 - 2.083 = 2.957\ \text{A}$$

Applying KVL for the loop $abga$, we get

$$2 \times 5.04 + 2 \times I_x - V_s = 0$$

Therefore, $V_s = 10.08 + 2 \times 2.957 = 15.99$ V

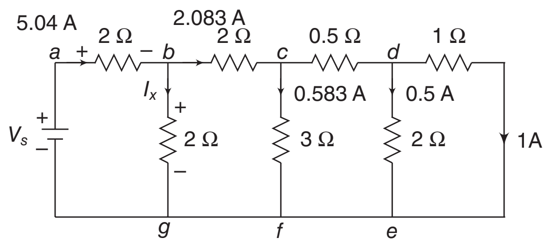


Figure E1.2(b)

Example 1.3

Determine the current I_L in the circuit shown in Figure E1.3. **[AU Nov/Dec, 2010]**

Solution

Assume the branch currents as shown in Figure 1.3.

Applying KVL to the three loops I, II and III, we get

$$-3I_2 - 5I_L - I_1 + 8 = 0$$

i.e., $I_1 + 3I_2 + 5I_L = 8$ (1)

$-3(I_2 - I_L) - (I_1 - I_L) - 6 + 5I_L = 0$

i.e., $-I_1 - 3I_2 + 9I_L = 6$ (2)

$+4 - 3(I_1 - I_2) + 3(I_2 - I_L) + 3I_2 = 0$

i.e., $-3I_1 + 9I_2 + 3I_L = -4$ (3)

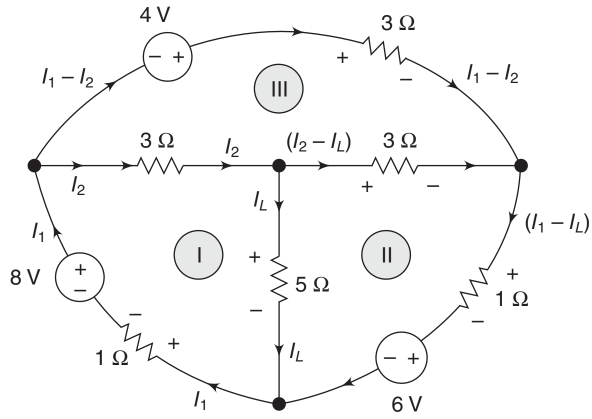


Figure E1.3

Writing the KVL Equations (1), (2) and (3) in matrix form, we have

$$\begin{bmatrix} 1 & 3 & 5 \\ -1 & +3 & 9 \\ -3 & 9 & -3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_L \end{bmatrix} = \begin{bmatrix} 8 \\ 6 \\ -4 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 1 & 3 & 5 \\ -1 & -3 & 9 \\ -3 & 9 & -3 \end{vmatrix} = 1[9 - 81] - 3[3 + 27] + 5[-9 - 9] = -252$$

$$\Delta_1 = \begin{vmatrix} 8 & 3 & 5 \\ 6 & -3 & 9 \\ -4 & 9 & -3 \end{vmatrix} = 8[9 - 81] - 3[-18 + 36] + 5[54 - 12] = -420$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{-420}{-252} = 1.667 \text{ A}$$

$$\Delta_2 = \begin{vmatrix} 1 & 8 & 5 \\ -1 & 6 & 9 \\ -3 & -4 & -3 \end{vmatrix} = 1[-18 + 36] - 8[3 + 27] + 5[4 + 18] = -112$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-112}{-252} = 0.444 \text{ A}$$

$$\Delta_L = \begin{vmatrix} 1 & 3 & 8 \\ -1 & -3 & 6 \\ -3 & 9 & -4 \end{vmatrix} = 1[12 - 54] - 3[4 + 18] + 8[-9 - 9] = -252$$

$$I_L = \frac{\Delta_3}{\Delta} = \frac{-252}{-252} = 1 \text{ A}$$

Therefore, $I_L = 1$ A

Example 1.4

Obtain the potential difference V_{AB} in the circuit shown in Figure E1.4 using Kirchhoff's laws.

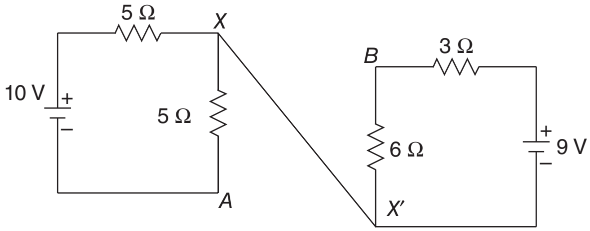


Figure E1.4(a)

Solution

As the nodes $X - X'$ shown in Figure E1.4(a) do not have any element connected in between, they can be simply shorted to a single node-X and the circuit can be redrawn as shown in Figure E1.4(b).

Figure E1.4(b) consists of two loops 1 and 2 with the currents I_1 and I_2 respectively. Assuming clockwise current direction and applying KVL for loop 1, we get

$$(5 + 5)I_1 = 10$$

or $$I_1 = 1\text{A}$$

Similarly, for loop 2,

$$(6 + 3)I_2 = 9$$

or $$I_2 = 1\text{A}$$

Voltage drop across $V_{BA} = V_{BX} + V_{XA}$

According to Ohm's law,

$$V_{BX} = V_{6\Omega} = 6 \times I_2 = 6 \times 1 = 6 \text{ V}$$

i.e., $$V_{XA} = V_{5\Omega} = 5 \times I_1 = 5 \times 1 = 5 \text{ V}$$

Therefore, $V_{BA} = 6 + 5 = 11$ V

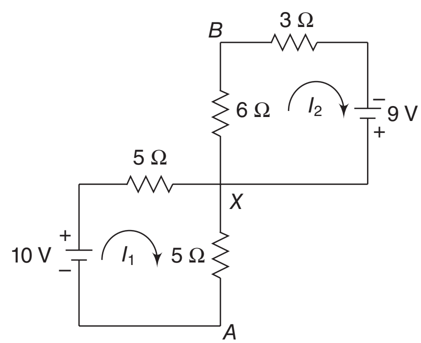


Figure E1.4(b)

1.5 Resistors Connected in Series

In a series circuit, all the components are connected such that there is only one closed path through which the current flows. The same current flows through all the components connected in series. If a series circuit consists of only pure resistors, then it is called a series resistor circuit. The voltage across each resistor can be determined by Ohm's law.

For example, consider the series circuit shown in Figure 1.13. It consists of the resistances R_1, R_2 and R_3 connected in series with a voltage source V_s across them. Due to this voltage source, a current I flows through the circuit.

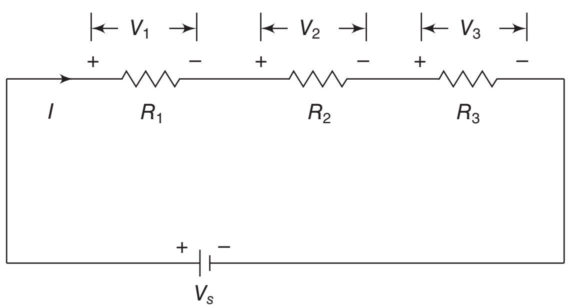


Figure 1.13 Resistors in series

The total or equivalent resistance of the circuit is given by

$$R_{\text{eq}} = R_1 + R_2 + R_3$$

Applying KVL for the circuit, we get

$$V_s = V_1 + V_2 + V_3 = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3) = IR_{\text{eq}}$$

Therefore, $I = \dfrac{V_s}{R_{\text{eq}}}$

Hence, the current in the series circuit is determined by the voltage and its equivalent resistance.

The equivalent resistance in a series circuit is equal to the sum of all resistances connected in series. If "n" number of varying resistors are connected in series, then

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots + R_n$$

When all resistors have equal values, $R_{\text{eq}} = nR$.

Application of Series Resistor Circuit

Resistors are connected in series in order to increase the total resistance of the circuit and hence to limit the current flow. For a fixed source voltage, V_s if the flow of current is limited by adding resistors in series, then it will result in the voltage division across the resistors.

Effects of Resistors Connected in Series

The effects of resistors connected in series are:

(i) The current remains the same through all the elements connected in series.
(ii) Voltage division occurs across the elements connected in series.
(iii) The voltage across different elements connected in series depends on its resistance value and circuit current.
(iv) If 'n' resistors are connected in series, then $R_{\text{eq}} = R_1 + R_2 + R_3 + \cdots + R_n$
(v) If 'n' resistors of equal values are connected in series, then $R_{\text{eq}} = nR$
(vi) Resistances, voltages and powers are additive in a series circuit.
(vii) The voltage applied is equal to the sum of voltage drops across different elements.

Limitations of Series Circuit

The limitations of series circuit are:

(i) If there is a discontinuity or break in any part of the circuit, no current will flow through any element of the circuit.
(ii) As the voltage gets divided by the addition of elements, the series circuit is not practical for supplying power in the household and industrial applications.
(iii) It is also not practical to connect the devices having different current ratings in series.

1.6 Resistors Connected in Parallel

A circuit is called parallel circuit if all the components in that circuit are connected in a manner that there are more than one path or branch through which the circuit current can flow. As the current has to flow in more than one branch, the current gets divided in a parallel circuit. But, the net voltage remains the same across all the branches connected in parallel.

Consider a simple circuit consisting of two resistors connected in parallel as shown in Figure 1.14(a). If a voltage V_s is applied to node-a of the circuit, then current I is divided into I_1 and I_2.

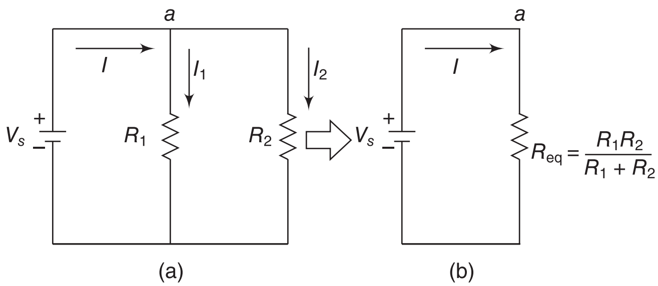


Figure 1.14 (a) Two resistors in parallel and (b) its equivalent

Applying *KCL* to the node-a, we get

$$I = I_1 + I_2$$

According to Ohm's law,

$$I_1R_1 = I_2R_2 = V_s$$

i.e., $$I_1 = \frac{V_s}{R_1} \text{ and } I_2 = \frac{V_s}{R_2}$$

Therefore, $$I = \frac{V_s}{R_1} + \frac{V_s}{R_2}$$

$$V_s = \frac{I}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = I\left(\frac{R_1R_2}{R_1 + R_2}\right) = IR_{eq}$$

Therefore, as shown in Figure 1.14(b), two parallel resistors R_1 and R_2 can be replaced with their equivalent single resistance $R_{eq} = \frac{R_1R_2}{R_1 + R_2}$.

Effects of Resistors Connected in Parallel

The effects of resistors connected in parallel are:

(i) The voltage remains the same across all the elements connected in parallel.

(ii) The current gets divided in a parallel circuit.

(iii) The current flows through each element connected in parallel depends on its resistance value and the voltage across it.

(iv) The equivalent resistance of a parallel circuit can be calculated by,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

(v) If 'n' resistors connected in parallel have equal resistance values, then the equivalent resistance of the circuit is $R_{eq} = \frac{R}{n}$.

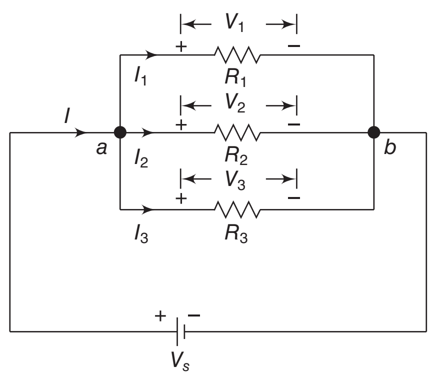


Figure 1.15 Three resistors in parallel

(vi) Branch currents, conductances and powers are additive in a parallel circuit.

Advantages of Parallel Circuits

The advantages of parallel circuits are as follows:

(i) If there is a discontinuity or break in any one branch, the current will still flow in the other branches of the parallel circuit.

(ii) The electrical appliances having same voltage rating but different power or current rating can be connected in parallel.

(iii) Hence, the parallel circuits are used in all household and industrial wirings for connecting the electrical appliances and equipments.

Example 1.5

Determine the current delivered by the source in the circuit shown in Figure E1.5(a).

[AU Apr/May, 2011]

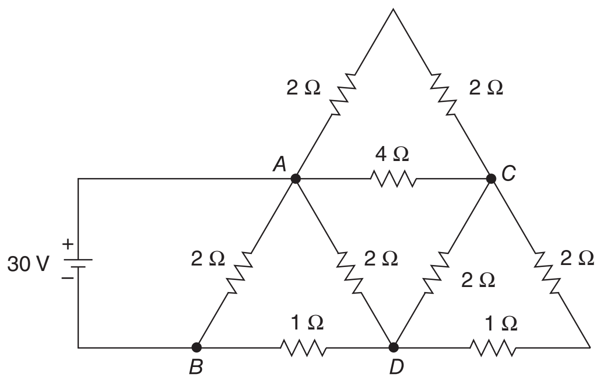


Figure E1.5(a)

Solution

Redrawing the given circuit shown in Figure E1.5(a), we have Figure E1.5(b).

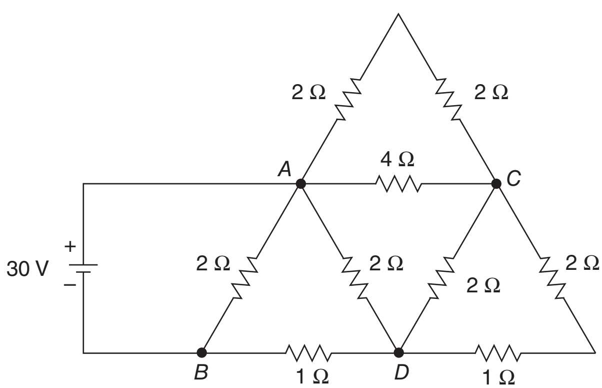


Figure E1.5(b)

Redrawing the circuit shown in Figure E1.5 (b), we have Figure E1.5 (c) and (d).

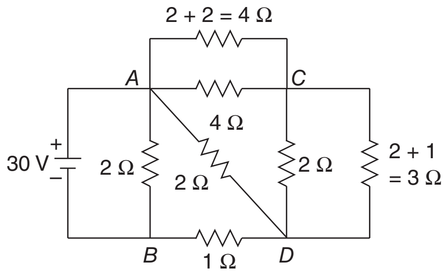


Figure E1.5(c)

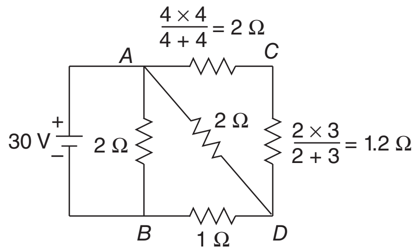


Figure E1.5(d)

The equivalent resistance R_{eq} across AB is given by

$$R_{eq} = [((2 + 1.2) \,||\, 2) + 1] \,||\, 2 = \frac{58}{55} = 1.0545$$

Therefore, $$I = \frac{30}{R_{eq}} = \frac{30}{1.0545} = 28.448 \text{ A}$$

1.7 Capacitors Connected in Series

Consider three capacitors of capacitance C_1, C_2 and C_3 connected in series as shown in Figure 1.16. Let V be the voltage applied across the series combination. Each capacitor carries the same amount of charge q. Let V_1, V_2 and V_3 be the voltage across the capacitors C_1, C_2 and C_3 respectively.

Therefore, $V = V_1 + V_2 + V_3$

The potential difference across each capacitor is given by

$$V_1 = \frac{q}{C_1}; V_2 = \frac{q}{C_2}; V_3 = \frac{q}{C_3}$$

Hence, the resultant voltage, $V = \dfrac{q}{C_1} + \dfrac{q}{C_2} + \dfrac{q}{C_3} = q\left[\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}\right]$

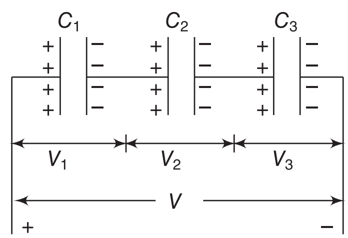


Figure 1.16 Capacitors in series

If C_s is the effective capacitance of the series combination, then charge q is present when a voltage V is applied across it.

i.e., $$V = \frac{q}{C_s}$$

Also, $$\frac{q}{C_s} = \frac{q}{C_1} + \frac{q}{C_2} + \frac{q}{C_3}$$

Therefore, $$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

If 'n' fixed capacitors are connected in series, then

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$$

1.8 Capacitors Connected in Parallel

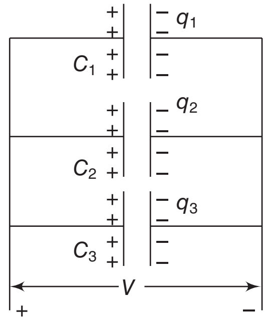


Figure 1.17 Capacitors in parallel

Consider three capacitors of capacitances C_1, C_2 and C_3 connected in parallel as shown in Figure 1.17.

Let this parallel combination be connected to a voltage, V. The voltage across each capacitor is the same. The charges on the three capacitors are given by

$$q_1 = C_1V,\ q_2 = C_2V,\ q_3 = C_3V$$

The total charge in the system of capacitors is

$$q = q_1 + q_2 + q_3$$

$$q = C_1V + C_2V + C_3V$$

But, we know that, $q = C_pV$, where C_p is the effective capacitance of the system

Hence, $C_pV = V(C_1 + C_2 + C_3)$

i.e., $C_p = C_1 + C_2 + C_3$

Therefore, the effective capacitance of the capacitors connected in parallel is the sum of the capacitances of the individual capacitors.

If 'n' number of fixed capacitors are connected in series, then $C_{\text{eq}} = C_1 + C_2 \cdots + C_n$

1.9 Inductors Connected in Series

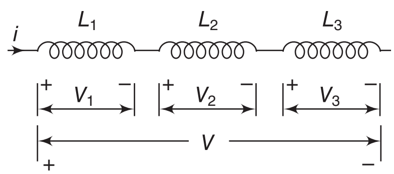


Figure 1.18 Inductors connected in series

Consider three inductors L_1, L_2 and L_3 are connected in series as shown in Figure 1.18. Let V be the voltage applied across the series combination. Each inductor carries the same amount of current i. Let V_1, V_2 and V_3 be the voltage across the inductors L_1, L_2 and L_3 respectively.

Therefore, $V = V_1 + V_2 + V_3$

The potential difference across each inductor is given by

$$V_1 = L_1\frac{di}{dt};\ V_2 = L_2\frac{di}{dt};\ V_3 = L_3\frac{di}{dt}$$

Hence, the resultant voltage, $V = L_1\dfrac{di}{dt} + L_2\dfrac{di}{dt} + L_3\dfrac{di}{dt} = \dfrac{di}{dt}(L_1 + L_2 + L_3)$

If L_s is the effective inductance of the series combination, then current di/dt is present when a voltage V is applied across it.

i.e., $V = L_s\dfrac{di}{dt}$

Also, $L_s\dfrac{di}{dt} = \dfrac{di}{dt}(L_1 + L_2 + L_3)$

Therefore, $L_s = L_1 + L_2 + L_3$

Therefore, the effective inductance of the inductors connected in series is the sum of the inductances of the individual inductors.

If 'n' number of fixed inductors are connected in series, then $L_{\text{eq}} = L_1 + L_2 + \ldots\ldots + L_n$.

1.10 Inductors Connected in Parallel

Consider three inductors L_1, L_2 and L_3 are connected in parallel as shown in Figure 1.19.

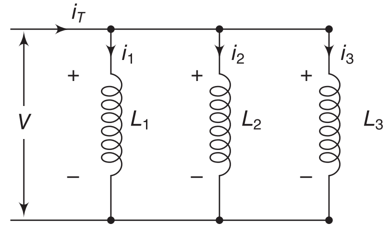


Figure 1.19 Inductors connected in parallel

Let this parallel combination be connected to a voltage, V. The voltage across each inductor is the same. The current on the three inductors are given by

$$\frac{di_1}{dt} = \frac{V}{L_1};\ \frac{di_2}{dt} = \frac{V}{L_2};\ \frac{di_3}{dt} = \frac{V}{L_3}$$

The total current in the system of inductors is

$$\frac{di_T}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt} + \frac{di_3}{dt} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

But, $\dfrac{di_T}{dt} = \dfrac{V}{L_p}$, where L_p is the effective inductance of the system

Hence,
$$\frac{V}{L_p} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3}$$

i.e.,
$$\frac{V}{L_p} = \frac{V}{L_1} + \frac{V}{L_2} + \frac{V}{L_3} = V\left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)$$

$$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

If 'n' number of fixed inductors are connected in parallel, then

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$$

1.11 Waveforms and RMS Value

AC Quantity

An AC quantity has a varying magnitude and angle with respect to time when compared with DC quantity which has a constant magnitude all the time.

Waveform

A waveform is a graphical representation of instantaneous value of any quantity plotted against time as represented in Figure 1.20.

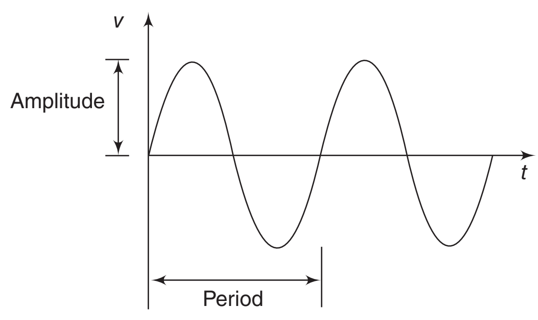


Figure 1.20 An AC waveform

Alternating Current

The current wave which reverses its direction at regularly recurring intervals is called alternating current.

Cycle

One complete cycle comprises of a set of positive and negative halves.

Amplitude

The maximum positive or negative value of an alternating quantity is called the amplitude or magnitude.

Frequency

The number of cycles per second of an alternating quantity is called frequency (f) and its unit is *cycles/second* or *Hertz* (Hz).

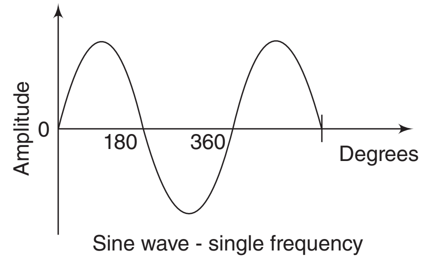


Figure 1.21 Phase measurement in a single waveform

Period (T)

Time period of an alternating quantity is the time required to complete one cycle. Time period is equal to the reciprocal of frequency and its unit is sec(s).

Phase

The phase represents a particular point in the cycle of a waveform, measured as an angle in *degrees*, as shown in Figure 1.21.

Phase Difference

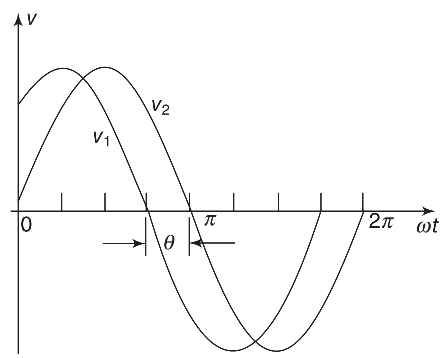


Figure 1.22 Phase difference of two waveforms

The term phase difference is used in comparison of the phase of two waveforms or alternating quantities.

The phase difference between two sinusoidal waveforms which have the same frequency is represented in Figure 1.22. The phase angle can be considered as a measure of the time delay between two periodic signals expressed as a fraction of the wave period. This fraction is normally expressed in units of angle, with a full cycle corresponding to 360°.

For example, inspection of waveforms shown in Figure 1.22 reveals that, since the voltage v_1 passes through zero cycle before v_2, the v_1 leads the v_2 by a phase difference of $\theta = \frac{\pi}{4} = \frac{360°}{8} = 45°$.

If, the alternating voltage v_2 is taken as a reference waveform, it can be represented mathematically as

$$v_2 = V_m \sin \omega t,$$

where, V_m is the magnitude of the waveform, and $\omega = 2\pi f$

As v_1 leads v_2 by θ, it can be expressed as

$$v_1 = V_m \sin(\omega t + \theta)$$

Effective or RMS Value

The Root Mean Square (RMS) value of an alternating current is the steady value of direct current (DC) which when flowing through a given circuit for a given time, produces the same amount of heat as would be produced by the alternating current flowing in the circuit for the same time.

RMS value of a wave can be obtained by the formula,

$$\text{RMS Value} = \sqrt{\frac{\text{Area under the squared curve for one cycle}}{\text{Time period}}}$$

RMS value of the alternating sinusoidal current is given by

$$I_{\text{RMS}} = \sqrt{\frac{I_m^2}{2}} = \frac{I_m}{\sqrt{2}} = 0.707 I_m$$

Similarly, RMS value of an AC voltage is given by

$$V_{\text{RMS}} = \sqrt{\frac{V_m^2}{2}} = \frac{V_m}{\sqrt{2}} = 0.707 V_m$$

Average Value

The average of all instantaneous values of an alternating quantity over one complete cycle is called average value. This can be determined by first obtaining the average value for a small interval of time and then integrating it over the curve. The average value of current is given by

$$I_{av} = \frac{1}{T}\int_0^T i\,dt$$

For an alternating sinusoidal current,

$$I_{av} = \frac{2I_m}{\pi} = 0.637 I_m, \text{ where } I_m \text{ is the maximum value of the current.}$$

For an alternating sinusoidal voltage,

$$V_{av} = \frac{2V_m}{\pi} = 0.637 V_m, \text{ where } V_m \text{ is the maximum value of the voltage.}$$

Form Factor and Peak Factor

Form factor (K_f) is defined as the ratio of RMS value to the average value.

$$\text{Form factor} = \frac{\text{RMS value}}{\text{Average value}}$$

Peak factor or cost factor (K_p) is defined as the ratio of peak value to the RMS value.

$$\text{Peak factor} = \frac{\text{Peak value}}{\text{RMS value}}$$

For a sinusoidal wave,

$$\text{Form factor } (K_f) = \frac{0.707 I_m}{0.637 I_m} = 1.11$$

$$\text{Peak factor } (K_f) = \frac{I_m}{\frac{I_m}{\sqrt{2}}} = \sqrt{2} = 1.414$$

Impedance

In a network, impedance is the measure of opposition to the flow of current or applied voltage. It is the extension of the concept of resistance to AC circuits. But, unlike resistance, which has only magnitude, the impedance possesses both magnitude and phase. When a DC current is supplied, the impedance cannot be distinguished from the resistance. Therefore, with a DC current, the resistance can be treated as impedance with zero phase angle.

1.11.1 Impedance of a Pure Resistive Circuit

Consider the circuit shown in Figure 1.23 (a) consisting of a resistor R connected across an alternating voltage source. Let $v = V_m \sin \omega t$ be the sinusoidal voltage applied across the resistance, resulting in a current i.

According to Ohm's law, $v = iR$ and

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t,$$

where $I_m = \dfrac{V_m}{R}$ represents the peak value of the circuit current.

From this relationship it can be concluded that the current in the resistor is in phase with the voltage as shown in Figure 1.23(b).

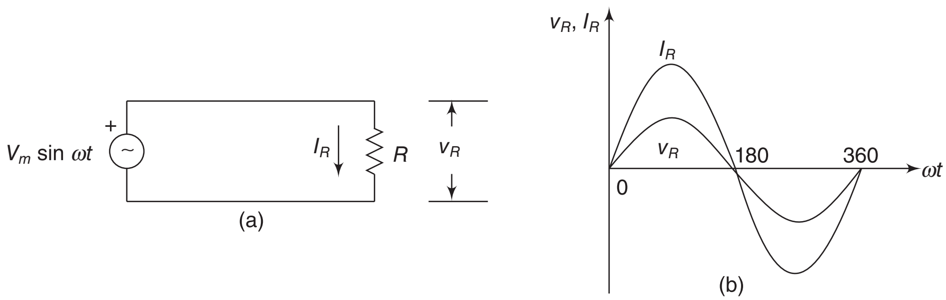


Figure 1.23 (a) A pure resistor circuit (b) v and i waveforms

As the impedance is the ratio of the voltage to current in an AC circuit, we get

$$Z = \frac{v}{i} = \frac{V_m \sin \omega t}{I_m \sin \omega t} = \frac{V_m \sin \omega t}{(V_m/R) \sin \omega t} = R$$

1.11.2 Impedance of a Pure Inductive Circuit

Consider the circuit shown in Figure 1.24(a) consisting of a pure inductor of self-inductance L is connected across an alternating voltage source.

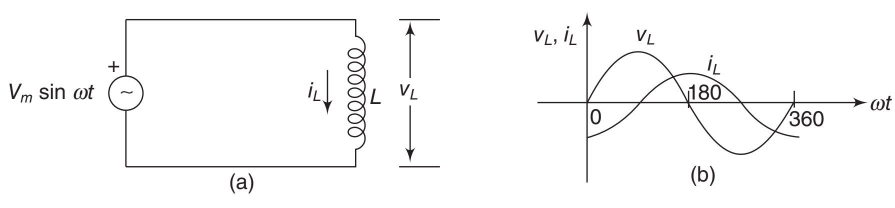


Figure 1.24 (a) A pure inductor circuit (b) v_L and i_L waveforms

Let $v = V_m \sin \omega t$ be the sinusoidal voltage applied across the inductor, resulting in a current i_L. The *emf* induced in the inductor is given by

$$v_L = L\frac{di_L}{dt}$$

Hence, $$i_L = \frac{1}{L}\int v dt = \frac{1}{L}\int V_m \sin \omega t dt = \frac{V_m}{L}\left(\frac{-\cos \omega t}{\omega}\right)$$

$$= -\frac{V_m}{\omega L}\cos \omega t = I_m \sin(\omega t - \pi/2), \text{ where } I_m = \frac{V_m}{\omega L}.$$

From this relationship, it can be concluded that the current in the inductor lags the voltage by an angle $\frac{\pi}{2}$ as shown in Figure 1.24(b).

The impedance, $$Z = \frac{V_m}{I_m} = \frac{V_m}{(V_m/\omega L)} = \omega L$$

The term $Z = \omega L$ denotes the inductive reactance (X_L).

1.11.3 Impedance of a Pure Capacitive Circuit

Consider the circuit shown in Figure 1.25(a) consisting of a pure capacitor of value *C Farad* is connected across an alternating voltage source.

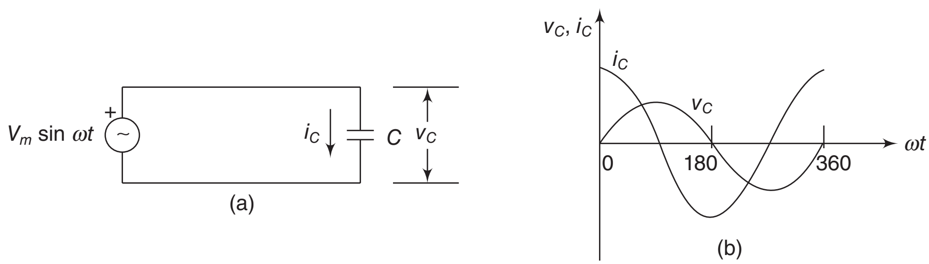


Figure 1.25 (a) A pure capacitor circuit (b) v_c and i_c waveforms

Let $v = V_m \sin \omega t$ be the sinusoidal voltage applied across the capacitor, resulting in a current i_c. The voltage across the capacitor is given by

$$v_C = \frac{1}{C}\int i_C dt$$

Hence, $$i_c = C\frac{dv}{dt} = C\frac{d}{dt}(V_m \sin \omega t)$$

$$= \omega C V_m \cos \omega t = I_m \cos \omega t, \text{ where, } I_m = \omega C V_m$$

$$i_c = I_m \sin(\omega t + \pi/2)$$

From this relationship, it can be concluded that the current in the capacitor leads the voltage by an angle $\frac{\pi}{2}$ as shown in Figure 1.25(b).

The impedance, $Z = \dfrac{V_m}{I_m} = \dfrac{V_m}{(\omega C V_m)} = \dfrac{1}{\omega C} = X_C$

The term $Z = \dfrac{1}{\omega C}$ denotes the capacitive reactance (X_C).

Complex Impedance

In an AC circuit, the voltage and current can be represented in the complex Euler form as

$$v = V_m \cos \omega t,\ v = V_m e^{j\omega t}$$

and $$i = I_m \cos(\omega t - \phi),\ i = I_m e^{j[\omega t - \phi]}$$

where the exponential term represents the Euler relationship for complex exponentials,

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

Hence, the impedance can be expressed in the complex form as

$$Z = \frac{V_m}{I_m} e^{-j\phi}$$

The general expression for impedance in rectangular form is given by $R + jX$, where R represents the physical or internal resistance and X represents the reactance of a capacitor or an inductor. The impedances of the individual components determined by the phase relationship between the current and voltage are given by

$$R \text{ for resistor, } j\omega L \text{ for inductor and } \frac{1}{j\omega C} \text{ or } \frac{-j}{\omega C} \text{ for capacitor.}$$

Impedance Combinations

Similar to resistances, impedances can be connected in series or parallel form. But, the phase relationship between the sources and impedances must be considered in the calculations. Hence, the equations derived for serial and parallel combination involve complex impedances. Combining the series impedances is a straight-forward procedure, as shown in Figure 1.26(a).

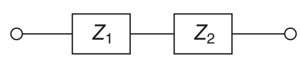


Figure 1.26 (a) Impedances in series

$$Z_{eq} = Z_1 + Z_2$$

The parallel combination of two impedances is shown in Figure 1.26 (b).

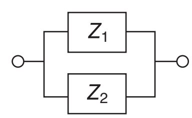


Figure 1.26 (b) Impedances in parallel

$$Z_{eq} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$Z_{eq} = \frac{Z_1 Z_1}{Z_1 + Z_2}$$

1.12 Phasor Diagram

Phasor diagrams are useful in the analysis of steady state AC circuits. The term phasor represents complex numbers, and diagram represents graphical projection of these numbers as vectors in a plane. The terms 'phase difference', 'lead', and 'lag' as well as 'in-phase' and 'out-of-phase' are used to specify the relationship or phase of one waveform with respect to the reference waveform.

This relationship can be expressed in time domain by using the generalised sinusoidal expression, $A(t) = A_m \sin(\omega t \pm \phi)$. But, expressing the relationship in the mathematical form alone makes it difficult to visualise the relationship between the waveforms.

Hence, the alternating sinusoidal currents can also be represented graphically in the phasor domain by means of a phasor diagram. This can be obtained by applying a rotating vector called phasor as shown in Figure 1.27.

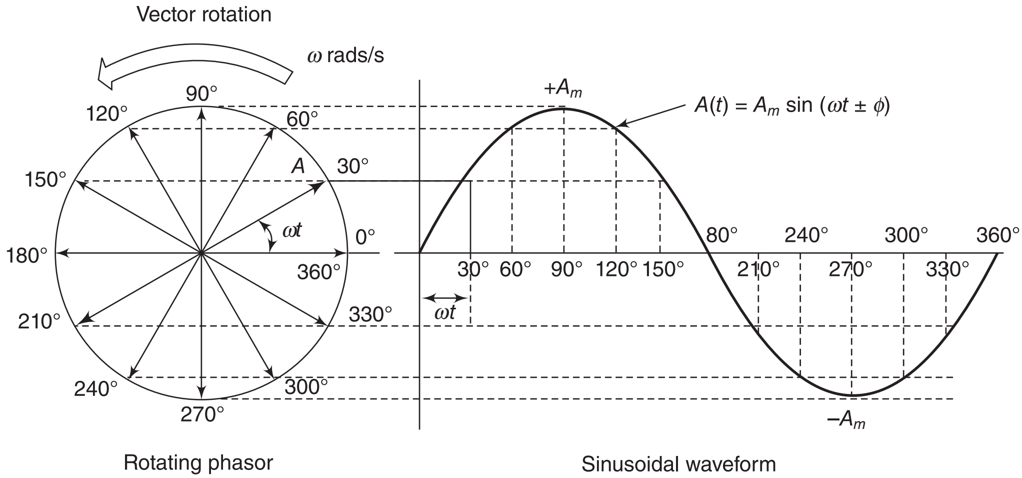


Figure 1.27 Graphical representation of the sinusoidal current

The process of carrying out certain complex operations can be simplified with these representations. For example, consider the AC circuit illustrated in Figure 1.28. This circuit consists of series elements R, L and C, and the same current I flows through all of these elements.

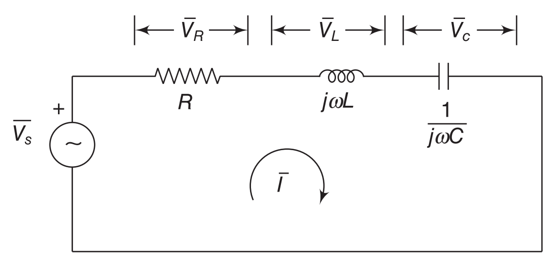


Figure 1.28 AC Circuit

Phasor Current ($\bar{I}$)

Since current $\bar{I}$ is common to all the elements in the circuit, it can be considered as a reference phasor, denoted as

$$i = I_m \sin \omega t$$

Hence, $\quad \bar{I} = I_m\angle 0^\circ \text{ or } |\bar{I}|\angle 0^\circ$

In this phasor representation, $\bar{I}$ is considered as reference, the phase angle of the current phasor is arbitrarily assigned to zero.

Phasor Voltage ($\bar{V}$)

The voltage phasors of this RLC circuit are given by $\bar{V}_R, \bar{V}_L$ and $\bar{V}_C$, and can be expressed in terms of common current I as

$$v_R = iR = (I_m \sin \omega t)R = RI_m \sin \omega t$$

Hence, $\quad \bar{V}_R = R\,|\bar{I}|\angle 0^\circ$

$$v_L = \frac{L di_L}{dt} = L(\omega I_m \cos \omega t) = \omega L I_m \sin(\omega t - 90^\circ)$$

Hence, $$\overline{V}_L = j\omega L \times I_m = \omega L|\overline{I}|\angle 90°$$

$$v_c = \frac{1}{C}\int idt = \frac{1}{C}\int I_m \sin \omega t dt = \frac{1}{\omega C} I_m(-\cos \omega t) = \frac{1}{\omega C} I_m \sin(\omega t - 90°)$$

Hence, $$\overline{V}_C = \frac{-j}{\omega C} \times I_m = \frac{1}{\omega C}|\overline{I}|\angle{-90°}$$

It is to be noted that in phasor representation imaginary terms '*j*' and '–*j*' are represented as 90° and –90° phase shifts from zero respectively.

The phasor representation and reason for difference in phase angles is further explained by means of sinusoidal responses represented in Figure 1.29(a).

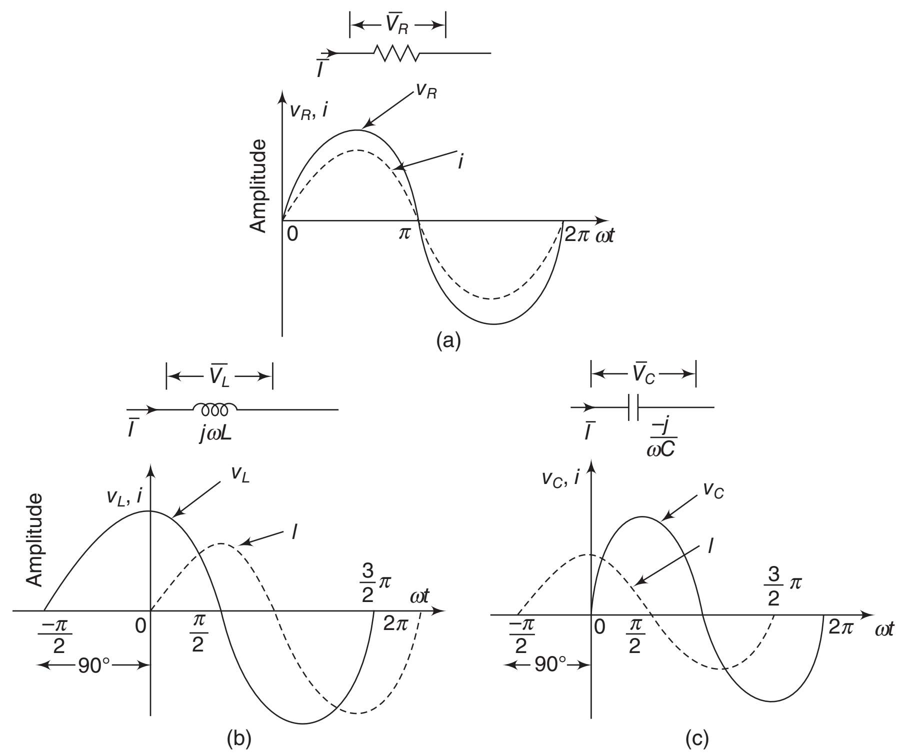


Figure 1.29 (a) $\overline{V}_R$ in Phase with $\overline{I}$, (b) $\overline{V}_L$ leads $\overline{I}$ by 90° and (c) $\overline{V}_C$ lags $\overline{I}$ by 90°

As shown in Figure 1.29(a), for a pure resistive element, the voltage across the resistor $\overline{V}_R$ is in phase with the current $\overline{I}$ flowing through it. Their magnitudes are related by Ohm's law.

The phase relationship for a pure inductor is illustrated in Figure 1.29(b). For an inductor, current $\overline{I}$ lags the voltage $\overline{V}_L$ by 90° and for a pure capacitor as shown in Figure 1.29(c), current $\overline{I}$ leads the voltage $\overline{V}_C$ by 90°. The phasor diagram for a pure resistor, inductor and capacitor is represented in Figs. 1.29(a) to (c).

Applying KVL to the circuits shown in Figure 1.28, we get

$$\overline{V}_s = \overline{V}_R + \overline{V}_L + \overline{V}_C$$

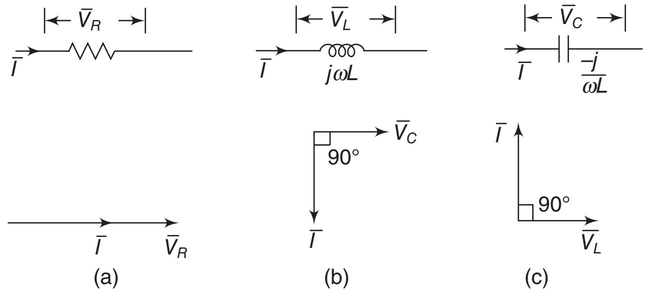


Figure 1.30 (a) $\bar{V}_R$ in phase with $\bar{I}$, (b) $\bar{I}$ Lags $\bar{V}_L$ by 90° and (c) $\bar{I}$ Leads $\bar{V}_C$ by 90°

The series impedance of this circuit is given by

$$Z = R + jX_L - jX_C$$

Case (1) If $X_L > X_C$, the net impedance is inductive and the net voltage $\bar{V}_s$ leads $\bar{I}$, as shown in Figure 1.31(a).

Case (2) If $X_L = X_C$, the net impedance is resistance. As the inductive and capacitive components are exactly same and opposite they cancel each other and the net voltage $\bar{V}_s$ is in phase with $\bar{I}$ as shown in Figure 1.31(b).

Case (3) If $X_L < X_C$, the net impedance is capacitive and the net voltage $\bar{V}_s$ lags $\bar{I}$ as shown in Figure 1.31(c).

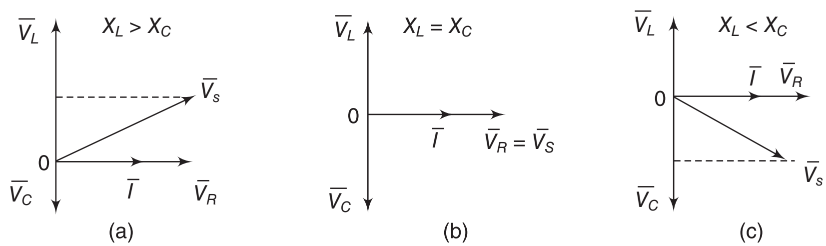


Figure 1.31 (a) when $X_L > X_C$ (b) when $X_L = X_C$ and (c) $X_L < X_C$

1.13 Instantaneous Real, Reactive and Apparent Power

An AC circuit contains AC sources and components. Table 1.1 shows different components used in an AC circuit and their voltage, current and power relationships.

A series RLC circuit with all circuit elements *R*, *L* and *C* connected in series is shown in Figure 1.32(a). As shown in the circuit, the resistor dissipates the power in the form of heat, and pure inductors or capacitors store the supplied energy in the form of magnetic or electric field.

The circuit can be simplified with a single load impedance Z_L as shown in Figure 1.32(b).

where, $Z_L = R + j\omega L - \dfrac{j}{\omega C}$.

In the representation of voltage and current in AC circuits, *v* and *i* are not considered as DC constants, and are represented as complex sinusoidal quantities consisting of both magnitudes and phase angles.

An alternating current is represented as

$$i = I_{\max} \sin \omega t$$

Table 1.1 *Different components used in an AC circuit and their voltage, current and power relationships*

Circuit Element	Impedance	Voltage (V)	Current (A)	Instantaneous Power (W)
Resistor R(Ω)	R	$v = Ri$	$i = \dfrac{v}{R}$	$p = i^2R$
Inductor $L(H)$	$j\omega L$	$v = L\dfrac{di}{dt}$	$i = \dfrac{1}{L}\int v\,dt + i(0^+)$ where $i(0^+)$ is the initial current	$p = Li\dfrac{di}{dt}$
Capacitor $C(F)$	$\dfrac{-j}{\omega C}$	$v = \dfrac{1}{C}\int i\,dt + v(0^+)$ where $v(0^+)$ is the initial voltage	$i = C\dfrac{dv}{dt}$	$p = Cv\dfrac{dv}{dt}$

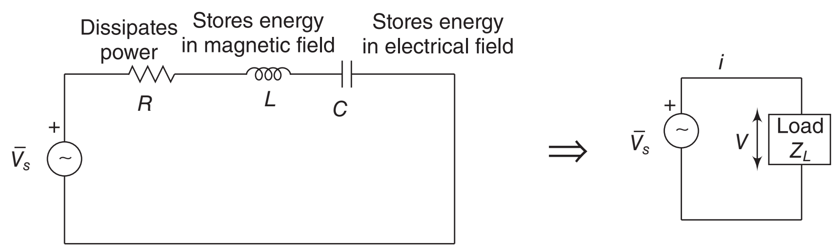


Figure 1.32 (a) A series RLC circuit with AC excitation and (b) its equivalent circuit

Similarly, in the representation of an AC voltage, if ϕ is the phase angle between current and voltage, then $v = V_{max}\sin(\omega t \pm \phi)$. In this case, ϕ is assigned with a (+)*ve* or (–)*ve* sign depending on whether the voltage v is leading or lagging the current i.

If the load is a pure resistor, then $\phi = 0°$, i.e., v is in phase with i

If the load is a pure inductor, then $\phi = 90°$, i.e., v leads i

If the load is a pure capacitor, then $\phi = -90°$, i.e., v lags i

Substituting the equations for v and i in the equation for power, $p = vi$, we get

$$P = V_{max}\sin(\omega t + \phi)I_{max}\sin\omega t$$

Power in an AC circuit is often expressed in three forms, (i) apparent power, (ii) average or active power and (iii) reactive power.

(i) Apparent Power

The apparent power in the AC circuit is defined as the product of applied voltage v and current i. It is called apparent power as it is simply calculated from the multiplication of known voltage and current values indicated by voltmeter and ammeter readings. The type of load connected to the circuit is not taken into consideration in the calculation of apparent power. It is symbolically represented by S, and its unit is volt-amperes (VA).

The apparent power is also called 'complex power' and is expressed as

$$S = VI^*$$

$$= \frac{V_{\max}}{\sqrt{2}} \times \frac{I_{\max}}{\sqrt{2}} = \frac{V_{\max} I_{\max}}{2}$$

(ii) Average or Active or Real Power

The active or average or real power is obtained by the product of apparent power (i.e., V_{rms} and I_{rms}) and the cosine of the angle between voltage and current. The active power is actually the 'real' or 'true' power of the circuit. It is represented by the symbol '*P*' and its unit is watt (W)

$$P = V_{\max} I_{\max} \sin \omega t \sin(\omega t + \phi)$$

Using the trigonometric identities and converting $V_{\max}$ and $I_{\max}$ values to corresponding V_{rms} and I_{rms} values, the above equation can be rewritten as

$$P = V_{\text{rms}} I_{\text{rms}} \cos \phi (1 - \cos 2\omega t) + V_{\text{rms}} \sin \theta (\sin 2\omega t)$$

If $V = V_{\text{rms}}$ and $I = I_{\text{rms}}$, then

$$\underbrace{P = VI \cos \phi}_{(1)} - \underbrace{VI \cos \phi}_{(2)} \cos 2\omega t + \underbrace{VI \sin \phi}_{(3)} \sin 2\omega t$$

where (1) indicates the average power and (2) indicates the peak power, as

Average power, $P_{av} = VI \cos \phi$, and

Peak power, $P_{\text{peak}} = VI \cos \phi$ or $VI \sin \phi$

By careful assessment of the equation for these terms, it can be concluded that, the average power is time independent. Both forms of peak power indicated by (2) have similar formats, and vary with a frequency twice that of applied voltage or current.

Average power can be written with respect to apparent power as

$$P = S \cos \phi$$

$$= V_{\text{rms}} \cdot I_{\text{rms}} \cdot \cos \phi = \frac{V_{\max} I_{\max}}{2} \cos \phi$$

(iii) Reactive Power

The reactive power is obtained by the product of apparent power (i.e., V_{rms} and I_{rms}) and the sine of the angle between voltage and current. It is represented by the symbol *'Q'* and its unit is *volt-ampere reactive (VAR)*.

$$Q = S \sin \phi$$

$$= V_{\text{rms}} I_{\text{rms}} \sin \phi = \frac{V_{\max} I_{\max}}{2} \sin \phi$$

1.14 Power, Power Factor and Energy

Power Triangle

In a vector domain, the equations associated with three types of power: (i) Apparent power (*S*) (ii) Average power (*P*) and (iii) Reactive power (*Q*) can be related to each other by

$$S = P + jQ$$

Case (i) Resistive load on active power

If the load is pure resistive, then $\phi = 90°$, and $P = |P|\angle\phi° = |P|\angle 0°$

Case (ii) Reactive load on reactive power:

If the load is pure inductive, then

$$\phi = 90°, \text{ and } Q_L = |Q_L|\angle\phi° = |Q_L|\angle 90°$$

If the load is pure capacitive, then

$$\phi = -90°, \text{ and } Q_c = |Q_c|\angle\phi° = |Q_c|\angle 90°$$

The phasor power S for inductive load is given by

$$S = P + jQ_L$$

This relationship can be graphically represented in vector domain as shown in Figure 1.33(a).

Similarly, for the capacitive load X_c, the apparent power S can be written as

$$S = P - jQ_c, \text{ (since } Q_c\ \angle{-90°} = -jQ_c)$$

This relationship can be graphically represented in vector domain as shown in Figure 1.33(b).

If the circuit has both inductive load reactive power (Q_L) and capacitive load reactive power (Q_c), then the difference between the reactive powers $Q_L - Q_c$ will be used in obtaining the reactive component (Q) of the power triangle.

For example, a series RLC circuit consists of both reactive terms X_L and X_c with corresponding reactive components Q_L and Q_c. If $X_L > X_c$, then the resultant Q will be in the direction of Q_L as shown in Figure 1.33(c).

In a more generalised form, the power triangle can be represented as shown in Figure 1.33(d).

Here average power, $P = S \cos\phi$ and reactive power, $Q = S \sin\phi$.

It is noted that the direction of Q is subject to change depending on the type of load used and the domination of either inductance or capacitance in the load.

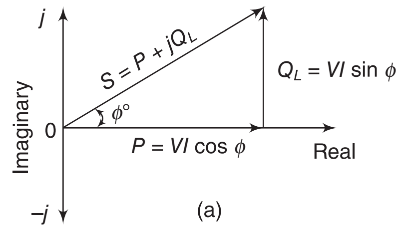


Figure 1.33(a) Graphical representation of phasor power S for inductive load

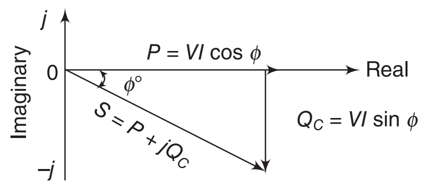


Figure 1.33(b) Graphical representation of phasor power S for capacitive load

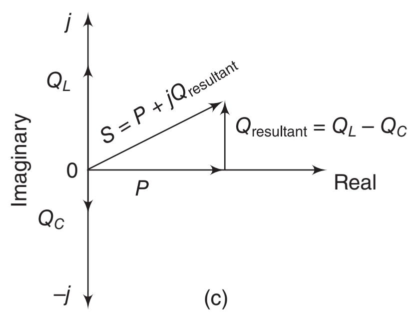


Figure 1.33(c) Graphical representation of phasor power S for inductive and capacitive loads

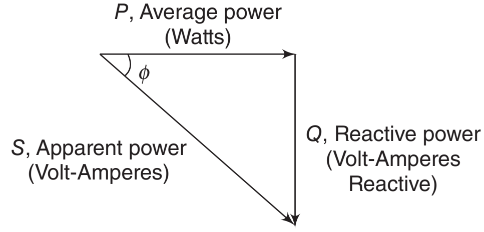


Figure 1.33(d) Graphical representation of power triangle

Power Factor

Power factor is defined as the cosine of the phase angle difference between the voltage and current. It is denoted as *p.f.* If ϕ is the phase angle between voltage and current, then,

$$p.f. = \cos\phi$$

where $\phi = |\phi_v - \phi_i|$

ϕ_v is the phase angle of voltage, v

ϕ_i is the phase angle of current, i

In the calculation of the phase difference between V and I, the symbol || indicates that only the difference in magnitude is taken into consideration by neglecting its polarity. This indicates that the calculation of power factor does not depend on whether V leads I or I leads V.

Energy

Energy is referred to as the amount of power consumed by the circuit components in unit time. Power in a circuit is delivered by the combination of supplied current and voltages.

$$\text{Energy} = \text{power} \times \text{time}$$
$$= pt \text{ (Watt hour)}$$

Energy can also be expressed in terms of kilowatt-hour (kWh) as

$$\text{Energy} = \frac{\text{power} \times \text{hour}}{1000}$$
$$= \frac{pt}{1000} \text{kiloWatt-hour}$$

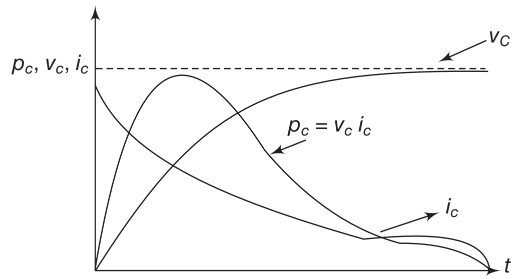


Figure 1.34 Power Response of a Capacitor

Energy Stored by a Capacitor

As stated earlier, power dissipation does not occur in an ideal capacitor. It stores energy its electric field between two conducting plates.

Power p_c in a capacitor at time t can be expressed as

$$p_c = v_c i_c$$

where, v_c = Voltage across the capacitor at time, t

i_c = Current through the capacitor at time, t.

At various instants of time, voltage response, current response and power response can be plotted as shown in Figure 1.34.

In a general form, energy stored in a capacitor can be expressed as

$$W_C = \frac{1}{2}Cv^2$$

From the above relationship, it can be concluded that the energy in a capacitor increases rapidly with the increase in the voltages due to the squared voltage term.

Energy Stored by an Inductor

As stated earlier, power dissipation does not occur in an ideal inductor. The energy is stored in its magnetic field. If the voltage v_L and current i_L characteristics of the inductor are plotted over a period of time t, then as shown in Figure 1.35, the power curve can be derived from the relationship, $p_L = v_L i_L$.

In a general form, energy stored in an inductor can be expressed as

$$W_L = \frac{1}{2}Li^2$$

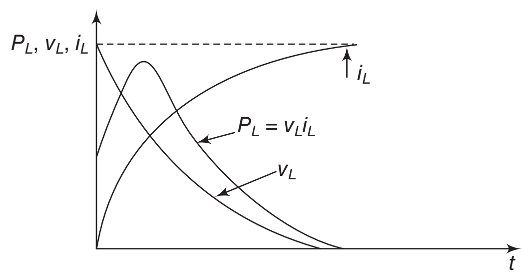


Figure 1.35 Voltage, current and power response of an inductor

From the above relationship, it can be concluded that the energy in an inductor increases rapidly with the increase in current, because of the squared current term.

Example 1.6

For the circuit shown in Figure E1.6 (a), calculate the energy stored by each capacitor

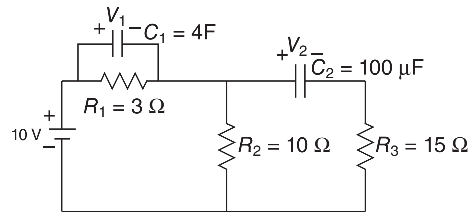


Figure E1.6(a)

Solution

Since the power supply shown in the given circuit is a DC source, the capacitor will act as an open terminal for a DC source. This is represented in Figure E1.6(b).

Current I can be calculated as

$$I = \frac{10}{3+10} = 0.769 \text{ A}$$

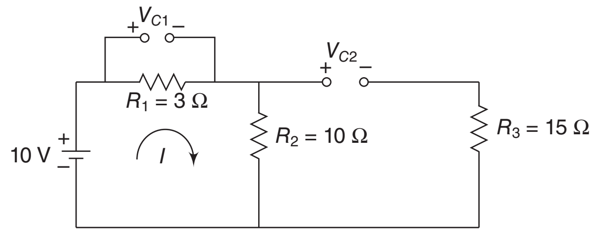


Figure E1.6(b)

Voltages across the capacitor C_1 and C_2 are

$$V_{C1} = IR_1 = 0.769 \times 3 = 2.307 \text{ V}$$

$$V_{C2} = IR_2 = 0.769 \times 10 = 7.69 \text{ V}$$

Energy stored in the capacitor C_1 and C_2 can be calculated as

$$W_{C1} = \frac{1}{2}CV_{C1}^2 = \frac{1}{2} \times 4 \times (2.307)^2 = 10.644 \text{ J}$$

$$W_{C2} = \frac{1}{2}CV_{C2}^2 = \frac{1}{2}(100 \times 10^{-6})(7.69)^2 = 2.958 \text{ mJ}$$

1.15 Voltage Division in DC Circuits

A series circuit is called a voltage divider circuit as only the voltage gets divided between the components connected in series and the current remains the same through them. According to Kirchhoff's Voltage Law, the sum of voltage drops across all the resistors in a series circuit is always equal to the applied voltage. This implies that the applied voltage gets proportionally divided among these resistors. The voltage division occurs across each resistor in a series circuit based on the magnitudes of individual resistances. Larger resistance will get larger portion of the applied voltage and vice versa.

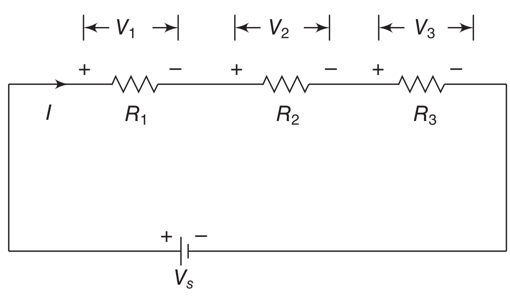


Figure 1.36 Voltage division in a series circuit

Consider an example for a series circuit as shown in Figure 1.36. This circuit consists of a voltage source V_s connected in series with three resistors R_1, R_2 and R_3. It is called a voltage divider circuit as the applied voltage V_s is proportionately divided into V_1, V_2 and V_3 among the resistors R_1, R_2 and R_3 respectively.

According to the Ohm's law, the voltage V_1 across resistor R_1 is

$$V_1 = IR_1$$

Similarly, $V_2 = IR_2$

and $V_3 = IR_3$

Inspection of Figure 1.36 reveals that, the current (I) remains the same through all these resistors and only the voltage gets divided.

Applying KVL to this circuit, the divided voltages can be summed up to the applied voltage V_s as

$$V_s = V_1 + V_2 + V_3$$
$$= IR_1 + IR_2 + IR_3$$

or
$$I = \frac{V_s}{R_1 + R_2 + R_3}$$

Using the above equation, voltages across R_1 is obtained as

$$V_1 = \frac{V_s}{R_1 + R_2 + R_3} R_1$$

Therefore, $V_1 = V_s \dfrac{R_1}{R_T}$, where $R_T = R_1 + R_2 + R_3$

Similarly, the voltages across the resistors R_2 and R_3 are

$$V_2 = IR_2 = \frac{V_s}{R_1 + R_2 + R_3} R_2 = V_s \frac{R_2}{R_T}$$

and
$$V_3 = IR_3 = \frac{V_s}{R_1 + R_2 + R_3} R_3 = V_s \frac{R_3}{R_T}$$

Hence, the generic form of the voltage division can be expressed as

$$V_x = V_s \frac{R_x}{R_T} = R_x \frac{V_s}{R_T}$$

The above equation yields the voltage division rule, which states that the voltage across a resistor in a series circuit is equal to the value of that resistor times the total applied voltage divided by the total resistance of the series circuit.

If a current I is flowing through a series circuit consisting of 'n' number of series resistors, R_1, R_2, $R_3 \ldots R_n$, then by applying KVL, we get

$$V_s = IR_1 + IR_2 + IR_3 + \cdots + IR_n$$
$$= I(R_1 + R_2 + R_3 + \cdots + R_n) = IR_T, \text{ where } R_T = R_1 + R_2 + R_3 + \cdots + R_n$$

According to voltage division, the voltage across 'n'th resistor is

$$V_n = V_s \times \frac{R_n}{R_1 + R_2 + R_3 + \cdots + R_n}$$

If all these 'n' number of number of resistors have equal values, i.e., $R_1 = R_2 = R_3 = \ldots R_n = R$, then the voltage across each resistor is equally divided as,

$$V_1 = V_2 = V_3 = \ldots V_n = \frac{V_s}{n}$$

It is to be noted that the voltage is directly proportional to the resistance, for two series resistors of different values, the largest valued resistor will receive the largest share of voltage and vice versa.

1.16 Voltage Division in AC Circuits

The voltage division rule for AC circuits is similar to that of DC circuits except that the resistances are replaced by impedances. For a network consisting of two impedances Z_1 and Z_2 as shown in Figure 1.37, the voltage division rule is given by,

$$\bar{V}_1 = \frac{Z_1 \bar{V}_S}{Z_1 + Z_2} \text{ and } \bar{V}_2 = \frac{Z_2 \bar{V}_S}{Z_1 + Z_2}$$

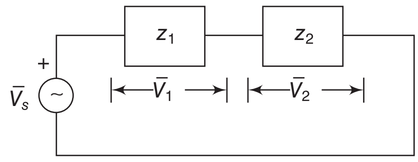


Figure 1.37 Voltage division in an AC circuit

where, $\bar{V}_S$ is the voltage applied to the circuit, $\bar{V}_1$ and $\bar{V}_2$ are the voltages across Z_1 and Z_2 respectively.

1.17 Current Division in DC Circuits

A parallel circuit is called a current divider circuit as only the current gets divided between the components connected in parallel and the voltage remains the same across them. According to Kirchhoff's Current Law, the net current divided by the parallel resistors connected to a node is equal to the net current flowing into that node. This implies that the supply current gets divided by the parallel resistors.

Consider an example for a parallel circuit as shown in the circuit of Figure 1.38. This circuit consists of a voltage source V_s and three resistors R_1, R_2 and R_3 connected in parallel between nodes 'a' and 'b'. It is called a current divider circuit as the current I at the node 'a' is divided among the resistors R_1, R_2 and R_3 as I_1, I_2 and I_3, respectively.

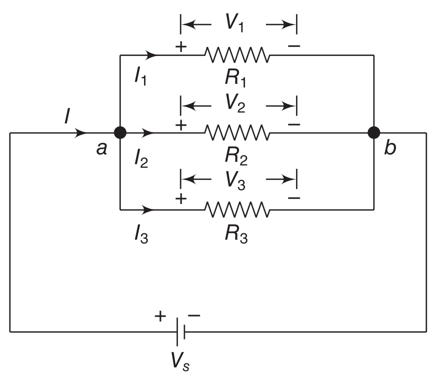


Figure 1.38 Current division in a parallel circuit

According to Ohm's law, the current I_1 through the resistor R_1 is $I_1 = \frac{V_s}{R_1}$ (since $V_1 = V_2 = V_3 = V_s$).

Similarly, $$I_2 = \frac{V_s}{R_2} \quad \text{and} \quad I_3 = \frac{V_s}{R_3}.$$

Inspection of Figure 1.38 reveals that, the voltages across all three resistors remain the same as V_s and only the currents get divided. Applying KCL, the divided currents can be summed up to obtain the net input current I.

i.e., $$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= \frac{V_s}{R_1} + \frac{V_s}{R_3} + \frac{V_s}{R_3} \\ &= V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = \frac{V_s}{R_T}, \end{aligned}$$

where $$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \text{ (or) } R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

and $$V_s = IR_T$$

Since the voltage is the same across all the components connected in parallel, we have

$$V_s = I_1R_1 = I_2R_2 = I_3R_3$$

In a generalised form, we have

$$V_s = I_xR_x$$

Substituting $V_s = IR_T$ in the above equation, we have

$$IR_T = I_xR_x$$

or $$I_x = I\frac{R_T}{R_x}$$

This expression is called current division rule, which states that the current through any branch in a parallel circuit is equal to the total resistance of the parallel circuit divided by the resistance value of the resistor and multiplied by the total current entering through the parallel circuit.

If a circuit consists of 'n' parallel resistors R_1, R_2, R_3 … R_n then the current division equation can be written in a generalised form as,

$$I = I_1 + I_2 + I_3 + \cdots + I_n$$

$$= V_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}\right) = \frac{V_s}{R_T}, \text{ where } R_T = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}.$$

or $$I = V_s(G_1 + G_2 + G_3 + \cdots + G_n), \text{ where conductance } G = \frac{1}{R}.$$

$$= V_sG_T, \text{ where, } G_T = G_1 + G_2 + G_3 + \cdots + G_n.$$

According to current division, the current through the 'n'th resistor is

$$I_n = I \times \frac{R_1 + R_2 + R_3 + \cdots + R_n}{R_n}$$

If all these resistors have equal values, i.e., $R_1 = R_2 = R_3 = \ldots = R_n = R$ then the current through each resistor is equally divided as

$$I_1 = I_2 = I_3 = \ldots = I_n = \frac{I}{n}$$

It is noted that the current is inversely proportional to the resistance. For two parallel resistors of different values, the smallest valued resistor will receive the largest share of current and vice versa.

1.18 Current Division in AC Circuits

The current division rule for AC circuits is similar to that of DC circuits except that the resistances are replaced by impedances. For a network consisting of two parallel branches with impedances Z_1 and Z_2 as shown in Figure 1.39, the current division rule is given by

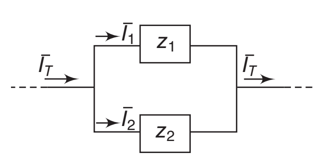


Figure 1.39 Current division in an AC circuit

$$\bar{I}_1 = \frac{Z_2\bar{I}_T}{Z_1+Z_2} \text{ and } \bar{I}_2 = \frac{Z_1\bar{I}_T}{Z_1+Z_2}$$

where, $\bar{I}_T$ is the total current applied to the parallel branches, $\bar{I}_1$ and $\bar{I}_2$ are the branch currents through Z_1 and Z_2 respectively.

Example 1.7

Find the current in each branch of the circuit and the total power consumed by the circuit of Figure E1.7(a). Assume $v(t) = 50\sin(\omega t + 45°)$V.

[AU May/June, 2009]

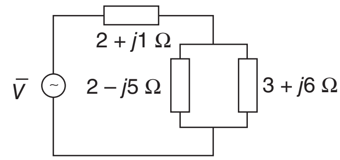


Figure E1.7(a)

Solution

Given

$$v(t) = 50\sin(\omega t + 45°) = V_m \sin(\omega t + 45°)\text{V}$$

Therefore, RMS voltage, $\bar{V} = \frac{V_m}{\sqrt{2}}\angle 45° = \frac{50}{\sqrt{2}}\angle 45° = 35.355\angle 45°$ V

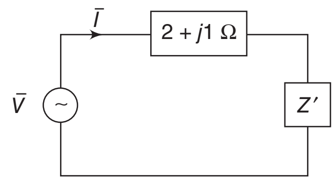


Figure E1.7(b)

To determine branch current, I:

Impedance Z' represented in Figure E1.7(b) is given by

$$Z' = (2-j5)\,||\,(3+j6) = \frac{(2-j5)(3+j6)}{2-j5+3+6}$$

$$= \frac{5.3851\angle -68.19° \times 6.7082\angle 63.43°}{5.099\angle 11.31°} = 7.84\angle -16.07°\ \Omega$$

$$\bar{I} = \frac{E}{2+j1+Z'} = \frac{35.355\angle 45°}{2+j1+6.8071-j1.9609}$$

$$= \frac{35.355\angle 45°}{8.8071-j0.9609} = \frac{35.355\angle 45°}{8.8593\angle -6.226°}$$

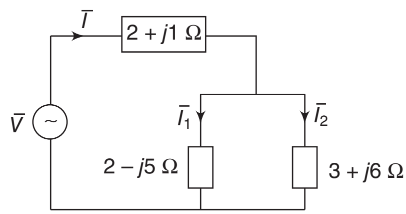


Figure E1.7(c)

To determine branch currents, $\bar{I}_1$ and $\bar{I}_2$:

The branch currents $\bar{I}_1$ and $\bar{I}_2$ are represented in Figure E1.7(c).

Using current division rule, we get

$$\bar{I}_1 = \bar{I} \times \frac{3+j6}{(2-j5+3+j6)}$$

$$= \frac{4\angle 51.226° \times 6.7082\angle 63.43°}{5.099\angle 11.31°}$$

$$= 5.2623\angle 103.346°\ \text{A}$$

$$\bar{I}_2 = \bar{I} \times \frac{2-j5}{(2-j5+3+j6)}$$

$$= \frac{4\angle 51.226° \times 5.3851\angle -68.19°}{5.099\angle 11.31°}$$

$$= 4.2244\angle -28.274°\ \text{A}$$

1.19 Nodal Analysis Method

A junction point in an electrical circuit is called a node. A voltage drop can be measured with respect to this point and another node acting as a reference point. Generally, grounded node is taken as the reference. If there is '*n*' number of nodes, then the number of independent KCL equations in nodal analysis is $(n-1)$ because one node acts as a reference node.

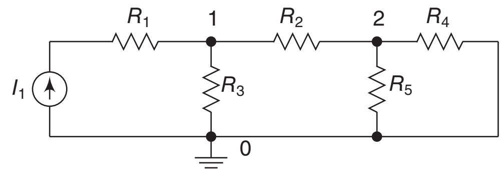


Figure 1.40(a)

Consider the example circuit shown in Figure 1.40(a).

The branch currents entering and leaving at node-1 can be marked as shown in Figure 1.40(b).

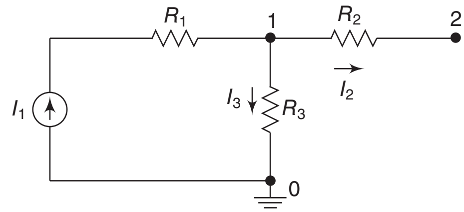


Figure 1.40(b)

According to KCL, the currents entering the node-1 = current leaving that node-1.

$$I_1 = I_2 + I_3$$

According to Ohm's law, $I_2 = \dfrac{V_1 - V_2}{R_2}$ and $I_3 = \dfrac{V_1 - V_0}{R_3}$

Therefore, $I_1 = \dfrac{V_1 - V_2}{R_2} + \dfrac{V_1 - V_0}{R_3}$ and since node-0 is grounded, $V_0 = 0$.

$$I_1 = \frac{V_1 - V_2}{R_2} + \frac{V_1 - 0}{R_3}$$

$$= \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_3}$$

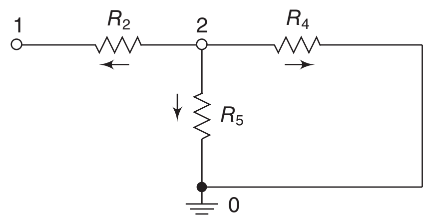


Figure 1.40(c)

Similarly, with reference to node-2 in Figure 1.40(c), assuming that all currents are leaving the node, we get

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_5} + \frac{V_2 - 0}{R_4} = 0$$

$$\frac{V_2 - V_1}{R_2} + V_2\left(\frac{1}{R_4} + \frac{1}{R_5}\right) = 0$$

The above nodal equations for nodes '1' and '2' are used to find out the voltages at each node.

Steps Involved in Nodal Analysis Method

Step 1 Identify all independent nodes wherever the current branches out and select a reference node.

Step 2 Write the nodal equation using KCL for all nodes except the reference node.

Step 3 Solve the nodal equations are then solved to find out nodal voltages and branch currents.

1.19.1 Super Nodal Analysis

The nodal analysis method makes the analysis simpler only if the circuit has only current sources. If voltage sources are present between any two nodes then it will be difficult for the nodal analysis method to be applied. In these cases, prior to nodal analysis, the voltage sources have to be converted into a current sources by source transformation. But, this is rather difficult procedure in certain cases.

In order to make the analysis simpler, a super node can be formed by combining two nodes connected by a voltage source. For example, consider the circuit shown in Figure 1.41. Inspection of this circuit reveals that totally three nodes are present including a ground node-0.

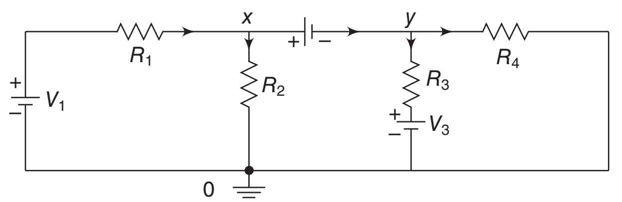


Figure 1.41 An example circuit

A voltage source is connected between nodes 'x' and 'y'. Hence, a super node can be formed by combining nodes 'x' and 'y' and writing the combined equation as

$$\frac{V_x - V_1}{R_1} + \frac{V_x}{R_2} + \frac{V_y - V_3}{R_4} + \frac{V_y}{R_3} = 0$$

Also, we have

$$V_x - V_y = V_2$$

Example 1.8

For the given circuit shown in Figure E1.8, write node voltage equations and determine currents in each branch for given network.

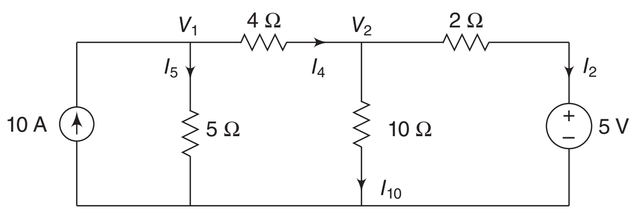


Figure E1.8

Solution

Applying Kirchhoff's current law at node V_1, we get

$$10 = \frac{V_1}{5} + \frac{V_1 - V_2}{4}$$

i.e., $$\frac{9}{20}V_1 - \frac{V_2}{4} = 10 \tag{1}$$

Applying KCL at V_2, we get

$$\frac{V_2 - V_1}{4} + \frac{V_2}{10} + \frac{V_2 - 5}{2} = 0$$

i.e., $$\frac{17}{20}V_2 - \frac{V_1}{4} = \frac{5}{2} \tag{2}$$

Solving the nodal Equations (1) and (2), we get

$$V_2 = 11.33 \text{ V}$$

$$V_1 = 25.8 \text{ V}$$

According to Ohm's law, the different branch currents are

$$I_5 = \frac{V_1}{5} = \frac{28.5}{5} = 5.7 \text{ A}$$

$$I_{10} = \frac{V_2}{10} = \frac{11.33}{10} = 1.133 \text{ A}$$

$$I_4 = \frac{V_1 - V_2}{4} = \frac{28.5 - 11.33}{4} = 4.2952 \text{ A}$$

and $$I_2 = \frac{V_2 - 5}{2} = \frac{11.33 - 5}{2} = 3.165 \text{ A}$$

Example 1.9

In the circuit shown in Figure E1.9(a), find the different node voltages and the currents I_1, I_2 and I_3.

[AU June, 2010]

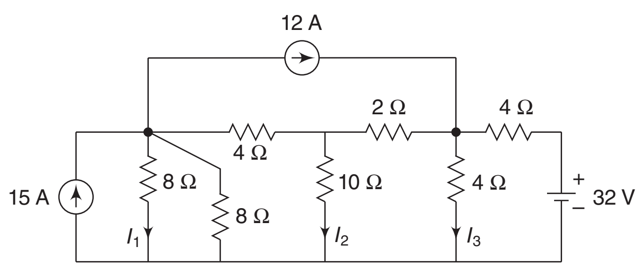


Figure E1.9(a)

Solution

Redrawing the circuit as shown in Figure E1.9(b), we get

(a) To determine currents:

Applying KCL at 3 nodes, we get

At node 1,

$$15 - I_1 - I_7 - I_4 - 12 = 0$$

i.e., $$I_1 + I_4 + I_7 = 3 \tag{1}$$

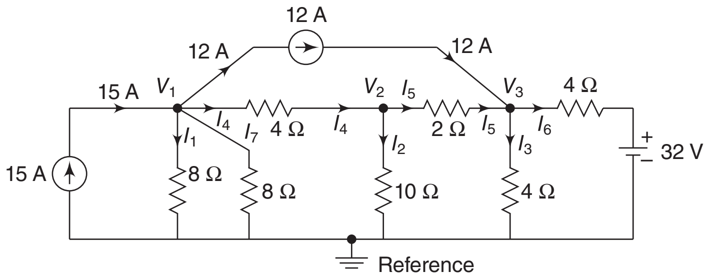


Figure E1.9(b)

At node 2,

$$I_4 - I_2 - I_5 = 0$$

i.e., $$I_2 - I_4 + I_5 = 0 \tag{2}$$

At node 3,

$$I_5 + 12 - I_3 - I_6 = 0$$

i.e., $$I_3 - I_5 + I_6 = 12 \tag{3}$$

Solving the above equations, we get

$$I_1 = \frac{V_1}{8}, I_2 = \frac{V_2}{10}, I_3 = \frac{V_3}{4}, I_4 = \frac{V_1 - V_2}{4}, I_5 = \frac{V_2 - V_3}{2}, I_6 = \frac{V_3 - 32}{4}, \text{and } I_7 = \frac{V_1}{8}$$

(b) To determine voltages:

At node 1,

$$\frac{V_1}{8} + \frac{V_1 - V_2}{4} + \frac{V_1}{8} = 3$$

i.e., $$0.5V_1 - 0.25V_2 = 3 \tag{4}$$

At node 2,

$$\frac{V_2}{10} - \frac{(V_1 - V_2)}{4} + \frac{(V_2 - V_1)}{4} = 0$$

i.e., $$-0.25V_1 + 0.85V_2 - 0.5V_3 = 0 \tag{5}$$

At node 3,

$$\frac{V_3}{4} - \frac{(V_2 - V_3)}{2} + \frac{(V_3 - V_{32})}{4} = 12$$

i.e., $$-0.5V_2 + V_3 = 20 \tag{6}$$

Solving the above equations, we get

$$V_1 = 18.1052 \text{ V}, V_2 = 24.2105 \text{ V}, V_3 = 32.1052 \text{ V}$$

$$I_1 = 2.2631 \text{ A}, I_2 = 2.42105 \text{ A}, I_3 = 8.0263 \text{ V}$$

Example 1.10

For the circuit shown in Figure E1.10, find the voltage across the 6 Ω resistor by using nodal analysis.

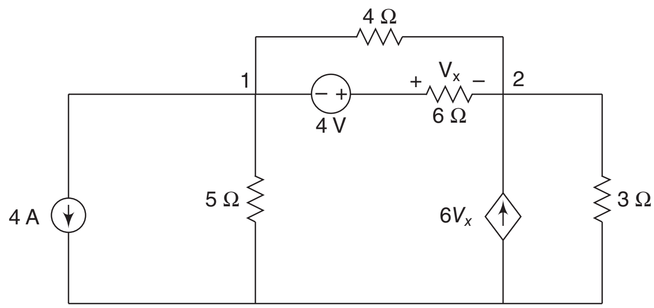


Figure E1.10

Solution

Applying KCL at node-1, we get

$$4+\frac{V_1}{5}+\frac{V_1+4-V_2}{6}+\frac{V_1-V_2}{4}=0$$

i.e., $$\frac{240+12V_1+10(V_1+4-V)+15(V_1-V_2)}{60}=0$$

or $$37V_1-25V_2=-280 \tag{1}$$

Applying KCL at node-2, we get

$$\frac{V_2-V_1-4}{6}+\frac{V_2-V_1}{4}-6V_x+\frac{V_2}{3}=0 \tag{2}$$

Inspecting the circuit in Figure E1.10, it is found that

$$V_1+4=V_x+V_2$$

$$V_x=V_1+4-V_2 \tag{3}$$

Substituting Equation (3) in Equation (1), we get

$$\frac{V_2-V_1-4}{6}+\frac{V_2-V_1}{4}-6(V_1+4-V)+\frac{V_2}{3}=0$$

i.e., $$\frac{2(V_2-V_1-4)+3(V_2-V_1)-72(V_1+4-V)+4V_2}{12}=0$$

or, $$-77V_1+81V_2=296 \tag{4}$$

Upon solving Equations (4) and (1), we get

$$V_1=-14.25 \text{ V and } V_2=-9.89 \text{ V}$$

From Equation(3), the voltage across 6 Ω resistor is

$$V_x=V_1+4-V_2$$
$$=-14.25+4+9.89=-0.36 \text{ V}.$$

Example 1.11

For the circuit shown in Figure E1.11(a), find the current through 5 Ω resistor using nodal method.

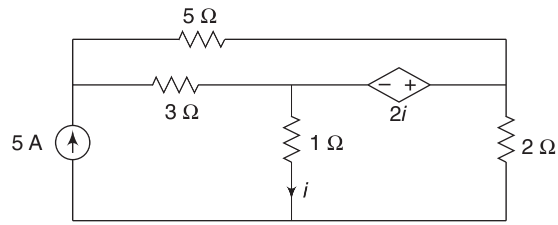


Figure E1.11(a)

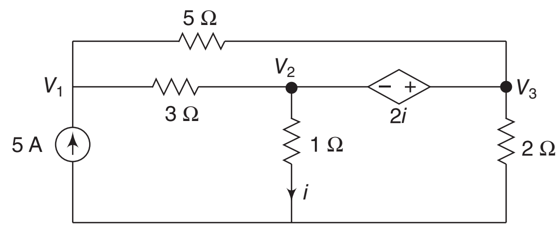


Figure E1.11(b)

Solution

Assume the nodal voltages V_1, V_2 and V_3 as shown in Figure E1.11(b).

Writing nodal equation at node '1',

$$\frac{V_1 - V_3}{5} + \frac{V_1 - V_2}{3} = 5$$

i.e., $8V_1 - 5V_2 - 3V_3 = 75$

Writing the equation for super node,

$$\frac{V_2 - V_1}{3} + V_2 + \frac{V_3 - V_1}{5} + \frac{V_3}{2} = 0$$

i.e., $-16V_1 + 40V_2 + 21V_3 = 0$

Between nodes '2' and '3', we have

$$V_3 - V_2 = 2i$$

According to Ohm's law,

$$i = \frac{V_2}{1} = V_2$$

Therefore, $V_3 - V_2 = 2V_2$

i.e., $V_3 = 3V_2$

Upon solving these nodal equations, we get

$$V_1 = 12.875 \text{ V}, V_2 = 2 \text{ V}, V_3 = 6 \text{ V}$$

The current passing through 5 Ω resistor is

$$I_{5\Omega} = \frac{V_1 - V_3}{5} = \frac{12.875 - 6}{5} = 1.375 \text{ A}$$

1.20 Mesh Analysis Method

Kirchhoff's laws are applied in the analysis and solving of electrical circuits. The method of solving a complex circuit can be simplified by using either mesh or nodal analysis method. Generally, KVL and KCL are used in deriving the mesh and nodal equations respectively. The mesh analysis method is discussed as follows.

The term loop or mesh represents a closed path in the circuit through which the current can flow in a circuit. Since the closed path or loop resembles a physical 'fence', it is called a mesh. The mesh current is a current that circulates around a mesh. If more than one mesh exists, then the current gets divided among them causing independent mesh current in every mesh. This results in independent KVL equation expressed around each of this mesh. For '*m*' number of independent meshes, a total of '*m*' number of KVL equations can be obtained around each mesh.

Mesh equations are derived from the current division in the meshes. A loop current is different from branch current. In order to demonstrate this difference, consider the circuit shown in Figure 1.42(a).

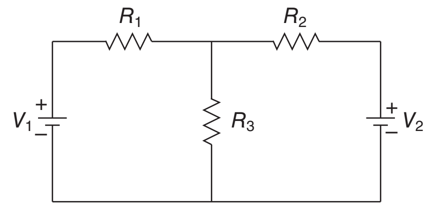


Figure 1.42 (a) An example circuit

The given circuit is redrawn with nodes as shown in Figure 1.42(b). On inspection of this circuit, we can identify two closed loops or meshes represented as mesh-1 (*abda*) and mesh-2 (*bcdb*). Here, I_1 and I_2 are the mesh currents flowing in mesh-1 and mesh-2 respectively.

As the branch 'bd' consisting of resistor R_3 is shared between the meshes 1 and 2, the resultant branch current is I_3. The magnitude of I_3 depends on the magnitudes of both the mesh currents I_1 and I_2. From the circuit shown in Figure 1.42(b), it is evident that the current directions of I_1 and I_2 in R_3 are opposite to each other. Therefore,

$$I_3 = I_1 - I_2$$

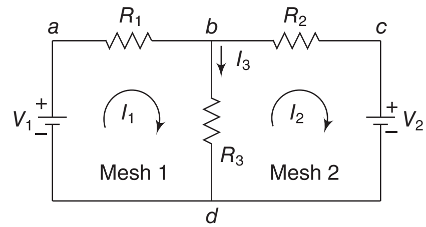


Figure 1.42 (b) Two closed loops

Considering the circuit in Figure 1.42(b) again, it can be noticed that there is a possibility of drawing one more closed path or mesh (*abcda*) through the elements $V_1 \rightarrow R_1 \rightarrow R_2 \rightarrow V_2$ and back to V_1 as shown in Figure 1.49(c). But, as all current divisions and related voltage drops are included in the equations of first two meshes '1' and '2' itself, drawing an additional mesh becomes unnecessary.

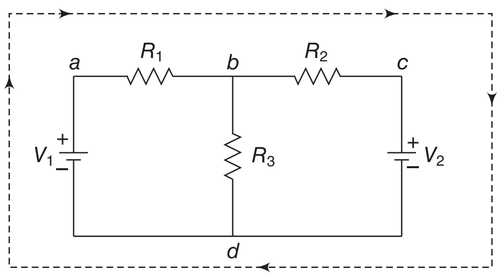


Figure 1.42 (c) Possible closed loop (*abcda*)

Assignment of the direction of mesh current is arbitrary. In order to simplify the procedure for writing the mesh equation, generally clockwise direction of current preferred.

In the process of writing the KVL equations for a mesh, the polarities of voltage drop across a component is determined by the assumed direction of the mesh current in that particular mesh.

If an element is located on the boundary between two meshes such as R_3 in Figure 1.42(d), the current flowing in the element is the algebraic sum of the currents flowing through it.

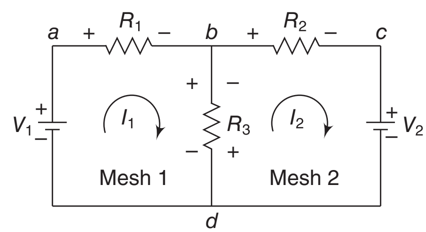


Figure 1.42(d)

Mesh Equations

Step 1 Make sure that the circuit considered for analysis has only voltage sources. If the circuit consists of current source, then use source transformation technique to convert it into a voltage source first.

Step 2 Assume the current direction and assign identification labels.

Step 3 Along the assumed direction of current, mark the polarities of voltage drops across each element.

Step 4 Assign the correct polarity to the voltage source.

Step 5 Write the mesh equation by taking polarities of each component into consideration; use KVL and equate the algebraic sum of voltage drops to zero in that particular mesh.

Step 6 For the shared branch, the algebraic sum of mesh currents flowing through it is considered. For example, while writing the equation for mesh-1, current through common branch *b-d* is considered to be $(I_1 - I_2)$, whereas while writing the equation for mesh-2, the current through the same branch is considered to be $(I_2 - I_1)$.

Step 7 Solve the mesh equations to determine the solution for unknown quantities.

To determine the current though I_3:
Applying KVL for mesh-1, we get

$$0 = I_1R_1 + (I_1 - I_2)R_3 - V_1$$

i.e.,
$$V_1 = I_1(R_1 + R_3) - I_2R_3$$

For mesh-2, the mesh equations are

$$0 = I_2R_2 + V_2 + (I_2 - I_1)R_3$$

i.e.,
$$-V_2 = I_2(R_2 + R_3) - I_1R_3$$

The equations for V_1 and V_2 are known as mesh equations, and have to be solved further to find solution for unknown mesh currents, I_1 and I_2. The branch current I_3 can be calculated from the difference between I_1 and I_2.

The mesh method of analysis is chosen if the circuit consists of more number of meshes than nodes. It is a preferred method in analysing only the planar circuits, i.e., the circuits drawn on a plane with no crossing branches.

1.20.1 Super Mesh Analysis

The mesh equation method makes the analysis of the circuits simpler, when only voltage sources are present. If the circuit consist of current source in any of its branches, then it will be difficult to apply mesh analysis technique directly. In that case, prior to mesh analysis the current source has to be converted into a voltage source by applying source transformation technique. Alternately, in order to simplify the analysis process, a super mesh can be formed by combining two meshes sharing a common current source.

For example, consider the circuit shown in Figure 1.43(a). Inspection of the circuit reveals that there are three possible meshes as shown in Figure 1.43(b). Let us assume the clockwise current directions for these meshes.

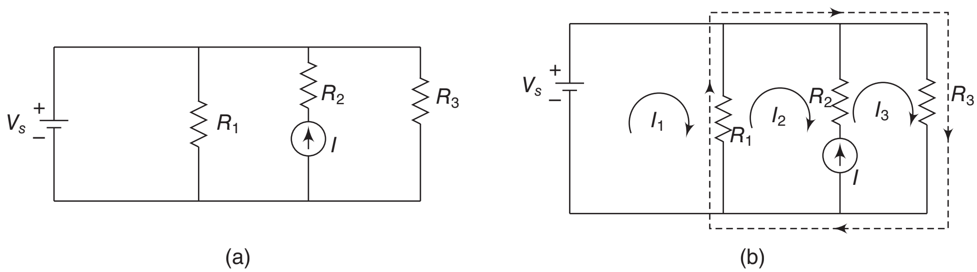


Figure 1.43 An example circuit

Applying KVL for the mesh-1, we get $V_s = R_1(I_1 - I_2)$. Inspecting the circuit, it can be noticed that a common current source is shared between the meshes 2 and 3. A super mesh can be formed by combining these two meshes and ignoring the branch consisting of current source in the traversal path as shown by dashed lines in Figure 1.43(b). The super mesh equation is written as

$$R_1(I_2 - I_1) + R_3I_3 = 0$$

The current driven by the current source I can be calculated as

$$I = I_3 - I_2$$

Thus, the current source shared by two meshes can be eliminated in the analysis by combining these meshes to form a super mesh.

Example 1.12

Determine the loop currents of the circuit shown in Figure E1.12(a). **[AU April/May, 2004]**

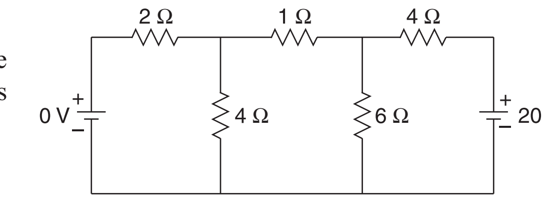


Figure E1.12(a)

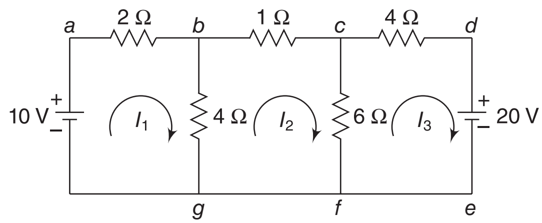


Figure E1.12(b)

Solution

As shown in Figure E1.12(b), the circuit consists of three meshes, 1 (*abga*), 2 (*bcfgb*) and 3 (*cdefc*) and the currents flowing in these meshes are I_1, I_2 and I_3 respectively.

Applying KVL for mesh-1 (*abga*), we get

$$2I_1 + 4(I_1 - I_2) - 10 = 0$$

i.e., $$6I_1 - 4I_2 = 10 \quad (1)$$

Applying KVL for mesh-2 (*bcfgb*), we get

$$1 \cdot I_2 + 6(I_2 - I_3) + 4(I_2 - I_1) = 0$$

i.e., $$-4I_1 + 11I_2 - 6I_3 = 0 \quad (2)$$

Applying KVL for mesh-3 (*cdefc*), we get

$$4I_3 + 20 + 6(I_3 - I_2) = 0$$

i.e., $$-6I_2 + 10I_3 = -20 \quad (3)$$

Writing the mesh Equation (1), Equation (2) and Equation (3) in matrix form, we have

$$\begin{bmatrix} 6 & -4 & 0 \\ -4 & 11 & -6 \\ 0 & -6 & 10 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -20 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 6 & -4 & 0 \\ -4 & 11 & -6 \\ 0 & -6 & 11 \end{vmatrix} = 6(110 - 36) + 4(-40 - 0) + 0 = 444 - 160 = 284$$

$$\Delta_1 = \begin{vmatrix} 10 & -4 & 0 \\ -0 & 11 & -6 \\ -20 & -6 & 10 \end{vmatrix} = 6(110 - 36) + 4(0 - 120) + 0 = 740 - 480 = 260$$

$$\Delta_2 = \begin{vmatrix} 6 & 10 & -6 \\ -4 & 0 & 10 \\ 0 & -20 & 10 \end{vmatrix} = 6(0 - 120) - 10(-40 + 0) + 0 = -720 - 400 = -320$$

$$\Delta_3 = \begin{vmatrix} 6 & -4 & 10 \\ -4 & 11 & 0 \\ 0 & -6 & -20 \end{vmatrix} = 6(-220 + 0) + 4(80 - 0) + 10(24 - 0) = -1320 + 320 + 240 = -760$$

Therefore, $$I_1 = \frac{\Delta_1}{\Delta} = \frac{260}{284} = 0.915 \text{ A}$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{-320}{284} = -1.126 \text{ A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-760}{284} = -2.676 \text{ A}$$

Since I_2 and I_3 have a negative sign, the actual current direction of I_2 and I_3 is opposite to the assumed clockwise direction.

Hence, $I_2 = 1.126$ A and $I_3 = 2.676$ A

Current through the 6 Ω resistor is

$$I_{6\,\Omega} = I_3 - I_2 = 1.55 \text{ A}$$

Example 1.13

Determine the value of R and current through it for the circuit shown in Figure E1.13(a), when the current is zero in the branch CD.

[AU April /May, 2004]

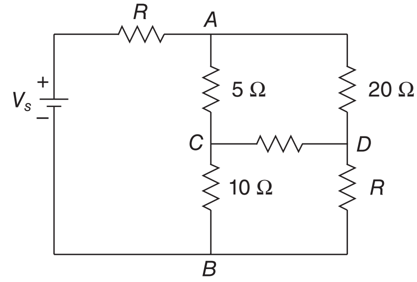


Figure E1.13(a)

Solution

Assuming the clockwise traverse current direction as shown in Figure E1.13(b), and applying KVL to mesh-1, we get

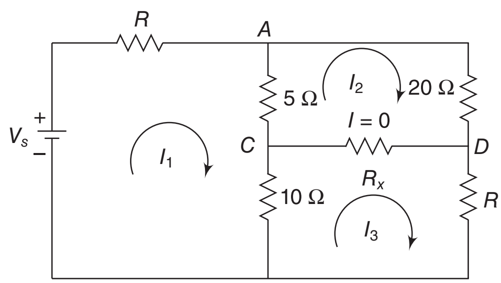


Figure E1.13(b)

$$RI_1 + 5(I_1 - I_2) + 10(I_1 - I_3) = V_s$$

i.e., $$(15 + R)I_1 - 5I_2 - 10I_3 = V_s \quad (1)$$

The current in the branch $CD = 0$.

$$I_2 - I_3 = 0$$

i.e., $$I_2 = I_3$$

Assume that resistance across the CD is R_x

Applying KVL to mesh-2, we get

$$20I_2 + R_x(I_2 - I_3) + 5(I_2 - I_1) = 0$$

Substituting $I_2 - I_3 = 0$, we get

$$20I_2 + 5I_2 - 5I_1 = 0$$

i.e., $$-5I_1 + 25I_2 = 0 \quad (2)$$

Applying KVL to mesh-3, we get

$$R_x(I_3 - I_2) + RI_3 + 10(I_3 - I_1) = 0$$

Substituting $I_2 - I_3 = 0$, we get

$$-10I_1 + (10 + R)I_3 = 0 \quad (3)$$

Writing the mesh Equations (1) to (3) in matrix form, we have

$$\begin{bmatrix} 15 + R & -5 & -10 \\ -5 & 25 & 0 \\ -10 & 0 & 10 + R \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 15+R & -5 & -10 \\ -5 & 25 & 0 \\ -10 & 0 & 10+R \end{vmatrix}$$

$$= (15+R)[25(10+R)-0]+5[-5(10+R)-0]-10[0+250]$$

$$= (15+R)(250+25R)+5(-50-5R)-2500$$

$$= 3750+250R+375R+25R^2-250-25R-2500$$

$$\Delta = 1000+600R+25R^2$$

$$\Delta_1 = \begin{vmatrix} V_s & -5 & -10 \\ 0 & 25 & 0 \\ 0 & 0 & 10+R \end{vmatrix} = V_s[250+25R]$$

$$= 250V_s + 25V_sR$$

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{250V_s + 25V_sR}{1000+600R+25R^2}$$

$$\Delta_2 = \begin{vmatrix} 15+R & V_s & -10 \\ -5 & 0 & 0 \\ -10 & 0 & 10+R \end{vmatrix} = (15+R)[0]-V_s[-50(10+R)-0]-10(0)$$

$$= 50V_s + 5V_sR$$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{50V_s + 5V_sR}{1000+600R+25R^2}$$

$$\Delta_3 = \begin{vmatrix} 15+R & -5 & V_s \\ -5 & 25 & 0 \\ -10 & 0 & 0 \end{vmatrix} = (15+R)(0)-5(0)+V_s(+250) = 250V_s$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{250V_s}{1000+600R+25R^2}$$

Since $I_2 - I_3 = 0$, we have $I_2 = I_3$

$$I_3 = \frac{250V_s}{1000+600R+25R^2} = I_2 = \frac{50V_s + 5V_sR}{1000+600R+25R^2}$$

$$250V_s = 50V_s + 5V_sR$$

Therefore, $R = 40\ \Omega$

and $\Delta = 1000 + 600 \times 40 + 25 \times 40^2 = 65000$

$$I_1 = \frac{250V_s + (25V_s)40}{65000} = 0.0192V_s$$

The current I_3 that flows through $R(= 40\ \Omega)$ is

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{250V_s}{65000} = 0.0005V_s$$

Example 1.14

Determine the current I_L in the circuit shown in Figure E1.14(a).

[AU Nov/Dec, 2010]

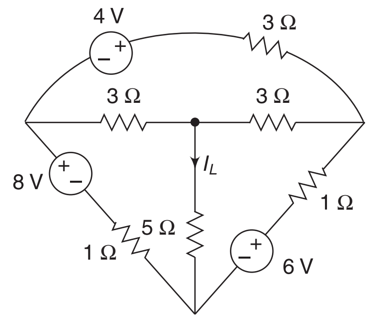


Figure E1.14(a)

Solution

The given circuit consists of three meshes.

Assume clockwise traverse current directions as shown in Figure E1.14(b). Applying KVL, we get three mesh equations as follows.

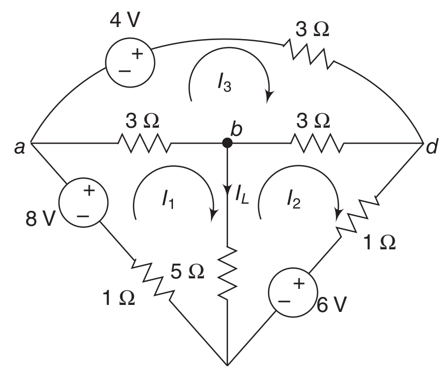


Figure E1.14(b)

For mesh-1 (*abca*),

$$3(I_1 - I_3) + 5(I_1 - I_2) + 1I_1 - 8 = 0$$

i.e., $$9I_1 - 5I_2 - 3I_3 = 8 \quad (1)$$

For mesh-2 (*bdcb*),

$$3(I_2 - I_3) + 1I_2 + 6 + 5(I_2 - I_1) = 0$$

i.e., $$-5I_1 + 9I_2 - 3I_3 = -6 \quad (2)$$

For mesh-3 (*adba*),

$$-4 + 3I_3 + 3(I_3 - I_2) + 3(I_3 - I_1) = 0$$

i.e., $$-3I_1 - 3I_2 + 9I_3 = 4 \quad (3)$$

Writing the mesh Equations (1) to (3) in matrix form, we have

$$\begin{bmatrix} 9 & -5 & -3 \\ -5 & 9 & -3 \\ -3 & -3 & 9 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 8 \\ -6 \\ 4 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 9 & -5 & -3 \\ -5 & 9 & -3 \\ -3 & -3 & 9 \end{vmatrix} = 252$$

$$\Delta_1 = \begin{vmatrix} 8 & -5 & -3 \\ -6 & 9 & -3 \\ 4 & -3 & 9 \end{vmatrix} = 420$$

$$\Delta_2 = \begin{vmatrix} 9 & 8 & -3 \\ -5 & -6 & -3 \\ -3 & 4 & 9 \end{vmatrix} = 168$$

$$\Delta_3 = \begin{vmatrix} 9 & -5 & 8 \\ -5 & 9 & -6 \\ -3 & -3 & 4 \end{vmatrix} = 308$$

Therefore, $I_1 = \frac{\Delta_1}{\Delta} = \frac{420}{252} = 1.667 \text{ A}$

$$I_2 = \frac{\Delta_2}{\Delta} = \frac{168}{252} = 0.667 \text{ A}$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{308}{252} = 1.222 \text{ A}$$

Hence, $I_L = I_1 - I_2 = 1$ A.

Example 1.15

For the circuit shown in Figure E1.15, using mesh analysis, determine the unknown voltage V_x which gives a voltage of 5 V across the 5 Ω resistor.

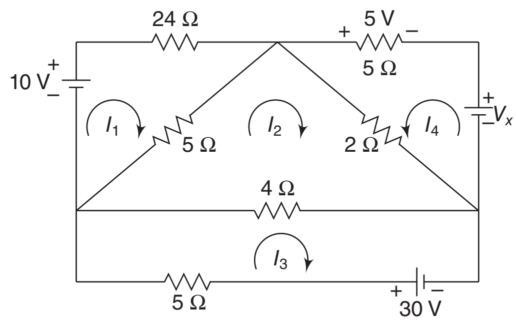


Figure E1.15

Solution

Inspecting the given circuit it can be noticed that the voltage across the 5 Ω resistor is 5 V. According to Ohm's law, $I_4 = I_{5\Omega} = \frac{5V}{5\Omega} = 1A$.

Applying KVL to this circuit according to the given current direction, we get four equations for I_1, I_2, I_3 and I_4

$$29I_1 - 5I_2 = 10 \quad (1)$$

$$-5I_1 + 11I_2 - 4I_3 + 2I_4 = 0 \quad (2)$$

$$-4I_2 + 9I_3 = 30 \quad (3)$$

$$2I_2 + 7I_4 = V_x \quad (4)$$

Writing the Equations (1) to (4) in matrix form, we get

$$\begin{bmatrix} 29 & -5 & 0 & 0 \\ -5 & 11 & -4 & 2 \\ 0 & -4 & 9 & 0 \\ 0 & 2 & 0 & 7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ 30 \\ V_s \end{bmatrix}$$

Applying Cramer's rule, wet get

$$\Delta = \begin{vmatrix} 29 & -5 & 0 & 0 \\ -5 & 11 & -4 & 2 \\ 0 & -4 & 9 & 0 \\ 0 & 2 & 0 & 7 \end{vmatrix}$$

$$= 29 \begin{vmatrix} 11 & -4 & 2 \\ -4 & 9 & 0 \\ 2 & 0 & 7 \end{vmatrix} + 5 \begin{vmatrix} -5 & -4 & 2 \\ 0 & 9 & 0 \\ 0 & 0 & 7 \end{vmatrix} = 29\{11(63) + 4(-28) + 2(-18)\} + 5\{-5)(63)\}$$

$$= 29\{693 - 112 - 36\} - 1575 = 14230$$

$$\Delta_4 = \begin{vmatrix} 29 & -5 & 0 & 10 \\ -5 & 11 & -4 & 0 \\ 0 & -4 & 9 & 30 \\ 0 & 2 & 0 & V_x \end{vmatrix}$$

$$= 29\begin{vmatrix} 11 & -4 & 0 \\ -4 & 9 & 30 \\ 2 & 0 & V_x \end{vmatrix} + 5\begin{vmatrix} -5 & -4 & 0 \\ 0 & 9 & 30 \\ 0 & 0 & V_x \end{vmatrix} - 10\begin{vmatrix} -5 & 11 & -4 \\ 0 & -4 & 9 \\ 0 & 2 & 0 \end{vmatrix}$$

$$= 29\{11(9V_x) + 4(-4V_x - 60)\} + 5\{-5(9V_x)\} - 10\{-5(-18)\}$$

$$= 29\{99V_x - 16V_x - 240\} - 225V_x - 900 = 29\{83V_x - 240\} - 225V_x - 900$$

$$= 2407V_x - 6960 - 225V_x - 900 = 2182V_x - 7860$$

Therefore, $I_4 = \dfrac{\Delta_4}{\Delta} = \dfrac{2182V_x - 7860}{14230}$

Substituting I_4 = 1 A, we have

$$1 = \frac{2182V_x - 7860}{14230}$$

Upon solving this equation, we get

$$V_x = 10.124 \text{ V}$$

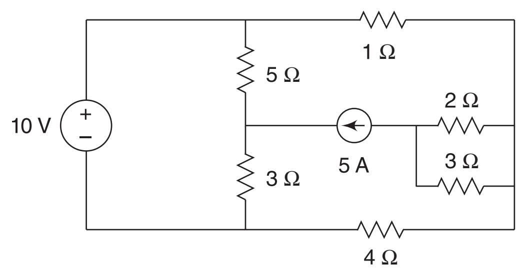


Figure E1.16(a)

Example 1.16

Find the mesh currents for the circuit shown in Figure E1.16(a) by applying super mesh analysis.

Solution

Step 1 The parallel combination of resistors 2 Ω and 3 Ω can be reduced to its equivalent value $R_{eq} = \dfrac{2 \times 3}{2+3} = 1.2\,\Omega$ as shown in Figure E1.16(b).

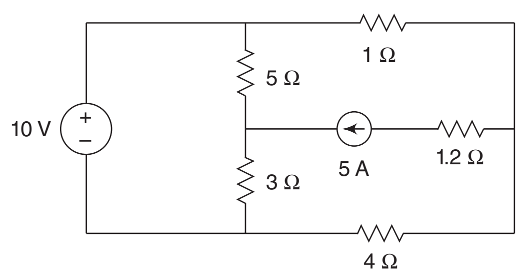


Figure E1.16(b)

Step 2 Assume the clockwise current directions for I_1, I_2 and I_3 assign nodes as shown in Figure E1.16(c).

Step 3 Write the KVL equations for each mesh, and combine the meshes sharing a current source to form a super mesh.

Applying KVL for the mesh-1 (*abcda*), we get

$$5(I_1 - I_2) + 3(I_1 - I_3) = 10$$

$$5I_1 - 5I_2 + 3I_1 - 3I_3 = 10$$

$$8I_1 - 5I_2 - 3I_3 = 10 \qquad (1)$$

As the meshes 2 (*befcb*) and 3 (*cfdc)* share a common 5 A current source between their nodes '*c*' and '*f*', these two meshes are combined to form a single super mesh.

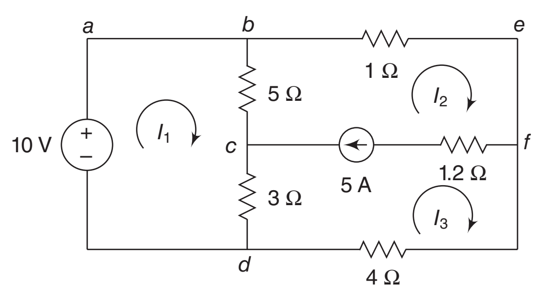


Figure E1.16(c)

The components connected in the shared branch *c-f* are not considered in the super mesh. Writing the KVL equation for this super mesh *efdb*, we get

$$5(I_2 - I_1) + 1I_2 + 4I_3 + 3(I_3 - I_1) = 0$$

i.e., $$-8I_1 + 6I_2 + 7I_3 = 0 \qquad (2)$$

Adding the mesh Equation (1) and Equation (2), we get

$$I_2 + 4I_3 = 10 \qquad (3)$$

Since the current source 5 A is equal to the difference between the current flowing in the meshes 2 and 3, we have

$$I_2 - I_3 = 5 \text{ or } I_2 = I_3 + 5 \qquad (4)$$

Substituting Equation (4) in Equation (3), we get

$$I_3 + 5 + 4I_3 = 0$$

$$I_3 = -1 \text{ A.}$$

The negative sign indicates that the actual current direction is opposite to the assumed current direction. Therefore, $I_3 = 1$ A.

and $$I_2 = I_3 + 5 = 6 \text{ A}$$

Example 1.17

For the circuit shown in Figure E1.17, find the current through 20 Ω using mesh analysis.

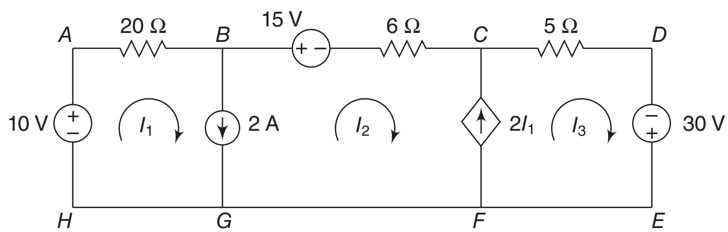


Figure E1.17

Solution

As the parallel branches consist of current sources, we can form a super mesh *ABCDEHA* and write the equation for that as

$$-10 + 20I_1 + 15 + 6I_2 + 5I_3 - 30 = 0$$

i.e., $$20I_1 + 6I_2 + 5I_3 = -25$$

In the parallel branches,

$$I_1 - I_2 = 2$$

$$I_3 - I_2 = 2I_1$$

Solving these equations, we get

$$I_{20\,\Omega} = I_1 = 1.146 \text{ A}$$

Example 1.18

For the network shown in Figure E1.18, obtain the current ratio $\overline{I}_1/\overline{I}_3$. **[AU April/May, 2011]**

Solution

Applying KVL for the three loops, we get

For Loop 1, $-j2I - 5I_1 + 5I_2 + V_1 = 0$

i.e., $-I_1(5 + j2) + 5I_2 = -V_1$

i.e., $I_1(5 + j2) - 5I_2 = V_1$ (1)

For Loop 2, $-I_2(-j4) - (j2)I_2 - 5I_2 + 5I_1 + j2I_3 = 0$

i.e., $5I_1 - I_2(5 - j2) + I_3(j2) = 0$ (2)

For Loop 3, $-5I_3 - j2I_3 + j2I_2 = 0$

i.e., $0I_1 + j2I_3 + (5 + j2)I_2 = 0$ (3)

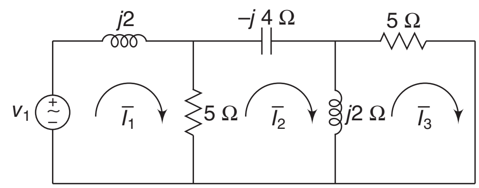


Figure E1.18

Using Cramer's rule, we get

$$\Delta = \begin{vmatrix} 5+j2 & -5 & 0 \\ 5 & -(5-j2) & j2 \\ 0 & j2 & -(5+j2) \end{vmatrix}$$

$$= (5+j2)(5+j2)(5-j2) - 25(5+j2) + 4(5+j2)$$

$$= 145 + j58 - 125 - j50 + 20 + j8 = 40 + j16 = 43.081\angle +21.801°$$

$$\Delta_1 = \begin{vmatrix} V_1 & -5 & 0 \\ 0 & -(5-j2) & j2 \\ 0 & j2 & -(5+j2) \end{vmatrix} = (5-j2)(5+j2)V_1 + 4V_1 = 33V_1$$

$$\Delta_3 = \begin{vmatrix} 5+j2 & -5 & V_1 \\ 5 & -(5-j2) & 0 \\ 0 & j2 & 0 \end{vmatrix} = 10jV_1 = 10V_1\angle +90°$$

Hence, $$\overline{I}_1 = \frac{\Delta_1}{\Delta} = \frac{33V_1\angle 0°}{43.081\angle +21.801°} = 0.7659V_1\angle -21.801° \text{ A}$$

and $$\overline{I}_3 = \frac{\Delta_3}{\Delta} = \frac{10V_1\angle +90°}{43.081\angle +21.801°} = 0.2321V_1\angle 68.199° \text{ A}$$

Therefore, $$\frac{\overline{I}_1}{\overline{I}_3} = \frac{0.7659V_1\angle -21.801°}{0.2321V_1\angle 68.199°} = 3.3\angle -90° \text{ A}$$

Example 1.19

For the circuit shown in Figure E1.19, determine the voltage across 10 Ω resistor.

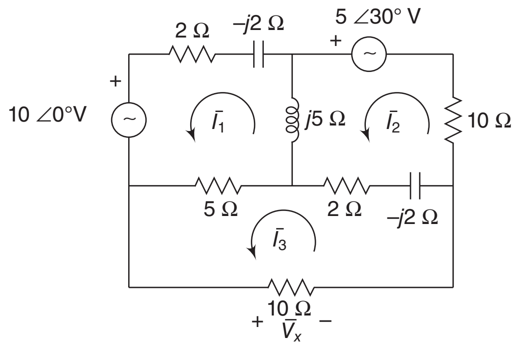


Figure E1.19

Solution

Applying KVL, for the circuit in the given current direction, we get

For mesh-1,

$$2\bar{I}_1 - j2\bar{I}_1 + j5(\bar{I}_1 + \bar{I}_2) + 5(\bar{I}_1 + \bar{I}_3) = 10\angle 0°$$

i.e., $$(7 + j3)\bar{I}_1 + j5\bar{I}_2 + 5\bar{I}_3 = 10\angle 0°$$

For mesh-2,

$$2 - j2(\bar{I}_2 - \bar{I}_3) + 10\bar{I}_2 + j5(\bar{I}_1 + \bar{I}_2) = 5\angle 30°$$

i.e., $$j5\bar{I}_1 + (12 + j3)\bar{I}_2 + (-2 + j2)\bar{I}_3 = 5\angle 30°$$

For mesh-3,

$$10\bar{I}_3 - j2(\bar{I}_3 - \bar{I}_2) + 2(\bar{I}_3 - \bar{I}_2) + 5(\bar{I}_3 + \bar{I}_1) = 0$$

i.e., $$5\bar{I}_1 + (-2 + j2)\bar{I}_2 + (17 - j2)\bar{I}_3 = 0$$

Writing these equations in matrix form, we have

$$\begin{bmatrix} 7+j3 & j5 & 5 \\ j5 & 12+j3 & -2+j2 \\ 5 & -2+j2 & 17-j2 \end{bmatrix} \begin{bmatrix} \bar{I}_1 \\ \bar{I}_2 \\ \bar{I}_3 \end{bmatrix} = \begin{bmatrix} 10\angle 0° \\ 5\angle 30° \\ 0 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{vmatrix} 7+j3 & j5 & 5 \\ j5 & 12+j3 & -2+j2 \\ 5 & -2+j2 & 17-j2 \end{vmatrix} = 1438 + j538 = 1535.346\angle 20.51°$$

$$\Delta_3 = \begin{vmatrix} 7+j3 & j5 & 10\angle 0° \\ j5 & 12+j3 & 5\angle 30° \\ 5 & -2+j2 & 0 \end{vmatrix} = \begin{vmatrix} 7+j3 & j5 & 10 \\ j5 & 12+j3 & 4.33+j2.5 \\ 5 & -2+j2 & 0 \end{vmatrix}$$

$$= -655.9 - j26.411 = 656.31\angle -177.69°$$

$$\bar{I}_3 = \frac{\Delta_3}{\Delta} = \frac{656\angle -177.69°}{1535.346\angle 20.51°} = 0.4275\angle -198.2° \text{ A}$$

The voltage across 10 Ω resistor, $\bar{V}_x = 10 \times \bar{I}_3 = 4.275\angle -198.2° \text{ A}$.

Two Mark Questions and Answers

1. **State the difference between a mesh and a loop.** **[AU April/May, 2013]**

   A loop is a closed path in a circuit where two nodes are not traversed twice except the initial point, which is also the final one. But in a loop, other paths can be included inside, whereas a mesh is a closed path in a circuit with no other paths inside it. In other words, it is a form of loop with no other loops inside it.

2. State Ohm's law and its limitation. **[AU April/May, 2013]**

Ohm's law states that, in a linear network, at constant temperature, the voltage across the conducting material is directly proportional to the current flowing through the material, i.e.,

$$V \propto I$$

i.e., $V = RI$

where R is the constant of proportionality and is called the *resistance* and its unit is *Ohm*.

Limitation of Ohm's law is that it is only valid at constant temperature. Also, it is not valid for non-linear elements.

3. Two capacitances C_1 and C_2 of values 10 μF and 5 μF, respectively, are connected in series. Determine the equivalent capacitance of the combination. **[AU April/May, 2014]**

Given C_1 = 10 μF and C_2 = 5 μF

Therefore, the equivalent capacitance of the combination is

$$C_{\text{eq}} = \frac{C_1 \times C_2}{C_1 + C_2} = \frac{10 \times 10^{-6} \times 5 \times 10^{-6}}{(10 \times 10^{-6}) + (5 \times 10^{-6})} = 3.33 \times 10^{-6} = 3.33\,\mu\text{F}$$

4. State Kirchoff's laws. **[AU April/May, 2012]**

Kirchhoff's Voltage Law states that the algebraic sum of the voltages around any closed loop or circuit is zero.

Kirchhoff's Current Law states that algebraic sum of currents at any node is zero, i.e., the total current entering a node is equal to the total current leaving that node.

5. Define super mesh. **[AU Nov/Dec, 2012]**

A super mesh is a mesh which can be formed by combining two meshes sharing a common source.

6. Define impedance and power factor of an alternating circuit. **[AU April/May, 2012]**

In a network, impedance is the measure of opposition to the flow of current or applied voltage. It is the extension of the concept of resistance to AC circuits. But, unlike resistance, which has only magnitude, the impedance possesses both magnitude and phase. When a DC current is supplied, the impedance cannot be distinguished from the resistance. Therefore, with a DC current, the resistance can be treated as impedance with zero phase angle.

Power factor is defined as the cosine of the phase angle difference between the voltage and current. It is denoted as *p.f.* If ϕ is the phase angle between voltage and current, then

$$p.f. = \cos\phi$$

where $\phi = |\phi_v - \phi_i|$

ϕ_v = phase angle of voltage, v and

ϕ_i = phase angle of current, i

Power factor is also defined as the ratio of active power to apparent power.

7. A bulb is rated as 230 V, 230 W. Determine the rated current and resistance of the filament. **[AU April/May, 2011]**

Given V = 230 V, P = 230 W

Since $P = \dfrac{V^2}{R}$, we get the resistance of the filament as

$$R = \frac{V^2}{P}$$

i.e., $$R = \frac{230^2}{230} = 230\ \Omega$$

Also, $P = VI$. Hence the rated current through the filament is

$$I = \frac{P}{V} = \frac{230}{230} = 1\ \text{A}$$

8. A coil having a resistance of 10 kΩ and inductance of 50 mH is connected to 10 V, 10 kHz power supply. Determine the impedance. [AU April/May, 2011]

Given $R = 10$ kΩ, $L = 50$ mH, $V = 10$ V and $f = 10$ kHz.

The inductive reactance of the coil is

$$X_L = 2\pi fL = 2\pi \times 10 \times 10^3 \times 50 \times 10^{-3} = 3141.59\ \Omega$$

Therefore, the impedance of the coil is

$$Z = R + jX_L = (10 \times 10^3) + j3141.59 = 10481.87\angle 17.44°$$

9. Is it possible to connect directly two voltage sources 2 V and 6 V in parallel? Justify your answer.

It is not possible to connect two voltage sources directly in parallel as it violates the KVL as shown in Figure UQ1.9.

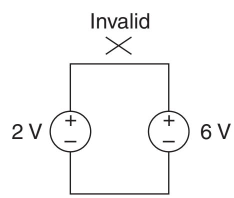


Figure UQ1.9

10. Is it possible to connect directly two current sources 5 A and 3 A in series? Justify your answer.

It is not possible to connect the current sources in the series as the KCL at node ‘a’ is violated in the example scenario shown in Figure UQ1.10.

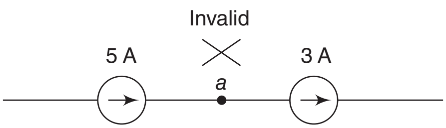


Figure UQ1.10

11. The resistance of two wires is 25 Ω when connected in series and 6 Ω when connected in parallel. Determine the resistance of each wire. [AU Nov/Dec, 2009]

Given $R_1 + R_2 = 25\ \Omega$ and $\dfrac{R_1 R_2}{R_1 + R_2} = 6\ \Omega$

Therefore, $R_1 R_2 = 6 \times 25 = 150$.

i.e., $$R_1 = \frac{150}{R_2}$$

Substituting $R_1 = \dfrac{150}{R_2}$ in $R_1 + R_2 = 25\ \Omega$, we get

$$\frac{150}{R_2} + R_2 = 25$$

i.e., $R_2^2 - 25R_2 + 150 = 0$

Solving the above equation, we get

$R_2 = 15\ \Omega$ or $10\ \Omega$.

When $R_2 = 15\ \Omega$, $R_1 = 10\ \Omega$

Similarly, when $R_2 = 10\ \Omega$, $R_2 = 15\ \Omega$

12. A series RLC circuit has $R = 25\ \Omega$, $L = 0.221$ mH and $C = 66.3\ \mu$F with frequency of 60 Hz. Determine the power factor. [AU Nov/Dec, 2009]

Given $R = 25\ \Omega$, $L = 0.221$ mH, $C = 66.3\ \mu$F and $f = 60$ Hz

The inductive reactance, $X_L = 2\pi fL = 2\pi \times 60 \times 0.221 \times 10^{-3} = 83.315\ \Omega$

The capacitive reactance, $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 60 \times 66.3 \times 10^{-6}} = 40\ \Omega$

Total reactance of the circuit is $X_T = X_L - X_C = 83.315 - 40 = 43.315\ \Omega$

Hence, the total impedance of the circuit is

$$Z = R + jX_T = 25 + j43.315 = 50.01\angle 60^\circ\ \Omega$$

Therefore, the power factor of the circuit it

$$\cos\phi = \frac{R}{Z} = \frac{25}{50.01} = 0.499$$

The circuit has lagging power factor since the inductive reactance is high when compared to capacitive reactance.

13. Determine the value of I for the diagram shown in Figure UQ1.13. [AU April/May, 2010]

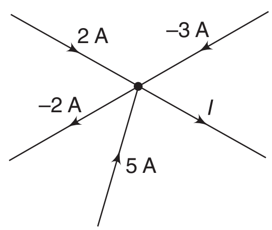


Figure UQ1.13

Applying KCL to the given node, we get

$$+2 + (-3) - (-2) + 5 - I = 0$$

i.e., $2 - 3 + 2 + 5 = I$

Therefore, $I = 6$ A

14. Determine the equivalent resistance between A and B shown in Figure UQ1.14(a). [AU April/May, 2010]

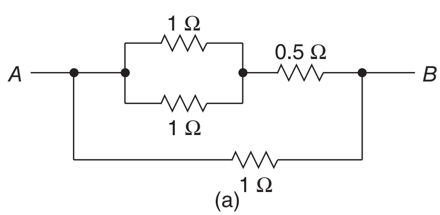


Figure UQ1.14(a)

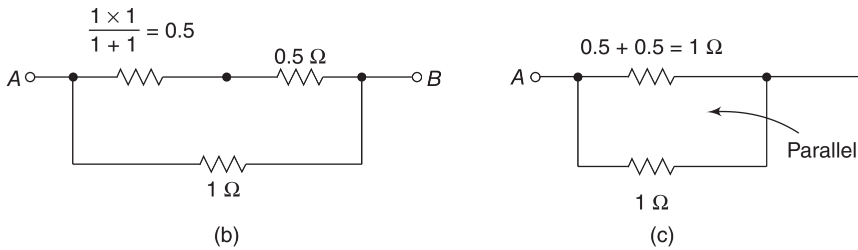


Figure UQ1.14(b) and (c)

From Figure UQ1.14, it is clear that the two 1 Ω resistances are in parallel. Therefore, its equivalent resistance is $\frac{1\times 1}{1+1}=\frac{1}{2}=0.5\,\Omega$.

Now, this 0.5 Ω is in series with 0.5 Ω. Hence, the equivalent resistance is 0.5 + 0.5 = 1 Ω.

Then this 1 Ω resistance is in parallel with 1 Ω resistance. Therefore, the total equivalent resistance between *A* and *B* is $\frac{1\times 1}{1+1}=\frac{1}{2}=0.5\,\Omega$.

15. How the resistance and inductive reactance are affected by change of frequency? [AU April/May, 2012]

The resistance is unaffected by the change of frequency because it is not dependent on frequency. The inductive reactance is directly proportional to frequency because inductive reactance, $X_L = \omega L = 2\pi fL$.

16. Draw the V-I characteristics of ideal voltage source. [AU April/May, 2009]

The V-I characteristics of ideal voltage source is shown in Figure UQ1.16.

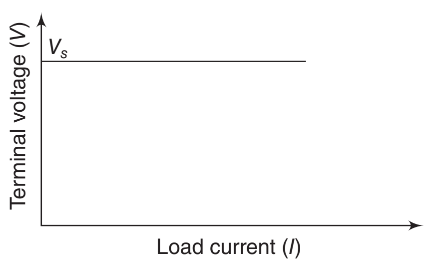


Figure UQ1.16

17. Define mesh analysis of a circuit. [AU Nov/Dec, 2012; April/May, 2011]

The mesh method analysis is a method that used in solving planar circuits for the currents at any place in the electrical circuit by writing mesh equations resulting from Kirchoff's Voltage Law.

18. Mesh current method is based on KCL and node voltage method is based on KVL – True or False. [AU Nov/Dec, 2011]

False

19. Three resistors R_1, R_2 and R_3 are connected in series. The power dissipated in R_1 is 50 W. The resistance of R_2 is 5 Ω. The current through the series circuit is 5 A. When the supply voltage is 100 V, determine the power dissipated in the circuit and the voltage across R_3.

[AU Nov/Dec, 2011]

Given $P = 50$ W, $R_2 = 5\ \Omega$, $I = 5$ A and $V = 100$ V.

The circuit based on the given data is shown in Figure UQ1.19.

It is known that, $P = I_2R_1$. Therefore, $R_1 = \dfrac{P}{I^2} = \dfrac{50}{5 \times 5} = 2\ \Omega$

The equivalent resistance of the circuit is $R_{eq} = R_1 + R_2 + R_3$.

i.e., $R_{eq} = 7 + R_3$ (1)

Using Ohm's law, we get

$$V = IR_{eq}$$

Substituting the given values, we get

$$R_{eq} = \frac{V}{I} = \frac{200}{5} = 20\ \Omega \tag{2}$$

Equating Equations (1) and (2), we get

$$R_3 = 13\ \Omega$$

Therefore, the voltage across R_3 is $IR_3 = 5 \times 13 = 65\ \Omega$. Also, the total power dissipated in the circuit is $P_T = VI = 100 \times 5 - 500$ W.

20. Define Nodal analysis of a circuit. **[AU April/May, 2012]**

The nodal method analysis is a method used in solving planar circuits for the voltages at any place in the electrical circuit by writing nodal equations resulting from Kirchoff's Current Law.

21. Determine the voltage across AB, V_{AB} across the resistor shown in Figure UQ1.21. **[AU Nov/Dec, 2011]**

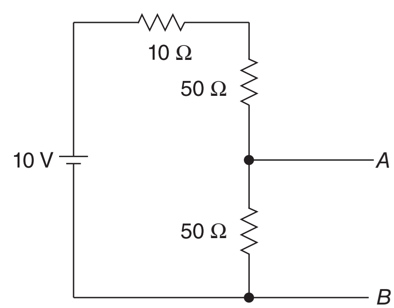


Figure UQ1.21

The equivalent resistance of the circuit is $R_{eq} = 110\ \Omega$.

Hence, the current through the circuit is

$$I = \frac{V}{R_{eq}} = \frac{10}{110} = 0.0909 \text{ A}$$

Therefore, $V_{AB} = I \times 50 = 0.0909 \times 50 = 4.5454$ V.

Review Questions

1. Discuss the applications and importance of Ohm's law with its limitations.
2. With the help of suitable examples, explain the concepts of Kirchhoff's current and voltage laws.
3. Derive the expressions for series and parallel combinations of resistors. Which combination is preferred in wiring applications? Justify your answer.
4. Compare the nodal analysis and mesh analysis methods.
5. Draw a sample circuit and explain the concepts of: (i) Circuit elements, (ii) Nodes, (iii) Junction points, (iv) Branches and (v) Meshes.

6. Explain the steps involved in applying: (i) Nodal analysis method, and (ii) Mesh analysis method for analysing the circuits.
7. What is the basis of selecting a super mesh analysis and super nodal analysis? Compare them with their practical implementation steps.
8. Explain the following AC circuit terminologies: (i) Peak value, (ii) Instantaneous value, (iii) Phase value, (iv) Average value, (v) RMS value, and (vi) Form factor.
9. With suitable waveforms, explain the need for phasor representation.
10. With the help of phasor diagrams and corresponding derivations, compare the voltage, current and impedance relationships for: (i) Resistor, (ii) Inductor, (iii) Capacitor, and (iv) RLC circuit.
11. Explain and compare the different types of powers with their corresponding equations and relationships.
12. Discuss the concepts of power factor for different load conditions such as R, L, C, R-L, and R-C.
13. How are resistance, capacitive and inductive reactances affected by the change of frequency?
14. Derive an expression for the current response of RLC series circuit with sinusoidal excitation. Comment on the phase-angle involved.
15. Discuss the characteristics of a series and parallel circuit.
16. List the advantages and applications of series and parallel circuits.
17. Why are domestic appliances connected in parallel?
18. Derive the equation for the current flowing through any resistance in a parallel circuit connected with '*n*' number of different resistances.
19. Explain the procedures to implement: (i) Current division rule and (ii) Voltage division rule.
20. State and explain Kirchhoff's laws.
21. With a suitable example, prove the validity of Kirchhoff's laws.
22. Derive the equation for the voltage across any resistance in a series circuit consisting of *n* number of different resistances.
23. Bring out the differences between: (i) Independent voltage source with dependent voltage source, and (ii) Independent current source with dependent current source.
24. State the properties of series and parallel circuits.
25. Draw and explain impedance triangle.
26. Determine the phasor representation of passive elements.
27. A certain resistor dissipates heat at the rate of 8 kJ/min. If a charge passes through the resistor at the rate of 313.5 C/min, what is the potential difference across its terminals?
28. The resistance of two wires is 25 Ω when connected in series and 6 Ω when they are joined in parallel. Calculate the resistance of each wire.
29. Find the equivalent resistance of the circuit shown in Figure Q1.30. Given $R_1 = R_4 = 10\ \Omega$ and $R_2 = R_3 = 5\ \Omega$.

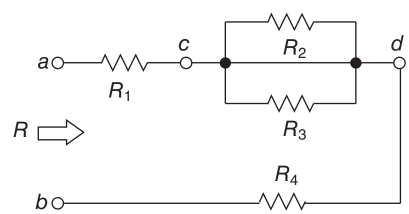


Figure Q1.30

30. When the resistor is placed across a 230 V power supply, the current through the resistor is 1 A. What is the value of additional resistor which can be connected in parallel to increase the current through the resistor to 15 A?
31. A parallel combination of three resistors of 100 Ω, 10 Ω and 5 Ω is connected to a series resistor of 30 Ω and a battery. If the internal resistance of the battery is 0.2 Ω and the potential difference across the 5 Ω resistor is 3 V, determine the voltage of the battery.
32. Find the equivalent resistance across the terminals *a* and *b* of the circuit shown in Figure Q1.32.

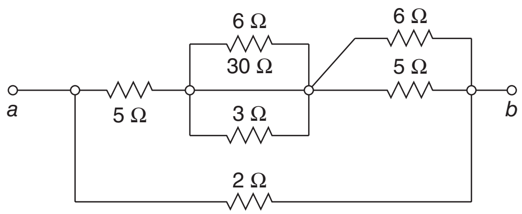


Figure Q1.32

33. Find R in the circuit of Figure Q1.33.

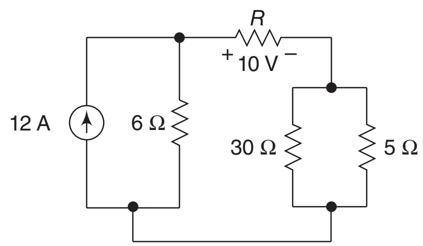


Figure Q1.33

34. For the network shown in Figure Q.1.34, find VS which makes $I_0 = 5.7$ mA. Use node voltage method.

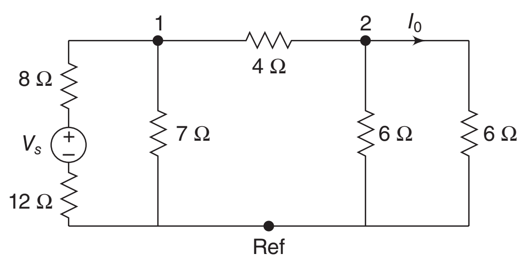


Figure Q1.34

35. Using the mesh current method, obtain the voltage V_x in the network of Figure Q1.35.

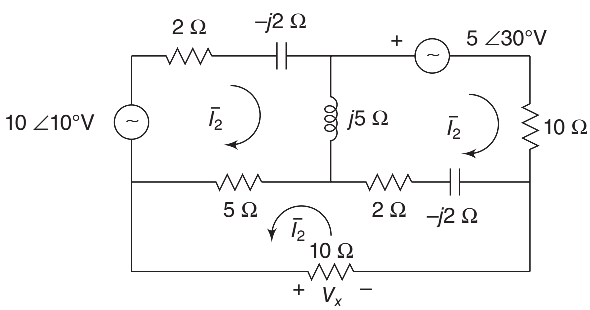


Figure Q1.35

36. In the network shown in Figure Q1.36, determine the current *I*.

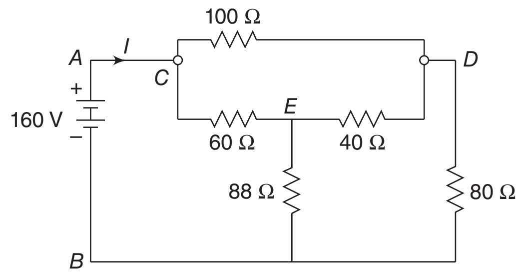


Figure Q1.36

37. For the circuit shown in Figure Q1.37, determine the total current I_T, phase angle and power factor.

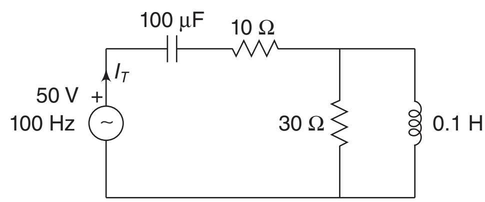


Figure Q1.37

38. Compute $\overline{V}_1$ and $\overline{V}_2$ in the circuit shown in Fig Q1.38, using nodal analysis.

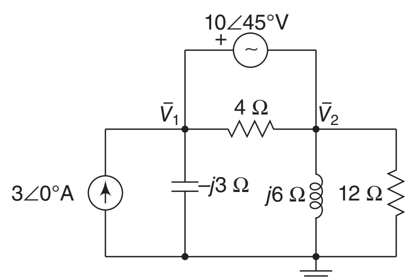


Figure Q1.38

39. In the circuit shown in Figure Q1.39, find the (i) equivalent resistance between the terminals *P* and *Q*, (ii) total current from the 240 V source, and (iii) current through the 18 Ω resistor.

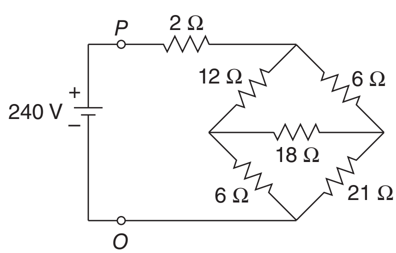


Figure Q1.39

40. For the network shown in Figure Q1.40, obtain the current ratio $\overline{I}_1/\overline{I}_2$ using mesh analysis.

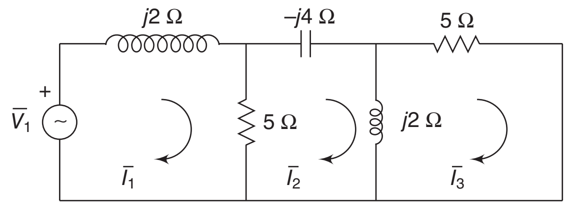


Figure Q1.40

41. A series RLC circuit has $R = 25\ \Omega$, $L = 0.221$ H and $C = 63$ mF with frequency of 60 Hz. Find the power factor.
42. A resistor R is connected in series with a parallel combination of two resistors having the values 20 Ω and 5 Ω individually. Calculate the value of R, if the total power dissipated in the circuit is 70 W when 50 V is applied to the circuit.
43. A resistance of 20 Ω is connected in parallel with an impedance of $(3 + j12)\ \Omega$. Calculate the total impedance of the circuit. Calculate different powers, if a voltage of $5\angle 60°$ V is applied to the circuit.
44. The voltages across two series-connected elements are $v_1 = 4 \sin \omega t$ V and $v_2 = 19 \sin(\omega t + 90°)$V. If the current flowing in the circuit is $(3 + j6)$ A, calculate the complex power.
45. The input power in a circuit is 350 W and the supply voltage is $v = 200 \sin (150 + 20°)$ V. If the voltage leads the current by 60°, find the complex power.
46. The resistance of two coils is 50 W if they are connected in series, and becomes 50 W if connected in parallel. Determine the individual resistance of the two coils.
47. Two coils connected in parallel across a 50 V, DC supply, derive a current of 15 A from the supply. If the power dissipation in one coil is 200 W, determine the resistance of each coil.
48. Find the current flow in the 1 Ω branch in the circuit of Figure Q1.48, using nodal analysis method.

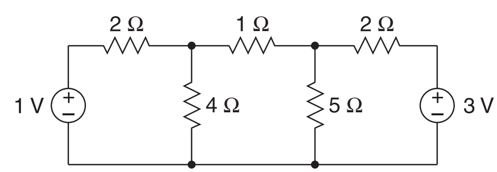


Figure Q1.48

49. Find the amount of total resistance between points A and B of the circuit shown in Figure Q1.49.

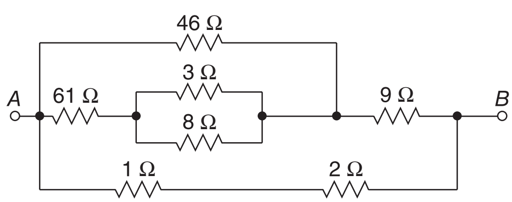


Figure Q1.49

50. Find the average power and energy transferred in one period of the sine function, having $i = 5 \sin \omega t$ mA and $v = 45 \sin \omega t$ V.
51. Derive an expression to find the phasor representation of $v = V_m \cos(\omega t + \phi)$ and $= I_m \sin \phi$.

CHAPTER 2

Network Theorems, Three-Phase Systems and Electrical Wiring

2.1 Introduction

Network theorem is a systematic method used to solve complex problems in circuit analysis. The different network theorems used to solve both AC and DC circuits are: (i) Thevenin's theorem, (ii) Norton's theorem, (iii) Superposition theorem, and (iv) Maximum power transfer theorem. In this chapter, these theorems are explained in detail with necessary examples. In the previous chapter, the circuits which are energised by two terminals called single-phase source are discussed. Generally, domestic loads are single-phase in nature. Thomas Edison had originally proposed that power can be distributed through DC networks. But Nikola Tesla and George Westinghouse strongly advocated the use of AC networks to distribute power. Almost all the electric power generated, transmitted and distributed is three-phase in nature. This chapter also deals with the generation, analysis of three phase systems with balanced and unbalanced loads. Further, the concepts related to star-delta conversion techniques, and house wiring and industrial wiring are discussed in this chapter.

2.2 Thevenin's Theorem

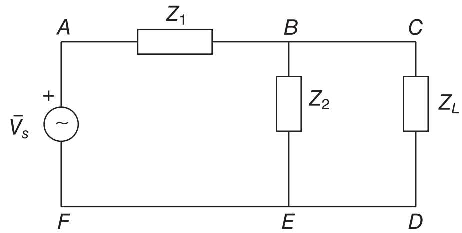


Figure 2.1(a) Circuit with source and impedances

Thevenin's theorem solves the complex circuits comprising of many sources and impedances by converting it into a simple equivalent circuit called Thevenin's equivalent circuit. Thevenin's theorem states that, "*any linear bilateral network consisting of many sources and impedances can be replaced with an equivalent circuit consisting of a Thevenin's voltage source, $\overline{V}_{\mathrm{Th}}$ in series with a Thevenin's impedance, Z_{Th} connected to load impedance, Z_L*". The equivalent circuit formed with $\overline{V}_{\mathrm{Th}}$, Z_{Th} and Z_L is called Thevenin's equivalent circuit.

Thevenin's voltage source, $\overline{V}_{\mathrm{Th}}$ is the open circuit voltage obtained by removing the load impedance Z_L. Thevenin's impedance, Z_{Th} is the equivalent impedance of the circuit obtained after short circuiting the available voltage sources and open circuiting the available current sources.

Thevenin's theorem is explained clearly using the circuit shown in Figure 2.1(a).

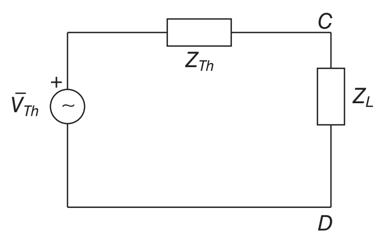


Figure 2.1(b) Thevenin's equivalent circuit

Using Thevenin's theorem, the above circuit is replaced by the Thevenin's equivalent circuit as shown in Figure 2.1 (b).

It is clear from Figure 2.1(a) and (b) that, the load impedance Z_L of the circuit gets unaffected.

The step-by-step procedure to determine $\bar{V}_{\text{Th}}$ and Z_{Th} for the circuit shown in Figure 2.1(b) is given as follows:

Step 1: Determination of $\bar{V}_{\text{Th}}$: It is obtained by removing the load impedance Z_L, i.e., by open circuiting the terminal CD as shown in Figure 2.1(c).

Using voltage division rule, we get the Thevenin's voltage as

$$\bar{V}_{\text{Th}} = \bar{V}_s \times \frac{Z_2}{Z_1 + Z_2}$$

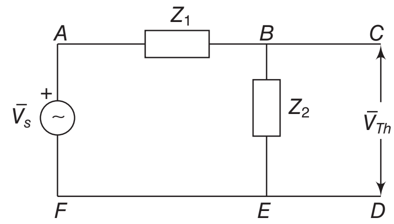


Figure 2.1(c) Determination of $\bar{V}_{Th}$

Step 2: Determination of Z_{Th}: Since a voltage source exists in the circuit, the source terminal AF is short circuited as shown in Figure 2.1(d).

Therefore, Z_{Th} is the equivalent impedance obtained from terminal CD as given by

$$Z_{\text{Th}} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

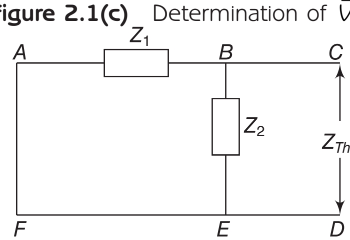


Figure 2.1(d) Determination of Z_{Th}

Step 3: Formation of Thevenin's Equivalent Circuit: Using $\bar{V}_{\text{Th}}$ and Z_{Th} obtained in the previous steps, the Thevenin's equivalent circuit is formed as shown in Figure 2.1(b).

2.2.1 Calculation of Thevenin's Impedance Z_{Th} for a Circuit with Dependent Source

When a dependent source exists in the complex circuit, the procedure to calculate Thevenin's impedance, Z_{Th} is different. Here, the dependent sources are unchanged when Z_{Th} is calculated.

The calculation of Z_{Th} for the circuit shown in Figure 2.2(a) is given as follows:

Step 1: As explained earlier, Thevenin's voltage V_{Th} is obtained by removing the load resistor R_L as shown in Figure 2.2(b).

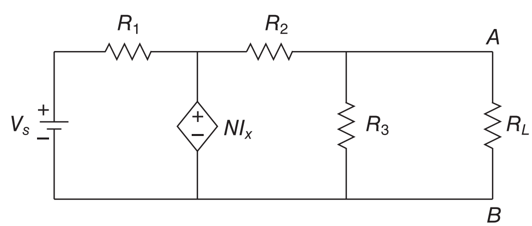


Figure 2.2(a) Circuit with independent source

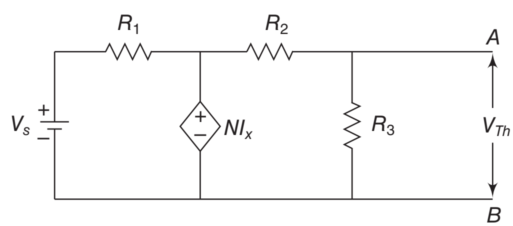


Figure 2.2(b) Determination of V_{Th}

Step 2: Similarly, the short circuit current I_N is determined by short circuiting the load terminal AB as shown in Figure 2.2(c).

Step 3: Now, Thevenin's impedance is calculated as

$$Z_{\mathrm{Th}} = R_{\mathrm{Th}} = \frac{V_{\mathrm{Th}}}{I_N}$$

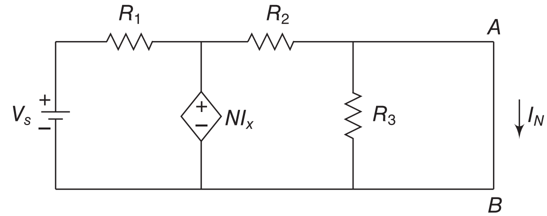


Figure 2.2(c) Determination of I_N

Alternate Method

The alternative approach that exists to determine Z_{Th} is explained as follows.

Step 1: A voltage source V_s' with a known value is connected in the place of load terminals as shown in Figure 2.2 (d).

Step 2: Using the known technique, the current I' is determined.

Step 3: Now, Thevenin's impedance is calculated as

$$Z_{\mathrm{Th}} = R_{\mathrm{Th}} = \frac{V_s'}{I'}$$

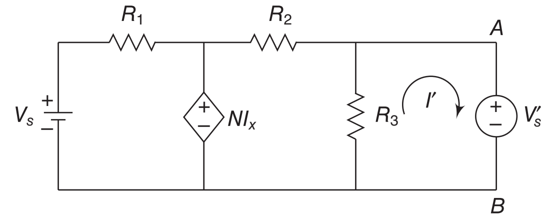


Figure 2.2(d)

Example 2.1

Using Thevenin's theorem for the circuit shown in Figure E2.1(a), determine the current through 5 Ω.

[AU April/May, 2010]

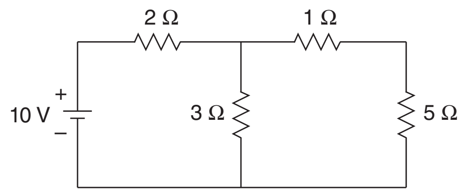


Figure E2.1(a)

Solution

Here the load resistance, $R_L = 5\ \Omega$.

(i) To determine Thevenin's voltage, V_{Th}:

The given circuit is redrawn as shown in Figure E2.1(b) by removing the load resistance, R_L.

Applying KVL to the circuit shown in Figure E2.1(b), we get

$$-2I - 3I + 10 = 0$$

Solving the above equation, we get

$$I = 2\ \mathrm{A}$$

Therefore, Thevenin's voltage for the given circuit is

$$V_{\mathrm{Th}} = V_{AB} = 3I = 6\ \mathrm{V}$$

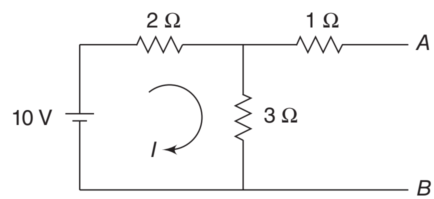


Figure E2.1(b)

(ii) To determine Thevenin's resistance, R_{Th}:

The circuit diagram to determine R_{Th} after short circuiting the voltage source is shown in Figure E2.1(c).

It is clear from Figure E2.1(c) that, 1 Ω resistor is in series with the parallel combination of the 2 Ω and 3 Ω. Therefore, the equivalent or Thevenin's resistance is given by

$$R_{\mathrm{Th}} = \frac{2 \times 3}{2+3} + 1 = \frac{6}{5} + 1 = 2.2\ \Omega$$

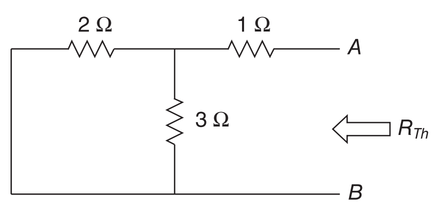


Figure E2.1(c)

(iii) To determine load current, I_L:

Thevenin's equivalent circuit with V_{Th}, R_{Th} and R_L for the given circuit is shown in Figure E2.1(d).

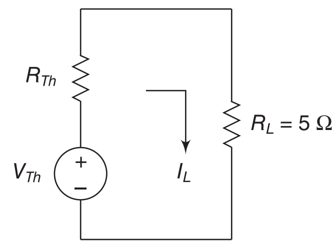


Figure E2.1(d)

Using KVL for the above circuit, we get the current through 5 Ω resistor as

$$I_L = \frac{V_{\text{Th}}}{R_L + R_{\text{Th}}}$$

Substituting the known values, we get

$$I_L = \frac{6}{5 + 2.2} = 0.8333\ \text{A}$$

Example 2.2

Determine the Thevenin's equivalent circuit of the network shown in Figure E2.2(a). Also, determine the current if load resistance of 100 Ω is connected across the terminal *AB*.

[AU Nov/Dec, 2005]

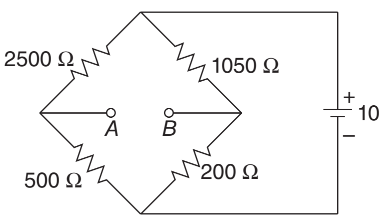


Figure E2.2(a)

Solution

Given $R_L = 100\ \Omega$

(i) To determine Thevenin's voltage, V_{Th}:

Here, the Thevenin's voltage, V_{Th} is the voltage across *AB*.

i.e., $$V_{\text{Th}} = V_{AB} = V_A - V_B \tag{1}$$

where V_A and V_B are the voltage drops across 2500 Ω and 1050 Ω respectively.

The given circuit is redrawn by indicating the different currents as shown in Figure E2.2(b) and its equivalent circuit is shown in Figure E2.2(c).

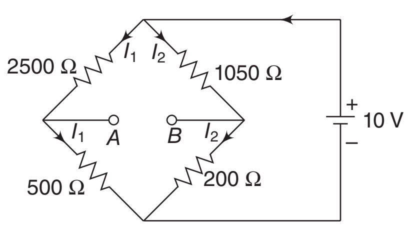


Figure E2.2(b)

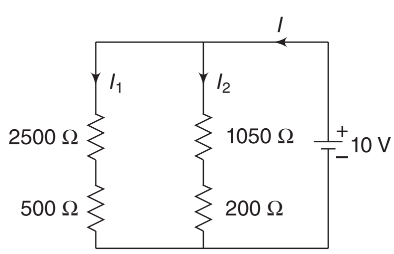


Figure E2.2(c)

Since the resistors connected in series can be added, the circuit shown in Figure E2.2(c) is redrawn as shown in Figure E2.2(d).

Since the resistors 3000 Ω and 1250 Ω are in parallel, we get

The current, $I = \dfrac{10}{3000 \| 1250} = \dfrac{10}{\left[\dfrac{3000 \times 1250}{3000 + 1250}\right]} = 0.01133 \text{ A}$

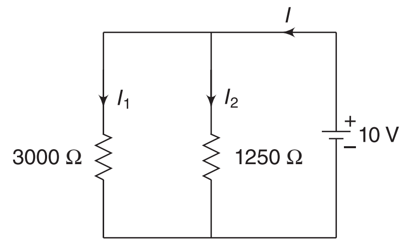


Figure E2.2(d)

Now, using the current division rule for the circuit shown in Figure E2.2(d), we get

$$I_1 = I \times \frac{1250}{(3000 + 1250)} = 0.01133 \times \frac{1250}{4250} = 0.003333 \text{ A}$$

$$I_2 = I \times \frac{3000}{(3000 + 1250)} = 0.01133 \times \frac{3000}{4250} = 0.008 \text{ A}$$

Hence, $V_A = 2500 \times 0.00333 = 8.325$ V and $V_B = 1050 \times 0.008 = 8.4$ V.

Therefore, the Thevenin's voltage is

$$V_{Th} = V_{AB} = 8.4 - 8.325 = 0.075 \text{ V}$$

(ii) To determine Thevenin's resistance, R_{Th}:

The circuit diagram and its subsequent reduced diagram to determine Thevenin's resistance, R_{Th} are shown in Figure E2.2 (e) to Figure E2.2(g).

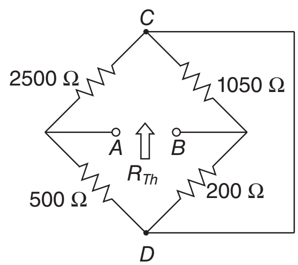


Figure E2.2(e)

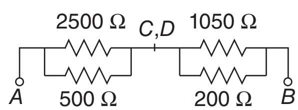


Figure E2.2(f)

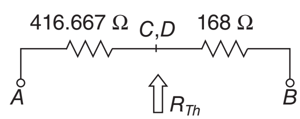


Figure E2.2(g)

Therefore, using Figure E2.2(g), we get

Thevenin's resistance, $R_{\text{Th}} = 416.667 + 168 = 584.67\ \Omega$

(iii) To calculate load current, I_L:

Thevenin's equivalent circuit with V_{Th}, R_{Th} and R_L for the given circuit is shown in Figure E2.2(h).

Using KVL for the above circuit, we get

The current through 5 Ω resistor as

$$I_L = \frac{V_{\text{Th}}}{R_L + R_{\text{Th}}}$$

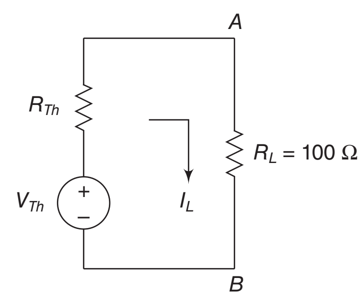


Figure E2.2(h)

Substituting the known values, we get

$$I_L = \frac{0.075}{584.67 + 100} = 1.0954 \times 10^{-4} = 109.54\ \mu\text{A}$$

Example 2.3

Using Thevenin's theorem, determine the current I_x as shown in Figure E2.3(a).

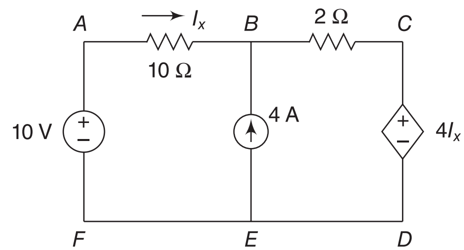


Figure E2.3(a)

Solution

Given $R_L = 10\ \Omega$.

(i) To determine Thevenin's equivalent voltage, V_{Th}:
It is known that to determine V_{Th}, the load resistance is to be removed. When the load resistance in the circuit shown in Figure E2.3(a) is removed, the current I_x becomes zero. Therefore, current dependent voltage source also becomes zero. Hence, the branch CD is short circuited. The circuit diagram after removing R_L and short circuiting the branch CD is shown in Figure E2.3(b).

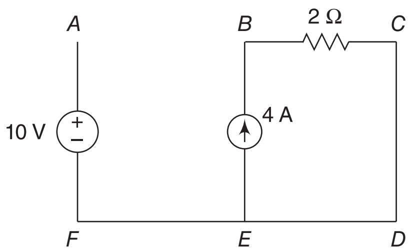


Figure E2.3(b)

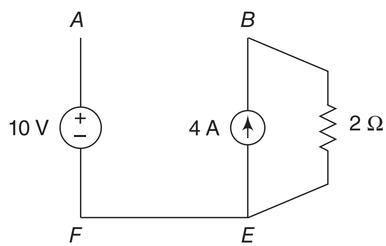


Figure E2.3(c)

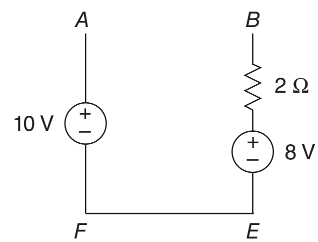


Figure E2.3(d)

Since the terminals CDE are at the same potential, the 2 Ω resistor is in parallel with the current source as shown in Figure E2.3(c). Now, using source transformation technique, this parallel combination shown in Figure E2.3(c) is converted as shown in Figure E2.3(d).

Applying KVL to the circuit shown in Figure E2.3(d), we get

$$V_{AB} + V_{BE} + V_{FA} = 0$$

$$V_{AB} + V_{2\Omega} + 8 - 10 = 0 \tag{1}$$

Since there exists an open circuit between A and B, no current flows through the loop $ABEFA$. Therefore, $V_{2\Omega} = 0 \times 2 = 0$. Hence, Equation (1) becomes

$$V_{AB} + 0 - 2 = 0$$

i.e., $$V_{AB} = 2\text{ V}$$

Therefore, Thevenin's voltage is

$$V_{\text{Th}} = 2\text{ V}.$$

(ii) To determine Thevenin's resistance, R_{Th}:
As the circuit has a dependent source, all the sources including independent sources is kept unaffected as it is shown in Figure E2.3(e).

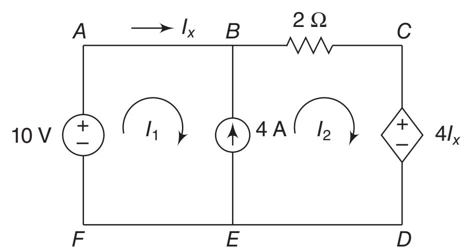


Figure E2.3(e)

In the circuit shown in Figure E2.3(e),

Short circuit current $I_N = I_x = I_1$ (2)

Hence, the current dependent voltage source is $4I_x = 4I_1$ (3)

Using Figure E2.3(e), we get

$$I_2 - I_1 = 4$$

i.e., $I_2 = 4 + I_1$ (4)

A super mesh exists in the circuit, because a current source is shared between two loops.

Therefore, applying KVL to the super mesh *ABCDEFA*, we get

$$2I_2 + 4I_x = 10 \quad (5)$$

Substituting Equation (2) and Equation (3) in Equation (4), we get

$$2(4 + I_1) + 4I_1 = 10$$

i.e., $6I_1 = 2$

Therefore, $I_1 = \frac{2}{6} = \frac{1}{3}\text{A}$

Using Equation (1), we get

short circuit current, $I_N = I_1 = \frac{1}{3}\text{A}$

Therefore, Thevenin's resistance, $R_{\text{Th}} = \frac{V_{\text{Th}}}{I_N} = \frac{2}{1/3} = 6\ \Omega$

(iii) To calculate load current, I_L:

Thevenin's equivalent circuit with V_{Th}, R_{Th} and R_L for the given circuit is shown in Figure E2.3(f).

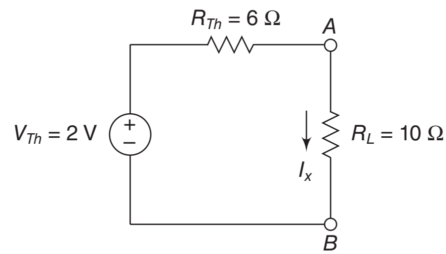


Figure E2.3(f)

Using KVL for the above circuit, we get the current through 5 Ω resistor as

$$I_x = I_L = \frac{V_{\text{Th}}}{R_L + R_{\text{Th}}}$$

Substituting the known values, we get

$$I_x = I_L = \frac{2}{6+10} = 0.125\ \text{A}$$

Example 2.4

Determine the Thevenin's equivalent circuit across the terminals *A* and *B* for the circuit shown in Figure E2.4(a)

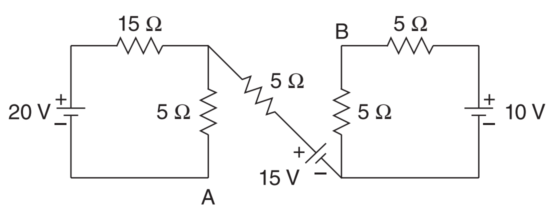


Figure E2.4(a)

Solution

The given circuit is redrawn as shown in Figure E2.4(b).

(i) To determine V_{Th}:

Applying KVL to the circuit shown in Figure E2.4(b), we get

$$(5 + 15)I_1 = 20$$

$$(5 + 5)\ I_2 = -10$$

Solving the above equations, we get

$$I_1 = \frac{20}{20} = 1\text{ A and } I_2 = -1\text{ A}$$

Therefore, $V_{\text{Th}} = V_{AB} = 15 - 5I_2 - 5I_1 = 15$ V

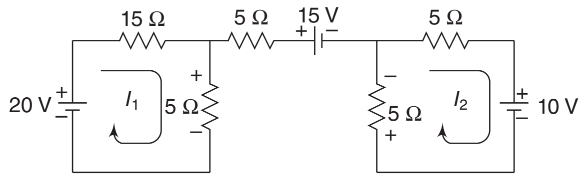


Figure E2.4(b)

(ii) To determine R_{Th}:
Once all the independent sources are deactivated, the circuit is redrawn as shown in Figure E2.4(c).

$$\text{Therefore, } R_{\text{Th}} = R_{AB} = \frac{15\times 5}{15+5} + 5 + \frac{5\times 5}{5+5} = 11.25\,\Omega$$

Hence, Thevenin's equivalent circuit for the given circuit is shown in Figure E2.4(d).

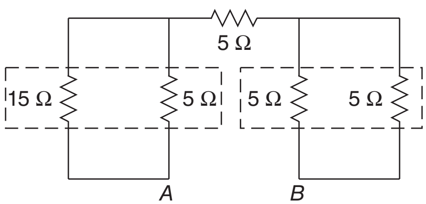


Figure E2.4(c)

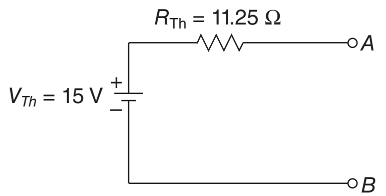


Figure E2.4(d)

Example 2.5

Find the Thevenin's equivalent circuit for the network shown in Figure E2.5(a) at terminals AB.

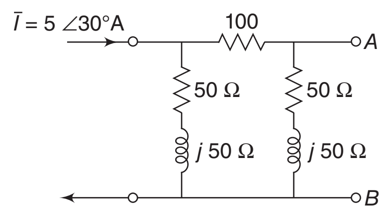


Figure E2.5(a)

Solution

By considering $\bar{I} = 5\angle 30°$ as the current source, the circuit is redrawn as shown in Figure E2.5(b).

(i) To determine $\bar{V}_{\text{Th}}$:
Using current division rule, we get

$$\bar{I}_2 = \bar{I} \times \frac{50 + j50}{50 + j50 + 100 + 50 + j50}$$

$$= 5\angle 30° \times \frac{50 + j50}{200 + j100}$$

$$= \frac{5\angle 30° \times 70.711\angle 45°}{223.606\angle 26.56°}$$

$$= 1.581\angle 48.44°\text{ A}$$

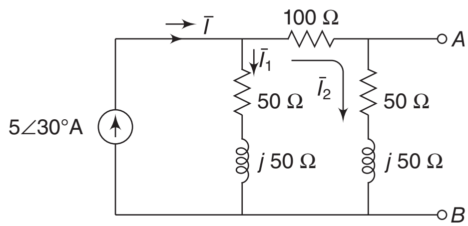


Figure E2.5(b)

Therefore, $\bar{V}_{\text{Th}} = I_2 \times (50 + j50) = 1.581\angle 48.44° \times 70.711\angle 45°$

$$= 111.8\angle 93.44°\text{ V}$$

(ii) To determine Z_{Th}:

The circuit obtained after open circuiting the current source is shown in Figure E2.5(c).

Therefore,
$$Z_{Th} = \frac{(50 + j50)(150 + j50)}{50 + j50 + 150 + j50} = \frac{70.711\angle 45° \times 158.11\angle 18.43°}{223.606\angle 26.56°}$$
$$= 50\angle 36.87°\ \Omega = 40 + j30\ \Omega$$

The Thevenin's equivalent circuit across A-B is shown in Figure E2.5(d).

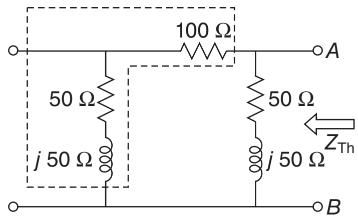


Figure E2.5(c)

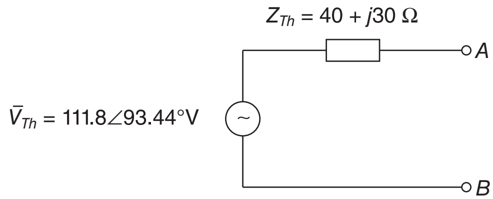


Figure E2.5(d)

2.3 Norton's Theorem

Norton's theorem states that, "*any linear bilateral circuit consisting of energy sources and impedances can be replaced with an equivalent circuit consisting of a Norton's current source, $\overline{I}_N$ in parallel with a Norton's impedance, Z_N connected to load impedance, Z_L.*" Similar to Thevenin's theorem, the Norton's theorem is used to simplify the analysis of the complex circuits.

Norton's current source, $\overline{I}_N$ is the short circuit current obtained by shorting the load impedance Z_L. Norton's impedance, Z_N determined by the same procedure which is used in determine, Z_{Th}. The equivalent circuit formed with $\overline{I}_N$, Z_N and Z_L is called Norton's equivalent circuit.

Norton's theorem is explained clearly using the circuit shown in Figure 2.3(a).

Using Norton's theorem, the above circuit is replaced by the Norton's equivalent circuit as shown in Figure 2.3 (b).

The step by step procedure to determine $\overline{I}_N$ and Z_N for the circuit shown in Figure 2.3(a) is given below.

Step 1: Determination of $\overline{I}_N$: It is obtained by short circuiting the load impedance Z_L at terminal CF as shown in Figure 2.3(c).

Since entire current flows through the short circuited branch CF, no current flows through Z_2. Therefore,

$$\overline{I}_N = \frac{\overline{V}_s}{Z_1}$$

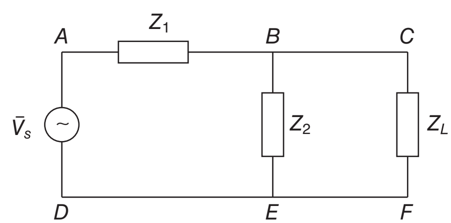


Figure 2.3(a) Circuit with source and impedances

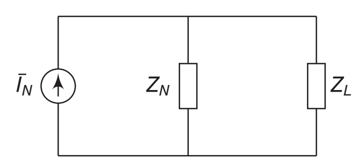


Figure 2.3(b) Norton's equivalent circuit

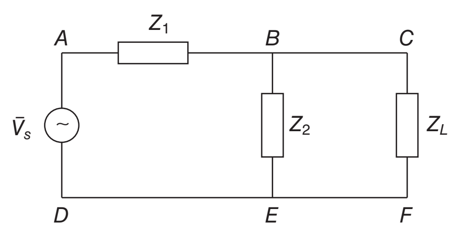


Figure 2.3(c) Short circuiting the load

Step 2: Determination of Z_N:

Since Z_N is determined using the same procedure, we get

$$Z_N = Z_{\text{Th}} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Step 3: Formation of Norton's equivalent circuit:

Using $\bar{I}_N$ and Z_N obtained in the previous steps, the Norton's equivalent circuit is formed as shown in Figure 2.3(b).

It is to be noted that, if dependent sources exist in the circuit, the same procedure followed in Thevenin's theorem is followed to determine Z_N.

Also, using source transformation technique, Thevenin's equivalent circuit can be transformed into Norton's equivalent circuit and vice versa as shown in Figure 2.3(d).

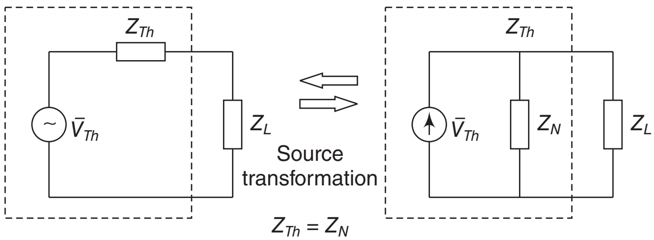


Figure 2.3(d) Transformation between Thevenin's and Norton's equivalent circuit

Example 2.6

Determine the current through *AB* in the network shown in Figure E2.6(a) using Norton's theorem. **[AU April/May, 2005]**

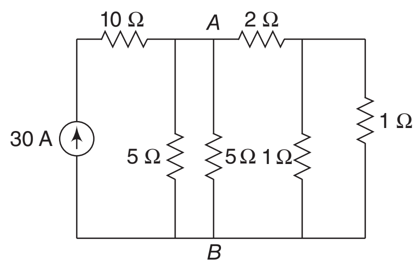


Figure E2.6(a)

Solution

Given $R_L = 5\ \Omega$.

(i) To find I_N:

The branch *AB* across which R_L is connected is short circuited as shown in Figure E2.6(b).

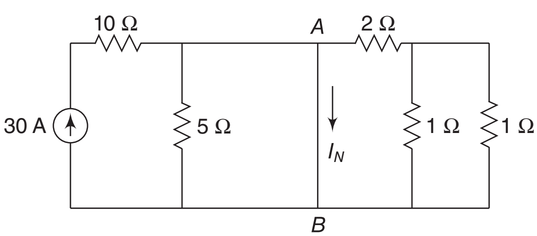


Figure E2.6(b)

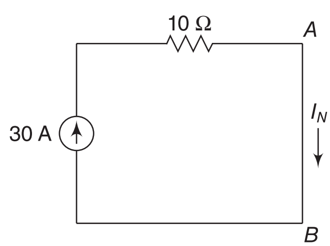


Figure E2.6(c)

Since all the current flows through the branch *AB*, the above circuit is redrawn as shown in Figure E2.6(c). Hence, Norton's current, $I_N = 30$ A.

(ii) To find R_N:

Since a current source exists in the given circuit, it is open circuited and is redrawn as shown in Figure E2.6(d) and its simplified circuit is shown in Figure E2.6(e).

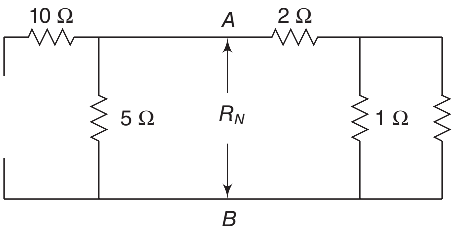


Figure E2.6(d)

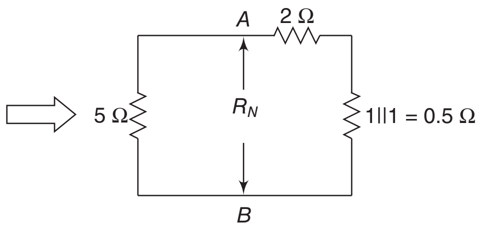


Figure E2.6(e)

Again the circuit shown in Figure E2.6(e) is simplified as shown in Figure E2.6(f).

Hence, $R_N = 2.5 \,||\, 5 = \dfrac{2.5 \times 5}{2.5 + 5} = 1.67\ \Omega$

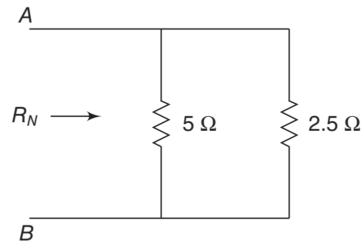


Figure E2.6(f)

(iii) To determine load current, I_L:

The Norton's equivalent circuit using I_N and R_N obtained in the previous steps is shown in Figure E2.6(g).

Using current division rule, the load current is given by

$$I_L = I_N \times \frac{R_N}{R_N + R_L} = \frac{30 \times 1.67}{(1.67 + 5)} = 7.5\text{ A}$$

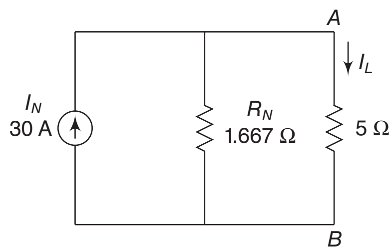


Figure E2.6(g)

Example 2.7

Obtain the Norton's equivalent for the circuit shown in Figure E2.7(a) across the terminal *AB*.

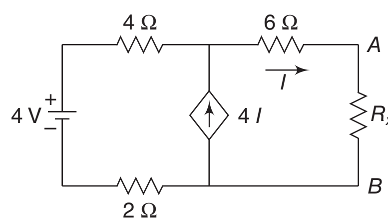


Figure E2.7(a)

Solution

Given $R_L = R_x$

(i) To find I_N:

The branch *AB* across which R_L is connected is short circuited as s hown in Figure E2.7(b). Also, the current flowing through 4 Ω and 2 Ω resistor is considered as I_x.

Applying KCL at node-C, we get

$$4I = I + I_x$$

i.e., $$I_x = 3I \tag{1}$$

Applying KVL to the super mesh *ECABDFE*, we get

$$-4I_x + 6I - 2I_x = 4 \tag{2}$$

Substituting Equation (1) in Equation (2), we get

$$-6(3I) + 6I = 4$$

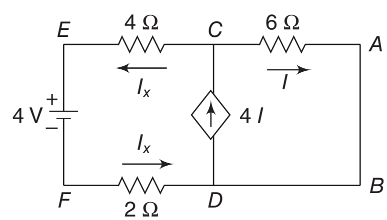


Figure E2.7(b)

i.e., $I = \frac{-4}{12} = \frac{-1}{3}$ A

Therefore, $I_N = I = \frac{-1}{3}$ A

Since the result is in negative, the direction of current is opposite to assumed direction.

(ii) To find R_N:

Since the given circuit has a dependent source, R_N is determined as

$$R_N = \frac{V_{\text{Th}}}{I_N}$$

To find V_{Th}:

The circuit is open circuited at the terminals AB as shown in Figure E2.7(c).

Since it is an open circuit, no current flows through the resistors 4 Ω and 6 Ω, and the voltage drops across them are zero. Also for the dependent current source, since $4I = 4 \times 0 = 0$, it is open circuited as shown in Figure E2.7(c).

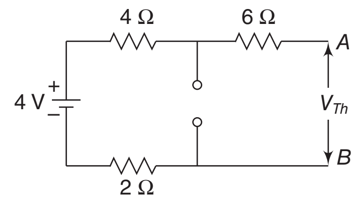


Figure E2.7(c)

Hence, $V_{\text{Th}} = 4$ V

Therefore, $R_N = \frac{4}{1/3} = 12\ \Omega$

Hence, the Norton's equivalent circuit is shown in Figure E2.7(d).

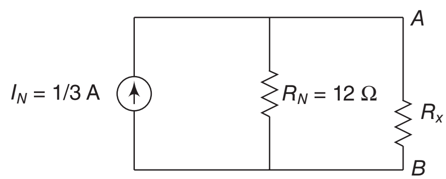


Figure E2.7(d)

Example 2.8

Determine the load current $\bar{I}_L$, using Norton's theorem, for the circuit shown in Figure E2.8(a).

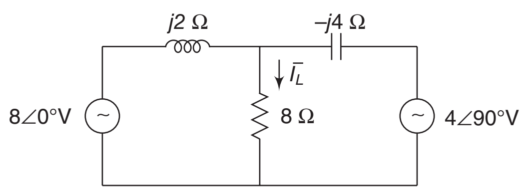


Figure E2.8(a)

Solution

Given $R_L = 8\ \Omega$

(i) To determine $\bar{I}_N$:

The given circuit diagram is redrawn as shown in Figure E2.8(b) by short circuiting the load resistor 8 Ω. Also, the currents $\bar{I}_1$ and $\bar{I}_2$ are assumed to flow in the direction as shown in Figure E2.8(b).

From Figure E2.8(b), we get

$$\bar{I}_1 = \frac{8\angle 0°}{j2} = 4\angle -90°\ \text{A} = -j4\ \text{A}$$

and $$\bar{I}_2 = \frac{4\angle 90°}{-j4} = \frac{4\angle 90°}{4\angle -90°} = 1\angle 180° = -1\ \text{A}$$

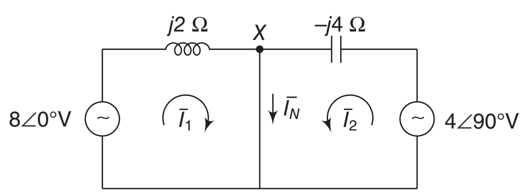


Figure E2.8(b)

Applying KCL at node "X", we get

$$\bar{I}_N = \bar{I}_1 + \bar{I}_2$$

Substituting the known values in the above equation, we get

$$\bar{I}_N = -1 - j4 \text{ A} = 4.123\angle{-104.04°} \text{ A}$$

(ii) To determine Z_N:
The load resistor is opened and the voltage source is short circuited as shown in Figure E2.8(c).

Therefore, $$Z_N = \frac{j2 \times -j4}{j2 - j4} = \frac{-j^2 8}{-j2} = \frac{-8}{j2} = -4\angle{-90°} = j4\ \Omega$$

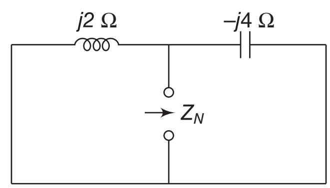


Figure E2.8(c)

(c) To determine load current, I_L:
The Norton's equivalent circuit is shown in Figure E2.8(d).
Using current division rule, we get

$$\bar{I}_L = \frac{\bar{I}_N Z_N}{Z_N + R_L} = \frac{4.123\angle{-104.04°}(j4)}{j4 + 8} = 1.844\angle{-40.61°} \text{ A}$$

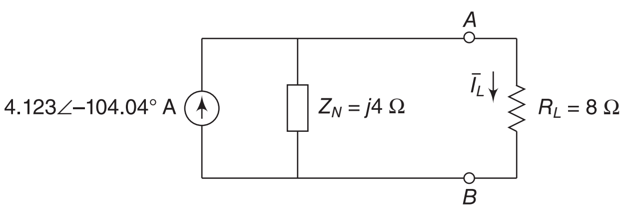


Figure E2.8(d)

2.4 Superposition Theorem

Superposition theorem states that, "*if a linear circuit consists of more than one independent sources, then the current flowing through any part of the circuit is equal to the algebraic sum of the currents produced by each independent source when it is considered separately.*" Superposition refers to the superposition of responses due to individual sources. In this theorem, a current source is replaced with an open circuit and a voltage source is replaced with a short circuit. If a circuit has a dependent source, then it is to be considered when each other source is considered.

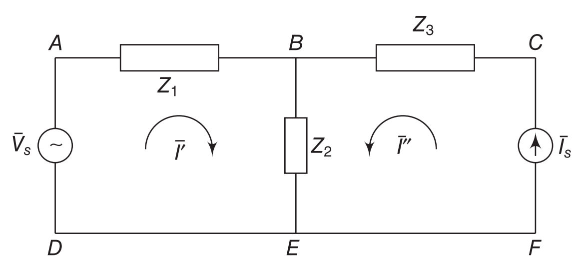


Figure 2.4 Complex circuit with two independent sources

Superposition theorem is explained clearly using the complex circuit shown in Figure 2.4.

Here, the circuit has two independent sources $\bar{V}_s$ and $\bar{I}_s$. The objective is to determine the current flowing through Z_2 using superposition theorem. Therefore, the current flowing through Z_2 has two components.

i.e., $$\bar{I} = \bar{I}' + \bar{I}''$$

where $\bar{I}'$ is the current flowing through Z_2 when source $\bar{V}_s$ alone is considered with open circuiting $\bar{I}_s$ and $\bar{I}''$ is the current flowing through Z_2 when source $\bar{I}_s$ alone is considered with short circuiting $\bar{V}_s$.

Example 2.9

Using superposition theorem, determine the voltage across 20 Ω resistance in the circuit shown in Figure E2.9(a).

[AU April/May, 2011]

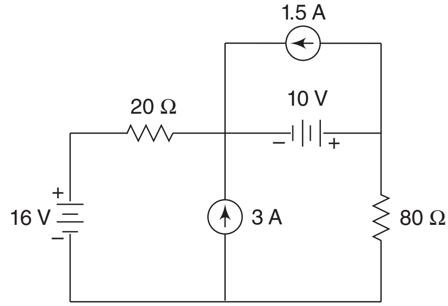


Figure E2.9(a)

Solution

Since there are four sources in the given circuit, the voltage across 20 Ω resistance is given by

$$V_T = V_1 + V_2 + V_3 + V_4 \qquad (1)$$

where V_1 is the voltage across 20 Ω resistance when 16 V source alone is considered, V_2 the voltage across 20 Ω resistance when 10 V source alone is considered, V_3 is the voltage across 20 Ω resistance when 3 A source alone considered and V_4 is the voltage across 20 Ω resistance when 1.5 A source alone is considered.

(i) To determine V_1:

Here 16 V source alone is considered. Therefore, the other two current sources are open circuited and 10 V source is short circuited. Hence, the circuit is redrawn as shown in Figure E2.9(b).

Using KVL, the current through 20 Ω resistance is given by

$$I' = \frac{16}{20+80} = 0.16 \text{ A}$$

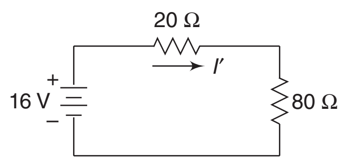


Figure E2.9(b)

Therefore, voltage across 20 Ω resistance due to 16 V source is

$$V_1 = I'R = 0.16 \times 20 = 3.2 \text{ V} \qquad (2)$$

(ii) To determine V_2:

Here 10 V source alone is considered. Therefore, the other two current sources are open circuited and 16 V source is short circuited. Hence, the circuit is redrawn as shown in Figure E2.9(c).

Using KVL, the current through 20 Ω resistance is given by

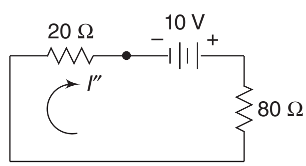


Figure E2.9(c)

$$I'' = \frac{10}{20+80} = 0.1 \text{ A}$$

Therefore, voltage across 20 Ω resistance due to 10 V source is

$$V_2 = I''R = 0.1 \times 20 = 2 \text{ V} \qquad (3)$$

(iii) To determine V_3:

Here, 3 A current soure alone is considered. Therefore, the other two voltage sources are short circuited and the 1.5 A current source is open circuited. Hence, the circuit is redrawn as shown in Figure E2.9(d).

Using current division rule, the current through 20 Ω resistance is given by

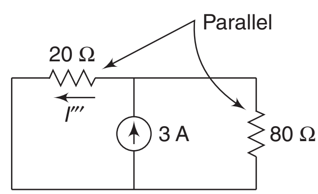


Figure E2.9(d)

$$I''' = 3 \times \frac{80}{20+80} = 2.4 \text{ A}$$

Therefore, voltage across 20 Ω resistance due to 3 A source is

$$V_3 = I''R = 2.4 \times 20 = 48 \text{ V} \quad (4)$$

(iv) To determine V_4:
Here, 1.5 A current soure alone is considered. Therefore, the other two voltage sources are short circuited and the 3 A current source is open circuited. Hence, the circuit is redrawn as shown in Figure E2.9(e).

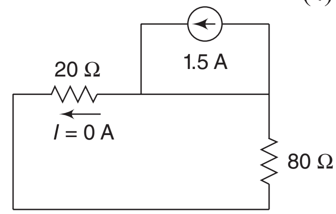


Figure E2.9(e)

Using KVL, the current through 20 Ω resistance is $I'''' = 0$ A

Therefore, voltage across 20 Ω resistance due to 1.5 A source is

$$V_4 = 0 \text{ V} \quad (5)$$

Substituting Equations (2) to (5) in Equation (1), we get

$$V_T = 3.2 + 2 - 48 + 0 = -42.8 \text{ V}$$

In the above equation, the negative sign is introduced in V_3 since the direction of current assumed in that case is different.

Example 2.10

Determine the current through 5 Ω resistors using superposition theorem for the circuit shown in Figure E2.10(a).

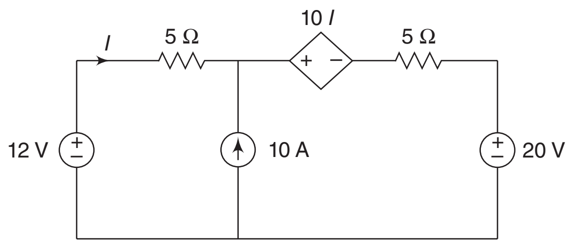


Figure E2.10(a)

Solution

Since there are three sources in the given circuit, the current through 5 Ω resistance is given by

$$I_{T1} = I_1' + I_2' + I_3' \quad (1)$$

where I_1' is the current through 5 Ω resistance when 12 V source alone is considered, I_2' the current through 5 Ω resistance when 20 V source alone is considered and I_3' is the current through 5 Ω resistance when 10 A source alone is considered.

Similarly, the current through 5 Ω resistance placed between dependent and independent voltage sources is given by

$$I_{T2} = I_1'' + I_2'' + I_3'' \quad (2)$$

where I_1'' is the current through 5 Ω resistance when 12 V source alone is considered, I_2'' is the current through 5 Ω resistance when 20 V source alone is considered and I_3'' is the current through 5 Ω resistance when 10 A source alone is considered.

(i) To determine I_1' and I_1'' :
Here, 12 V source alone is considered. Therefore, the other two sources, i.e., voltage and current sources are short and open circuited respectively. Hence, the circuit is redrawn as shown in Figure E2.10(b).

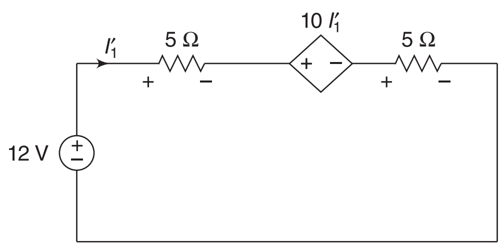


Figure E2.10(b)

Using KVL, we get

$$5I_1' + 10I_1' + 5I_1' = 12$$

Solving the above equation, we get

$$I_1' = I_1'' = \frac{12}{20} = 0.6 \text{ A} \tag{3}$$

(ii) To determine I_2' and I_2'' :

Here, 10 A source alone is considered. Therefore, the other two voltage sources are short circuited respectively. Hence, the circuit is redrawn as shown in Figure E2.10(c).

From Figure E2.10(c), we get

$$I_2 - I_1 = 10$$

i.e., $$I_2 = I_1 + 10 \tag{4}$$

Using KVL to the super mesh, we get

$$5I_1 + 10I_1 + 5I_2 = 0$$

Substituting Equation (4) in the above equation, we get

$$I_1 = I_2' = \frac{-50}{20} = -2.5 \text{ A} \tag{5}$$

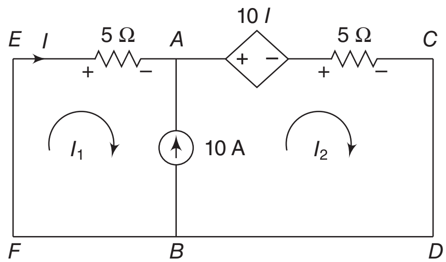


Figure E2.10(c)

Substituting I_1 = –2.5 A in Equation (4), we get

$$I_2 = I_2'' = -2.5 + 10 = 7.5 \tag{6}$$

(iii) To determine I_3' and I_3''

Here, 20 V voltage source alone is considered. Therefore, the other sources i.e., voltage and current sources are short and open circuited respectively and the circuit is redrawn as shown in Figure E2.10(d)

Using KVL, we get

$$5I_3' + 10I_3' + 5I_3' = -20$$

Solving the above equation, we get

$$I_3' = I_3'' = -\frac{20}{20} = -1 \text{ A} \tag{7}$$

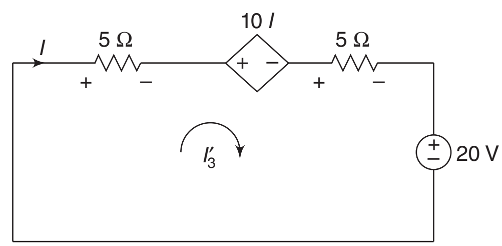


Figure E2.10(d)

Therefore,

$$I_{T1} = I_1' + I_2' + I_3' = -2.9 \text{ A}$$

and $$I_{T2} = I_1'' + I_2'' + I_3'' = 7.1 \text{ A}$$

Example 2.11

Using superposition theorem, determine the current in the 4 Ω resistor in the network shown in Figure E2.11(a).

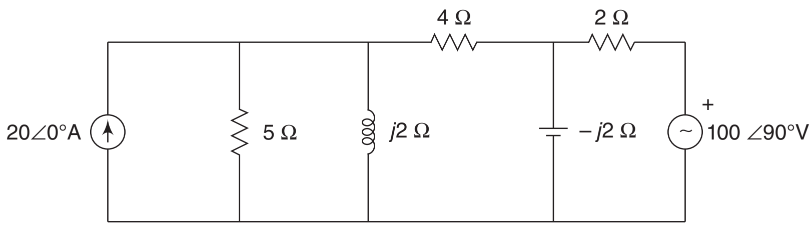


Figure E2.11(a)

Solution

Since there are two sources in the given circuit, the current through 4 Ω resistance is given by

$$I_T = I_1' + I_2' \tag{1}$$

where I_1' is the current through 4 Ω resistance when 20 A source alone is considered and I_2' is the current through 4 Ω resistance when 100V source alone is considered.

(i) To determine I_1':

Here, 20 A source alone is considered. Therefore, the voltage source is short circuited. Hence, the circuit is redrawn as shown in Figure E2.11(b).

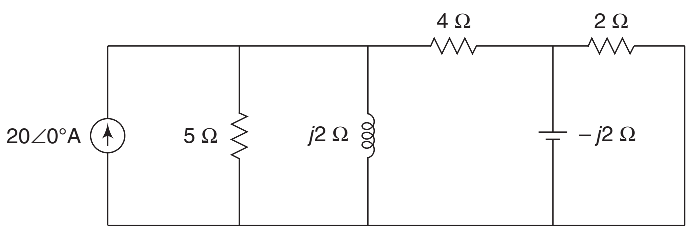


Figure E2.11(b)

The above circuit can be replaced with the circuit as shown in Figure E2.11(c).

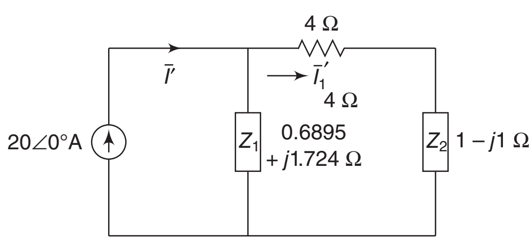


Figure E2.11(c)

Here, Z_1 is the equivalent impedance of the parallel combination of 5 Ω and $j2$ Ω.

i.e.,
$$Z_1 = \frac{5 \times j2}{5 + j2} = \frac{j10}{5.3851\angle 21.801°} = \frac{10\angle 90°}{5.3851\angle 21.801°}$$
$$= 1.8569\angle 68.2°\ \Omega = 0.6895 + j1.724\ \Omega$$

Similarly, Z_2 is the equivalent impedance of parallel combination of $-j2$ Ω and 2 Ω.

i.e.,
$$Z_2 = \frac{2 \times (-j2)}{2 - j2} = \frac{4\angle -90°}{2.8284\angle -45°} = 1.4142\angle -45°\ \Omega$$
$$= 1 - j1\ \Omega$$

Now, using current division rule, we get

$$\bar{I}_1' = \bar{I}' \times \frac{Z_1}{Z_1 + 4 + Z_2}$$
$$= 20\angle 0° \times \frac{1.8569\angle 68.2°}{0.6895 + j1.724 + 4 + 1 - j1}$$
$$= \frac{20\angle 0° \times 1.8569\angle 68.2°}{5.6895 + j0.724}$$
$$= \frac{37.138\angle 68.2°}{5.7333\angle 7.254°} = 6.475\angle 60.94°\ \text{A} = 3.14 + j5.661\ \text{A} \tag{2}$$

(ii) To determine I_2':

Here 100∠90° V source alone is considered. Therefore, the current source is open circuited and the circuit is redrawn as shown in Figure E2.11(d).

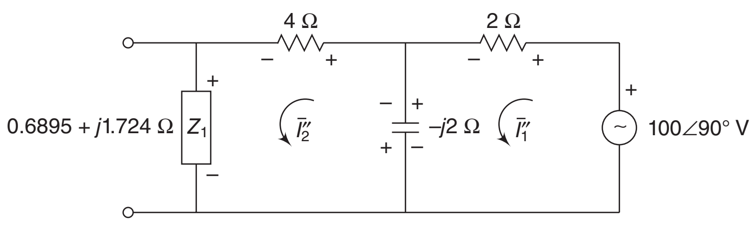


Figure E2.11(d)

Using KVL, we get

$$2\bar{I}_1'' - j2[\bar{I}_1'' - \bar{I}_2''] = 100\angle 90° \text{ i.e., } [2 - j2]\bar{I}_1'' + j2\bar{I}_2'' = 100\angle 90° \tag{3}$$

$$4\bar{I}_2'' + Z_1\bar{I}_2'' + (-j2)(\bar{I}_2'' - \bar{I}_1'') = 0 \text{ i.e., } j2\bar{I}_1'' + (4.6895 - j0.276)\bar{I}_2'' = 0 \tag{4}$$

Representing Equation (3) and Equation (4) in matrix form, we get

$$\begin{bmatrix} 2 - j2 & j2 \\ j2 & 4.6895 - j0.276 \end{bmatrix} \begin{bmatrix} \bar{I}_1'' \\ \bar{I}_2'' \end{bmatrix} = \begin{bmatrix} 100\angle 90° \\ 0 \end{bmatrix}$$

Applying Cramer's rule, we get

$$\Delta = \begin{bmatrix} 2-j2 & j2 \\ j2 & 4.6895-j0.276 \end{bmatrix}$$
$$= 9.379 - j0.552 - j9.379 - 0.552 + 4$$
$$= 12.827 - j9.931 = 16.222\angle -37.74°$$
$$\Delta_2 = \begin{vmatrix} 2-j2 & 100\angle 90° \\ j2 & 0 \end{vmatrix} = (100\angle 90°)(-j2)$$
$$= (100\angle 90°)(2\angle -90°) = 200\angle 0°$$

Therefore, $$\bar{I}_2'' = \frac{\Delta_2}{\Delta} = \frac{200\angle 0°}{16.222\angle -37.74} = 12.329\angle 37.74° = 9.748 + j7.547\text{A}$$

Hence using Equation (1), we get

$$I_T = (3.14 + j5.661) - (9.748 + j7.547) = -6.608 - j1.886 \text{ A} = 6.872\angle -164.07° \text{ A}$$

2.5 Maximum Power Transfer Theorem

Maximum power transfer theorem states that, "*Maximum power will be transferred to the load impedance Z_L by a circuit, if the load impedance Z_L is the conjugate of the circuit impedance Z_S when viewed from the output terminals.*" This theorem is also known as "Jacobi's law".

For example, consider the circuit shown in Figure 2.5 to explain the maximum power transfer theorem. This circuit has variable load impedance Z_L connected to its output terminal *BD* and is given by

$$Z_L = R_L + jX_L$$

When Z_L is removed, the circuit impedance when viewed from output terminal is

$$Z_s = Z_{\text{Th}} = R_s + jX_s$$

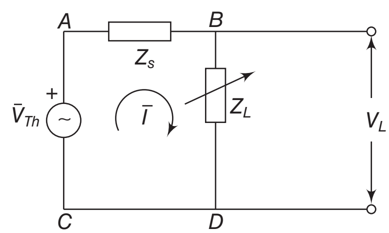


Figure 2.5

The current flowing in the circuit, I is

$$\bar{I} = \frac{\bar{V}_{\text{Th}}}{Z_s + Z_L} = \frac{\bar{V}_{\text{Th}}}{R_s + jX_s + R_L + jX_L}$$

Therefore, the power delivered to the load impedance is

$$P = |\bar{I}|^2 R_L = \frac{|\bar{V}_{\text{Th}}|^2 R_L}{(R_s + R_L)^2 + (X_s + X_L)^2} \tag{2.1}$$

Maximum power is transferred from the source to the load when the differentiation of the power with respect to load impedance is zero, i.e.,

$$\frac{dP}{dZ_L} = 0$$

or, $$\frac{\partial P}{\partial X_L} = 0 \tag{2.2}$$

and $$\frac{\partial P}{\partial R_L} = 0 \tag{2.3}$$

Considering the partial differentiation of power with respect to reactance X_L, we get

$$\frac{\partial P}{\partial X_L} = \frac{-2|\bar{V}_{\text{Th}}|^2 R_L(X_s + X_L)}{[(R_s + R_L)^2 + (X_s + X_L)^2]^2} = 0$$

Therefore, $X_s + X_L = 0$

i.e., $X_s = -X_L$

Hence the maximum power is delivered to the load when the load reactance is equal and opposite to the source reactance. Substituting this condition in Equation (2.1), we get

$$P = \frac{|\bar{V}_{\text{Th}}|^2 R_L}{(R_s + R_L)^2}$$

Similarly, when partial differentiation of power with respect to resistance R_L is considered, we get

$$\frac{\partial P}{\partial R_L} = \frac{|\bar{V}_{\text{Th}}|^2 (R_s + R_L)^2 - 2|\bar{V}_{\text{Th}}|^2 R_L(R_s + R_L)}{(R_s + R_L)^4} = 0$$

or $|\bar{V}_{\text{Th}}|^2 (R_s + R_L) - 2|\bar{V}_{\text{Th}}|^2 R_L = 0$

Therefore, $R_s = R_L$

Hence, maximum power will be transferred to the load impedance Z_L by a circuit, if the impedance of Z_L is the conjugate of the circuit impedance Z_s.

i.e., when $Z_L = Z_s^*$, i.e., $R_L + jX_L = R_s - jX_s$

Therefore, the net equivalent impedance of the circuit is expressed as

$$R_L + jX_L + R_L - jX_L = 2R_L$$

Power delivered to the load, $P = |\bar{I}|^2 R \text{ (or) } \dfrac{|\bar{V}|^2}{R}$

Substituting $R = 2R_L$ and $|\bar{I}| = \dfrac{|\bar{V}_{\text{Th}}|}{2R_L}$ in the above equation, we get

The maximum power transferred to the load, $P_{\max} = \dfrac{|\bar{V}_{\text{Th}}|^2}{4R_L}$

Example 2.12

Find the value of R_L at which maximum power is transferred to R_L and hence the maximum power transferred to R_L in the circuit shown in Figure E2.12(a). **[AU April/May, 2010]**

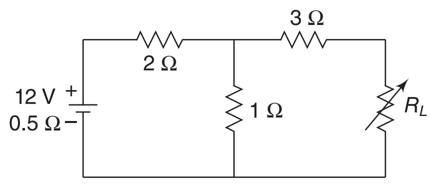


Figure E2.12(a)

Solution

For the maximum power to be transferred, $R_L = R_{\text{Th}}$

The 12 V voltage source has an inherent series internal resistance of 0.5 Ω.

The Thevenin's equivalent resistance is found by shorting the voltage source as shown in Figure E2.12(b), (c) and (d).

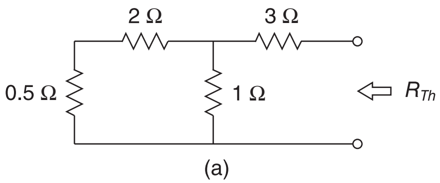


Figure E2.12(b)

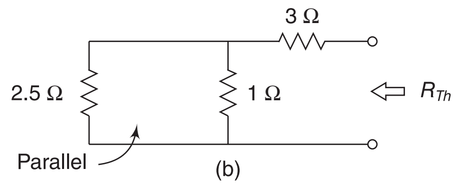


Figure E2.12(c)

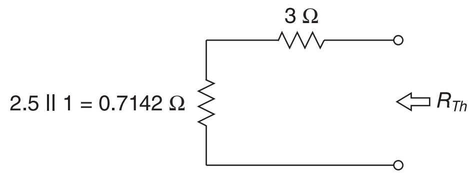


Figure E2.12(d)

Therefore, $R_{\text{Th}} = 3 + 0.7142 = 3.7142\ \Omega = R_L$

To determine the maximum power, first determine I applying KVL to the equivalent circuit shown in Figure E2.12(e) and then determine the open circuit voltage V_{Th} removing R_L.

$$0.5I + 2I + I = 12$$

Therefore, $I = 3.4285$ A

and $V_{\text{Th}} = 3.4285 \times 1 = 3.4285$ V

Hence, $$P_{\max} = \frac{V_{\text{Th}}^2}{4R_L} = \frac{(3.4285)^2}{4 \times 3.7142} = 0.7912 \text{ W}$$

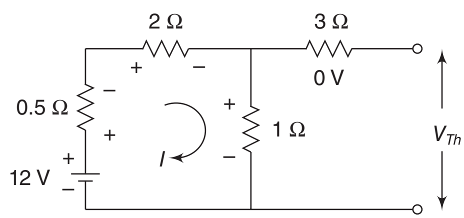


Figure E2.12(e)

Example 2.13

Determine the maximum power delivered to the load impedance Z_L in the circuit shown in Figure E2.13(a).

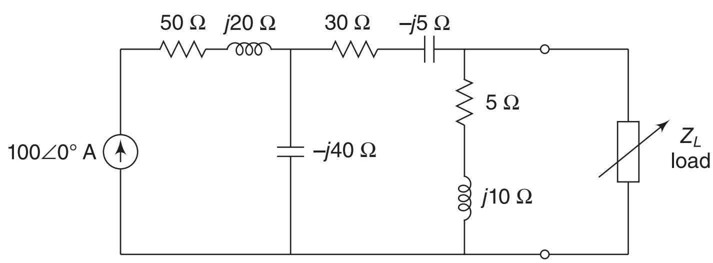


Figure E2.13(a)

Solution

The maximum power transfer occurs when $Z_L = Z_S^*$. To calculate Z_s, the voltage source is replaced by a short circuit and Z_L is removed as shown in Figure E2.13(b).

Therefore,
$$Z_S = \frac{(5+j10)\times(30-j45)}{5+j10+30-j45}$$
$$= \frac{11.18\angle 63.43^\circ \times 54.08\angle -56.31^\circ}{49.50\angle -45^\circ}$$
$$= 12.21\angle 52.17^\circ$$
$$= 7.497 + j9.64\ \Omega$$

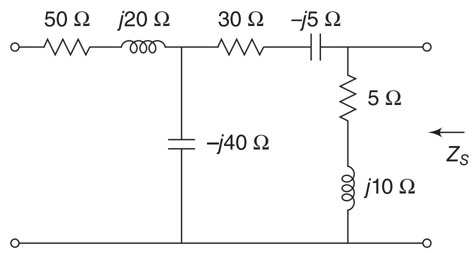


Figure E2.13(b)

Hence, for maximum power transfer,
$$Z_L = Z_s^* = 7.494 - j9.64\ \Omega$$

To find the value of maximum power $P_{\max}$, V_{Th} is calculated by open circuiting Z_L as shown in Figure E2.13(c).

According to current division rule,
$$\bar{I}_1 = 100\angle 0^\circ \times \frac{-j40}{-j40+30-j5+5+j10}$$
$$= \frac{100\angle 0^\circ \times 40\angle -90^\circ}{35-j35}$$
$$= \frac{4000\angle -90^\circ}{49.497\angle -45^\circ}$$
$$= 80.812\angle -45^\circ\ \text{A}$$

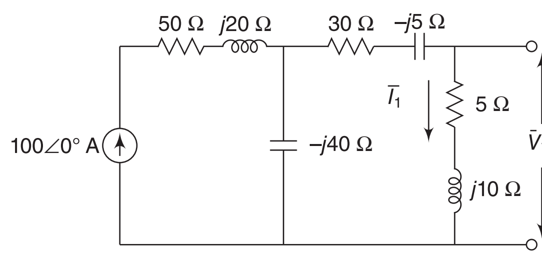


Figure E2.13(c)

Therefore,
$$\bar{V}_{\text{Th}} = \bar{I}_1 \times (5+j10) = 80.812\angle -45^\circ \times 11.18\angle 63.43^\circ$$
$$= 903.48\angle 18.435^\circ\ \text{V}$$

Hence, the value of maximum power transferred,
$$P_{\max} = \frac{|\bar{V}_{\text{Th}}|^2}{4R_L} = \frac{(903.45)^2}{4\times 5.424} = 32.62\ \text{kW}$$

Example 2.14

In the network shown in Figure E2.14(a), find the value of Z_L so that the power transfer from the source is maximum. Also, find the value of $P_{\max}$.

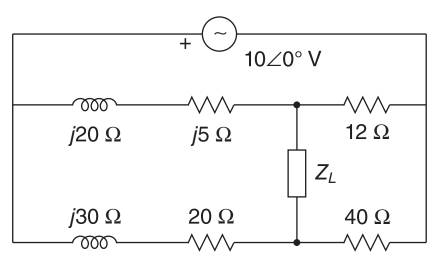


Figure E2.14(a)

Solution

The maximum power transfer occurs when $Z_L = Z_s^*$. To calculate Z_s, the voltage source is replaced by a short circuit and Z_L is removed as shown in Figure E2.14(b).

Therefore,

$$Z_s = Z_{xy} = \frac{12(15 + j20)}{12 + 15 + j20} + \frac{40(20 + j30)}{40 + 20 + j30} = \frac{180 + j240}{27 + j20} + \frac{800 + j1200}{60 + j30}$$

$$= \frac{300\angle 53.13^\circ}{33.6\angle 36.52^\circ} + \frac{1442.22\angle 56.3^\circ}{67.082\angle 26.56^\circ} = 8.928\angle 16.61^\circ + 21.499\angle 29.74^\circ$$

$$= 8.555 + j2.55 + 18.66 + j10.66 = 27.215 + j13.21\,\Omega$$

Hence, for maximum power transfer, $Z_L = Z_s^* = 27.215 - j13.21\,\Omega$

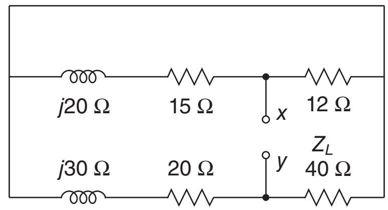


Figure E2.14(b)

To find the value of maximum power P_{max}, $\overline{V}_{Th}$ is calculated by open-circuiting Z_L as shown in Figure E2.14(c).

According to voltage division rule,

$$\overline{V}_x = \frac{15 + j20}{15 + j20 + 12} \times 10\angle 0^\circ$$

$$= \frac{150 + j200}{27 + j20}$$

$$= \frac{250\angle 53.13^\circ}{33.6\angle 36.528^\circ}$$

$$\overline{V}_x = 7.44\angle 16.602^\circ \text{ V} = 7.129 + j2.125 \text{ V}$$

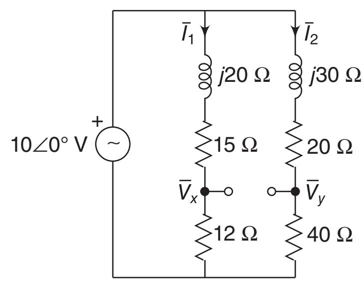


Figure E2.14(c)

and

$$\overline{V}_y = \frac{(20 + j30)}{20 + j30 + 40} \times 10\angle 0^\circ$$

$$= \frac{200 + j300}{60 + j30}$$

$$= \frac{360.55\angle 56.30^\circ}{67.08\angle 26.56^\circ}$$

$$= 5.374\angle 29.74^\circ \text{ V} = 4.66 + j2.66 \text{ V}$$

Therefore,

$$\overline{V}_{Th} = \overline{V}_x - \overline{V}_y = 7.129 + j2.125 - 4.66 - j2.66$$

$$= 2.469 - j0.535 \text{ V} = 2.526\angle -12.23 \text{ V}$$

Hence, the value of maximum power transferred, $P_{max} = \dfrac{|\overline{V}_{Th}|^2}{4R_L} = \dfrac{(2.526)^2}{4 \times 27.215} = 58.61\text{ mW}$

2.6 Introduction to Three-Phase Supply

The electrical power is generated, transmitted, and distributed in sinusoidal form the commercial, industrial and domestic applications. In general, two types of electrical power can be generated: (i) Single-phase power and (ii) Poly-phase power. The main disadvantage of single-phase power supply is that it can carry only a reasonable amount of power. But poly-phase system is normally used to generate, transmit and distribute bulk electric power. This chapter deals with three-phase system which is a type of poly-phase system. Also, the generation of three-phase electric power, the relationship between voltage and current and their power measurements are also discussed.

In an electrical power system, there are two types of systems namely, single-phase and poly-phase systems. Single-phase system consists of two wires where the current flows through one wire and returns through another wire when it is energised by two terminals. Generally, most of the homes and small industries where the capacity of motor is not greater than 5 horse power, single-phase systems are used. But nowadays three-phase system which is a type of poly-phase system is used to generate, transmit and distribute electrical energy.

In a three-phase system, three conductors can carry three alternating electrical quantities at same frequency. The electrical quantity in these three conductors reach the same peak amplitude at different time instance is shown in Figure 2.6.

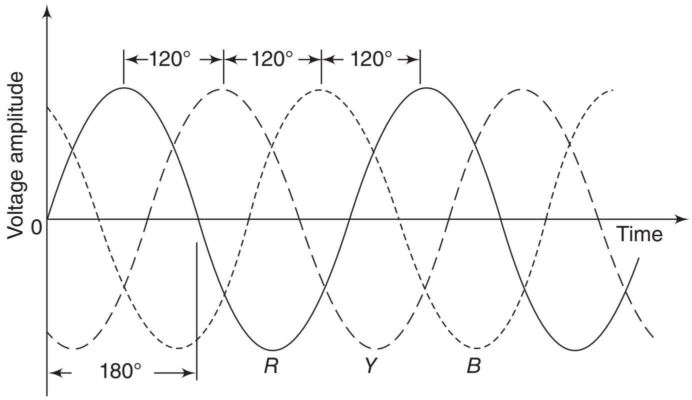


Figure 2.6 Three-phase power system of an electrical quantity

If any one of the alternating quantities is taken as reference, the other two are delayed by one-third and two-thirds of a cycle of the alternating quantity, i.e., 120° apart. Therefore, three-phase power systems can be viewed as the combination of three separate single-phase systems with 120° phase difference.

2.6.1 Advantages of Three-Phase Power System

The advantages of three-phase system over a single-phase system are:

(i) Power to weight ratio of three-phase system is high when compared to single-phase system, i.e., for the same electrical power, size of energy source required in three-phase system is less when compared to single-phase system.

(ii) For transmission of electrical power, the requirement of conductor material in three-phase system is less when compared to single-phase system.

(iii) Instantaneous power is always constant in three-phase system when compared to pulsating power in single-phase system.

(iv) Three-phase system will have better power factor and efficiency when compared to single-phase system.

(v) For a given size of the system, higher output power can be obtained in three-phase system when compared to single-phase system.

(vi) Efficient, reliable, economical and have better regulation when compared to single-phase system.

(vii) When a fault occurs in a single line, the other two lines can be used to transmit the power to the load.

(viii) Motor which operate on three-phase supply does not require any starting device like capacitor to run it.

(ix) Torque produced in the motor using three-phase supply is uniform when compared to pulsating torque produced by using single-phase supply.

(x) Three-phase system can be used to supply the electrical energy for single-phase load.

(xi) Three-phase supply can be rectified into DC supply with very low ripple factor when compared to single-phase supply.

(xii) Parallel operation is easy in three-phase system when compared to single-phase system.

2.7 Basics of Three-Phase Power System

The colour codes of the wires used in three-phase system vary from country to country. In India, Red (R), Yellow (*Y*) and Blue (*B*) the colour codes used in three-phase system. The two different configurations by which the three wires in three-phase system connected are: (i) Star (*Y*) connection and (ii) Delta (Δ) connection. The different types of three-phase power system are: (i) three-phase, three wire system and (ii) three-phase, four wire system. The fourth wire in three-phase four wire system is the neutral wire represented in black colour. It is known that, the three-phase power system can be used as source and load.

As a source, three-phase power system can be used as either three or four wire star connection or three wire delta connection. Similarly, as a load, depending upon the application, the type of connection and configuration of three-phase power system varies. The different terms used in three-phase power system are described in the following section:

(i) **Phase:** A branch of the circuit in three-phase system is known as phase.

(ii) **Line:** The wire which connects the source and load is known as transmission line or line.

(iii) **Neutral:** The fourth wire in the three-phase system where all the phases in star connection are connected together is known as neutral.

(iv) **Phase Voltage:** The voltage which is measured between a line and neutral or the voltage across a particular phase is called as phase voltage. It is represented as $\overline{V}_{RN}, \overline{V}_{YN}$ and $\overline{V}_{BN}$ or simply $\overline{V}_R, \overline{V}_Y$ and $\overline{V}_B$.

(v) **Line Voltage or Line-to-Line Voltage:** The voltage which is measured between any two lines in a three-phase power system is known as line voltage. It is represented as $\overline{V}_{RY}, \overline{V}_{YB}$ and $\overline{V}_{BR}$ and is given by $\overline{V}_{RY} = \overline{V}_R - \overline{V}_Y$, $\overline{V}_{YB} = \overline{V}_Y - \overline{V}_B$ and $\overline{V}_{BR} = \overline{V}_B - \overline{V}_R$ respectively.

(vi) **Line Currents:** The currents flowing through a particular line are called line currents represented by $\overline{I}_R, \overline{I}_Y$ and $\overline{I}_B$.

(vii) **Phase Current:** The current flowing through a single-phase or branch of the system is called as phase current. It is represented as $\overline{I}_{RY}, \overline{I}_{YB}$ and $\overline{I}_{BR}$ and is given by $\overline{I}_{RY} = \overline{I}_R - \overline{I}_Y$, $\overline{I}_{YB} = \overline{I}_Y - \overline{I}_B$, $\overline{I}_{BR} = \overline{I}_B - \overline{I}_R$ respectively.

(viii) **Load Impedance:** For a star-connected load, the impedance between the line and neutral is called load or line impedance and for a delta-connected load, the impedance between two lines is called load or phase impedance.

(ix) **Phase Sequence:** The time order or the sequence in which the electrical quantity in the three-phase system reach their respective maximum values is known as phase sequence. If the phase sequence of a particular system is *RYB*, then it indicates that *R* phase reaches the maximum value of electrical quantity at first and then followed by *Y* phase and *B* phase.

(x) **Balanced Condition:** The condition for having a balanced source or balanced load is given below.

- (a) **Balanced Source:** A three-phase system is said to be balanced source, if the phase voltage of each phase has same magnitude and frequency and phase difference between the lines is 120°.
- (b) **Balanced Load:** A three-phase system is said to be a balanced load if the impedance is same for all the phase either in star or delta connection.

(xi) **Unbalanced Condition:** If the load impedance differs in one or more phases, then the three-phase system is said to be an unbalanced load. This unbalanced condition leads to change in line and phase currents.

(xii) **Three-Phase Source:** If the three-phase system is used to generate three-phase power supply, then it is said to be three-phase source.

(xiii) **Three-Phase Load:** If the three-phase system uses the three-phase supply to perform certain functions, then it is said to be three-phase load.

The schematic diagrams of three-phase star-connected power system with three wire and four wire are shown in Figures 2.7 (a) and (b) respectively.

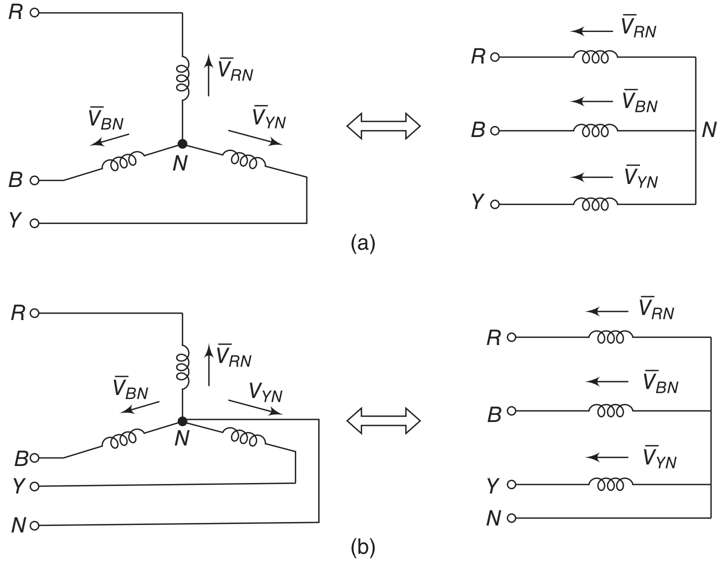


Figure 2.7 Schematic diagram of star-connected three-phase system

The schematic diagram of three-phase delta-connected power system is shown in Figure 2.8.

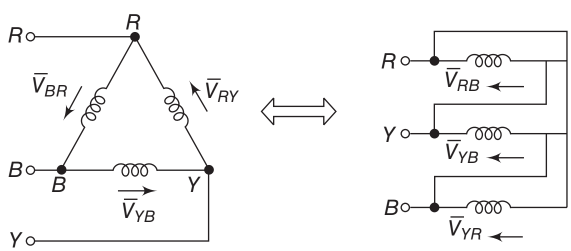


Figure 2.8 Schematic diagram of delta-connected three-phase system

The relation between the phase voltages in star-connected balanced three-phase system is given by

$$\left.\begin{aligned} \bar{V}_{RN} &= |\bar{V}|\angle 0^\circ \\ \bar{V}_{YN} &= |\bar{V}|\angle -120^\circ \\ \bar{V}_{BN} &= |\bar{V}|\angle +120^\circ \text{ or } |\bar{V}|\angle -240^\circ \end{aligned}\right\} \quad (2.4)$$

Similarly, the relation between the line voltages in delta-connected balanced three-phase system is given by

$$\left.\begin{aligned} \bar{V}_{RY} &= |\bar{V}|\angle 0° \\ \bar{V}_{YB} &= |\bar{V}|\angle -120° \\ \bar{V}_{BR} &= |\bar{V}|\angle +120° \text{ or } |\bar{V}|\angle -240° \end{aligned}\right\} \tag{2.5}$$

The vector diagram of a balanced star and delta connected power system using the above equations is shown in Figures 2.9(a) and (b) respectively.

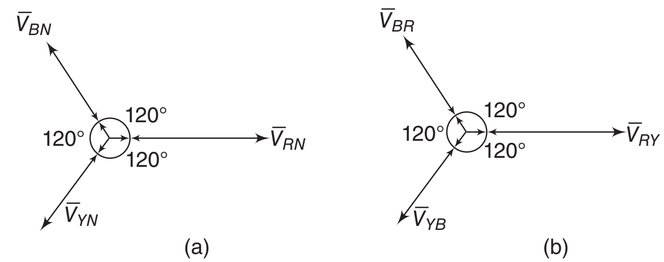


Figure 2.9 Vector diagram of balanced (a) star (b) delta-connected system

2.8 Generation of Three-Phase Voltages

The primary requirement in three-phase system before analysing the balanced and unbalanced condition is the generation of three-phase voltages. The three-phase AC generator or an alternator is used to generate the three-phase voltages. The two main components of AC generator or an alternator is the field and armature in which either field or armature is stationary. Therefore, the alternator configurations by which the three-phase voltage can be generated are: (i) Stationary field with rotating armature and (ii) Rotating field with stationary armature. The instantaneous phase voltages of the three-phase system when connected in star is given by

$$\begin{aligned} \bar{V}_{RN} &= V_m \sin(\omega t) \\ \bar{V}_{YN} &= V_m \sin(\omega t - 120°) \\ \bar{V}_{BN} &= V_m \sin(\omega t - 240°) = V_m \sin(\omega t + 120°) \end{aligned} \tag{2.6}$$

where V_m is the maximum value of voltage in volt.

Adding all the instantaneous voltage given in Equation (2.6), we get

$$\begin{aligned} \bar{V}_{RN} + \bar{V}_{YN} + \bar{V}_{BN} &= V_m \sin \omega t + V_m \sin(\omega t - 120°) + V_m \sin(\omega t + 120°) \\ &= V_m[\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 120° + \cos \omega t \sin 120°] \\ &= V_m[\sin \omega t + 2 \sin \omega t \cos 120°] = V_m\left[\sin \omega t + 2 \sin \omega t\left(\frac{-1}{2}\right)\right] = 0 \end{aligned}$$

Therefore, $$\bar{V}_{RN} + \bar{V}_{YN} + \bar{V}_{BN} = 0 \tag{2.7}$$

It is clear from Equation (2.7) that the phasor addition of all the phase voltages at any instant in three-phase balanced star-connected system is always zero. Similarly, if the instantaneous line voltages of the three-phase system when connected in delta connection are added, we get $\bar{V}_{RY} + \bar{V}_{YB} + \bar{V}_{BR} = 0$.

2.9 ANALYSIS OF THE THREE-PHASE SYSTEM

The different three-phase systems for which the relationship between phase and line voltages, phase and line currents, power and phasor diagram are discussed as follows:

(i) Three-phase, balanced star-connected source
(ii) Three-phase, balanced delta-connected source
(iii) Three-phase, balanced star-connected load
(iv) Three-phase, balanced delta-connected load
(v) Three-phase, unbalanced delta-connected load
(vi) Three-phase, unbalanced four wire star-connected load
(vii) Three-phase, unbalanced three wire star-connected load

2.9.1 Three-Phase, Balanced, Star-Connected Source **[April/May, 2015]**

The circuit diagram for a three-phase balanced star-connected source with phase sequence *RYB* is shown in Figure 2.10.

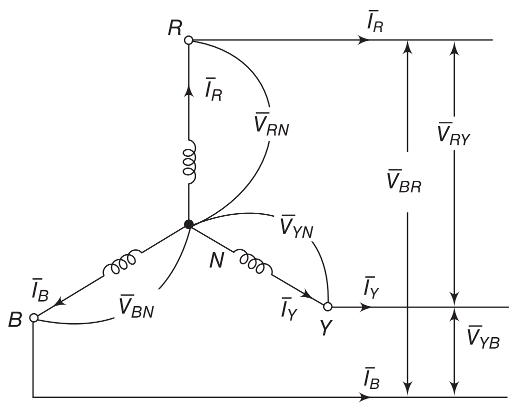


Figure 2.10 Circuit diagram for a three-phase balanced star-connected source

In a balanced system, all the magnitude of phase voltages, line voltages, phase currents and line currents are equal which is represented by

$$|\bar{V}_{RN}| = |\bar{V}_{YN}| = |\bar{V}_{BN}| = |\bar{V}_{\text{ph}}|, |\bar{V}_{RY}| = |\bar{V}_{YB}| = |\bar{V}_{BR}| = |\bar{V}_L| \tag{2.8}$$

$$|\bar{I}_R| = |\bar{I}_Y| = |\bar{I}_B| = |\bar{I}_L|, |\bar{I}_{RY}| = |\bar{I}_{YB}| = |\bar{I}_{BR}| = |\bar{I}_{\text{ph}}| \tag{2.9}$$

Relationship Among Line and Phase Quantities

(i) Current Relationship

For a three-phase balanced star-connected source, the line current and phase current are same. Therefore,

$$\bar{I}_{RY} = \bar{I}_R; \bar{I}_{YB} = \bar{I}_Y; \bar{I}_{BR} = \bar{I}_B \tag{2.10}$$

From Equations (2.9) and (2.10), we can conclude that in a balanced star-connected three-phase source, phase current is equal to the line current as given by

$$\bar{I}_{\text{ph}} = \bar{I}_L \tag{2.11}$$

(ii) Voltage Relationship

It is known that, $\bar{V}_{RY} = \bar{V}_{RN} - \bar{V}_{YN}$

Using parallelogram law of addition and vector diagram shown in Figure 1.6, we get

$$|\bar{V}_{RY}| = \sqrt{|\bar{V}_{RN}|^2 + |\bar{V}_{YN}|^2 + 2|\bar{V}_{RN}||\bar{V}_{YN}|\cos 60°}$$

Using Equation (2.8) in the above equation and solving, we get

$$|\bar{V}_{RY}| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \tag{2.12}$$

Similarly, we get

$$|\bar{V}_{YB}| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \text{ and } |\bar{V}_{BR}| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \tag{2.13}$$

Therefore, using Equation (2.8), Equation (2.12) and Equation (2.13), we get the relation between the line and phase voltages as

$$|\bar{V}_L| = \sqrt{3}\,|\bar{V}_{\text{ph}}| \tag{2.14}$$

Hence, it can be concluded that in a star-connected balanced three-phase source, the line voltage is $\sqrt{3}$ times the phase voltage or phase voltage is $\frac{1}{\sqrt{3}}$ times the line voltage. It is to be noted that the angle between phase voltage and line voltage is 30°.

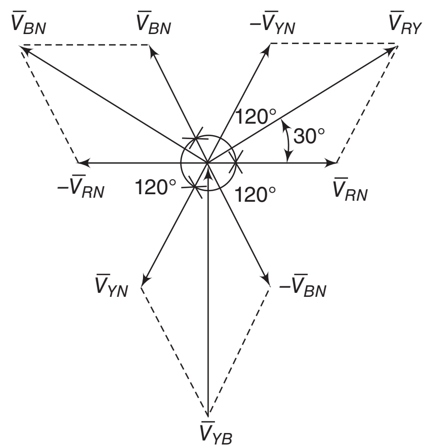


Figure 2.11 Vector diagram for a three-phase balanced star-connected source

(iii) Vector Diagram

The vector diagram for a three-phase balanced star-connected source by considering the phase voltage as reference is shown in Figure 2.11.

(iv) Power Relationship

The real power produced per phase in the system is

$P_{\text{ph}} = |\bar{V}_{\text{ph}}||\bar{I}_{\text{ph}}|\cos\phi$.

Therefore, the total real power produced in the system is given by

$$P = 3|\bar{V}_{\text{ph}}||\bar{I}_{\text{ph}}|\cos\phi \tag{2.15}$$

Using Equation (2.11) and Equation (2.14), we get

$$P = 3\frac{|\bar{V}_L|}{\sqrt{3}}|\bar{I}_L|\cos\phi = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi \text{ (Watts)} \tag{2.16}$$

Similarly, the total reactive power, Q, and total apparent power, S, produced in the system are given by

$$Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\sin\phi \qquad \text{(VAR)} \tag{2.17}$$

$$S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \qquad \text{(VA)} \tag{2.18}$$

2.9.2 Three-Phase, Balanced, Delta-Connected Source [April/May, 2015]

The circuit diagram for a three-phase balanced delta-connected source with phase sequence *RYB* is shown in Figure 2.12.

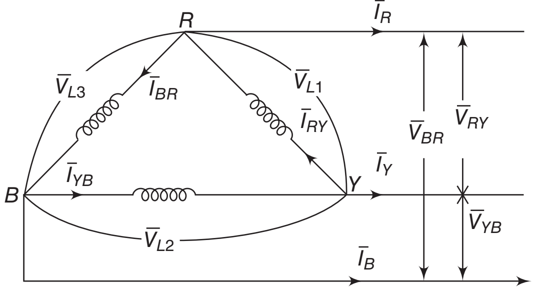


Figure 2.12 Circuit diagram for a three-phase balanced delta-connected source

Relationship Among Line and Phase Quantities

(i) Current Relationship

Applying Kirchhoff's current law at the node *R* in Figure 2.12, we have

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR}$$

Using the vector diagram shown in Figure 2.13 and applying parallelogram law of addition, we get

$$|\bar{I}_R| = \sqrt{|\bar{I}_{RY}|^2 + |\bar{I}_{BR}|^2 + 2\,|\bar{I}_{RY}||\bar{I}_{RB}|\cos 60°}$$

Solving the above equation by using Equation (2.9), we get

$$|\bar{I}_R| = \sqrt{3}\,|\bar{I}_{\text{ph}}| \tag{2.19}$$

Similarly, we get

$$|\bar{I}_Y| = \sqrt{3}\,|\bar{I}_{\text{ph}}| \text{ and } |\bar{I}_B| = \sqrt{3}\,|\bar{I}_{\text{ph}}| \tag{2.20}$$

Therefore, using Equation (2.9), Equation (2.19) and Equation (2.20), we get the relation between the line and phase currents as

$$|\bar{I}_L| = \sqrt{3}\,|\bar{I}_{\text{ph}}| \tag{2.21}$$

Hence, it can be concluded that in a delta-connected balanced three-phase source, the line current is $\sqrt{3}$ times the phase current or phase current is $1/\sqrt{3}$ times the line current. It is to be noted that the phase angle between phase current and line current is 30°.

(ii) Voltage Relationship

Applying Kirchhoff's voltage law to the loop consisting of $\bar{V}_R$ and $\bar{V}_{RY}$, we have

$$\bar{V}_{RN} = \bar{V}_{RY},\ \bar{V}_{YN} = \bar{V}_{YB} \text{ and } \bar{V}_{BN} = \bar{V}_{BR} \tag{2.22}$$

Using Equations (2.8) and (2.22), we can conclude that, in a balanced delta-connected three-phase source, phase voltage is equal to the line voltage as given in Equation (2.26).

$$\bar{V}_{\text{ph}} = \bar{V}_L \tag{2.23}$$

(iii) Vector Diagram

The vector diagram for a balanced delta-connected source by considering the phase currents as reference vector is shown in Figure 2.13.

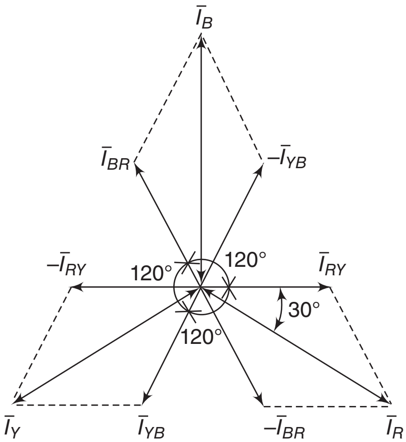


Figure 2.13 Phasor diagram for a balanced delta-connected source

(iv) Power Relationship

The real power produced per phase in the system is $P_{\text{ph}} = |\overline{V}_{\text{ph}}||\overline{I}_{\text{ph}}| \cos \phi$.

Therefore, the total real power produced in the system is given by

$$P = 3|\overline{V}_{\text{ph}}||\overline{I}_{\text{ph}}| \cos \phi \tag{2.24}$$

Using Equation (2.21) and Equation (2.23), we get

$$P = 3|\overline{V}_L| \frac{|\overline{I}_L|}{\sqrt{3}} \cos \phi = \sqrt{3}\,|\overline{V}_L||\overline{I}_L| \cos \phi \text{ (Watts)} \tag{2.25}$$

Similarly, the total reactive power, Q, and total apparent power, S, produced in the system are given by

$$Q = \sqrt{3}\,|\overline{V}_L||\overline{I}_L| \sin \phi \qquad \text{(VAR)} \tag{2.26}$$

$$S = \sqrt{3}\,|\overline{V}_L||\overline{I}_L| \qquad \text{(VA)} \tag{2.27}$$

2.9.3 Three-Phase, Balanced, Star-Connected Load **[April/May, 2015]**

The circuit diagram for a three-phase balanced star-connected load with phase sequence *RYB* is shown in Figure 2.14.

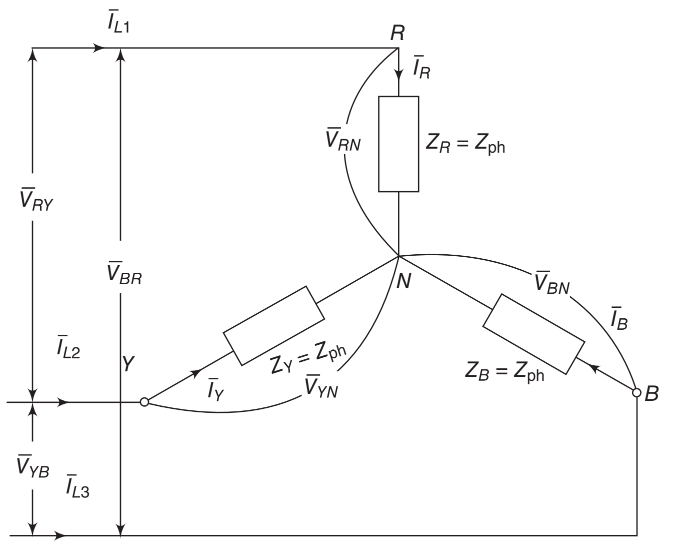


Figure 2.14 Circuit diagram for a three-phase balanced star-connected load

Relationship among phase current, phase voltage and load impedance:

Let Z_R, Z_Y and Z_B be the load impedances in *R*, *Y* and *B* phases respectively. But in a balanced load condition, all the load impedances are equal to the load impedance per phase, Z_{ph} as represented by

$$Z_R = Z_Y = Z_B = Z_{\text{ph}} \tag{2.28}$$

The current, voltage and power relationship between line and phase quantity explained in section 2.9.1 is applicable to the balanced three-phase star-connected load, i.e.,

$$\bar{I}_{\text{ph}} = \bar{I}_L, |\bar{V}_L| = \sqrt{3}\,|\bar{V}_{\text{ph}}|, P = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi, Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\sin\phi$$

and

$$S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \tag{2.29}$$

The relation between phase current, phase voltage and load impedance per phase is given by

$$\bar{I}_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{Z_{\text{ph}}}$$

Load Impedance

If the load has lagging, leading and unity power factor in nature, then the load impedance is given by $Z_{\text{ph}} = R_{\text{ph}} + jX_{\text{ph}}$, $Z_{\text{ph}} = R_{\text{ph}} - jX_{\text{ph}}$ or $Z_{\text{ph}} = R_{\text{ph}}$ respectively.

Power Factor

The power factor of the given three-phase star-connected balanced load is cos ϕ.

Phasor Diagram

The phasor diagram for a three-phase balanced star-connected load with lagging and leading power factor load is shown in Figures 2.15(a) and (b) respectively.

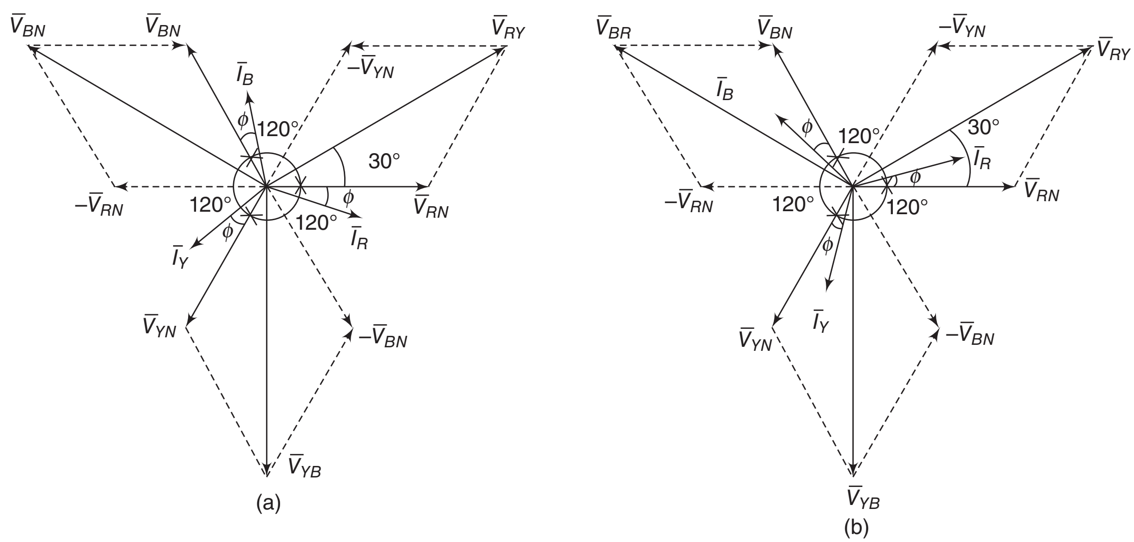


Figure 2.15 Phasor diagram for a three-phase balanced star-connected load with (a) lagging and (b) leading power factor

2.9.4 Three-Phase, Balanced, Delta-Connected Load

The circuit diagram for a three-phase balanced delta-connected load with phase sequence *RYB* is shown in Figure 2.16.

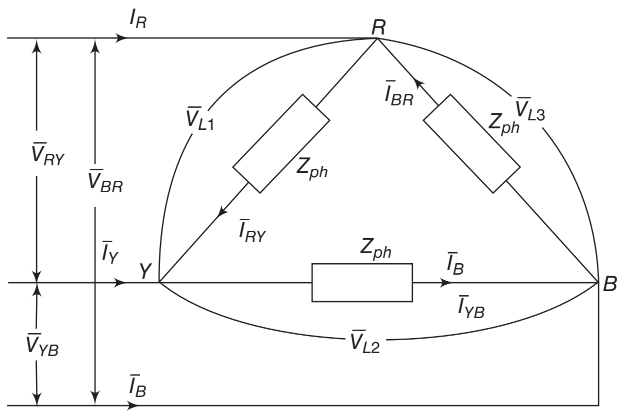


Figure 2.16 Circuit diagram for a three-phase balanced delta-connected load

Relationship among phase current, phase voltage and load impedance:
The current, voltage and power relationship between line and phase quantity explained in section 2.9.2 is applicable to the balanced three-phase delta-connected load, i.e.,

$$\bar{V}_{\text{ph}} = \bar{V}_L,\ |\bar{I}_L| = \sqrt{3}\,|\bar{I}_{\text{ph}}|,\ P = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi,\ Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\sin\phi$$

and $$S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L| \tag{2.30}$$

The relation between phase currents, phase voltage and load impedance per phase is given by

$$\bar{I}_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{Z_{\text{ph}}}$$

Load Impedance

If the load has lagging, leading or unity power factor in nature, then the load impedance is given by $Z_{\text{ph}} = R_{\text{ph}} + jX_{\text{ph}}$, $Z_{\text{ph}} = R_{\text{ph}} - jX_{\text{ph}}$ or $Z_{\text{ph}} = R_{\text{ph}}$ respectively.

Power Factor

The power factor of the given three-phase delta-connected balanced load is $\cos\phi$.

Phasor Diagram

The phasor diagram for a three-phase balanced delta-connected load with lagging and leading power factor load is shown in Figures 2.17(a) and (b) respectively.

2.9.5 Three-Phase Unbalanced Delta-Connected Load **[May/June, 2014]**

The circuit diagram for a three-phase unbalanced delta-connected load with phase sequence *RYB* is shown in Figure 2.18.

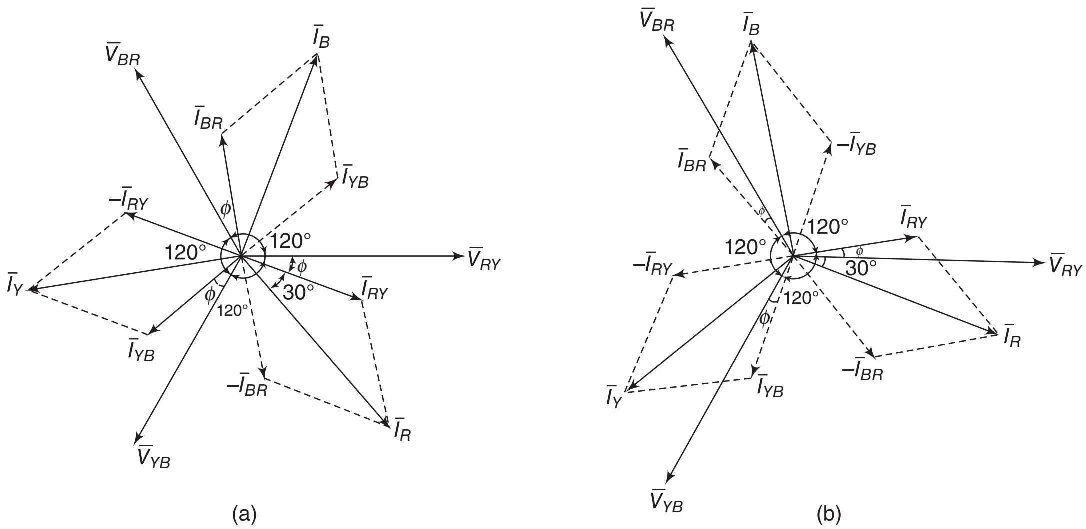


Figure 2.17 Phasor diagram for a three-phase balanced delta-connected load with (a) lagging and (b) leading power factor

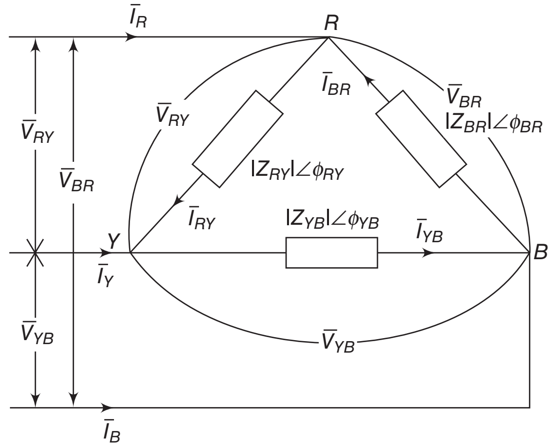


Figure 2.18 Three-phase unbalanced delta-connected load

The load impedance across *R-Y*, *Y-B* and *B-R* terminals are given by $|Z_{RY}|\angle\phi_{RY}$, $|Z_{YB}|\angle\phi_{YB}$ and $|Z_{BR}|\angle\phi_{BR}$ respectively. It is known that in delta connection, the phase and line voltages are same. Since there is change in load impedance, there will be changes only in the line and phase currents.

The phase and line currents in the system are given by

$$\bar{I}_{RY} = \frac{\bar{V}_{RY}}{|Z_{RY}|\angle\phi_{RY}} = \frac{\bar{V}_{RY}}{Z_{RY}},\ \bar{I}_{YB} = \frac{\bar{V}_{YB}}{|Z_{YB}|\angle\phi_{YB}} = \frac{\bar{V}_{YB}}{Z_{YB}}$$

and
$$\bar{I}_{BR} = \frac{\bar{V}_{BR}}{|Z_{BR}|\angle\phi_{BR}} = \frac{\bar{V}_{BR}}{Z_{BR}} \tag{2.31}$$

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR},\ \bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} \text{ and } \bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} \tag{2.32}$$

Also, the total real power, reactive power and apparent power for the unbalanced delta-connected load is given Equations (2.33) to (2.35) respectively.

$$P = |V_{RN}||I_{RY}|\cos\phi_{RY} + |V_{YN}||I_{YB}|\cos\phi_{YB} + |V_{BN}||I_{BR}|\cos\phi_{BR} \tag{2.33}$$

$$Q = |V_{RN}||I_{RY}|\sin\phi_{RY} + |V_{YN}||I_{YB}|\sin\phi_{YB} + |V_{BN}||I_{BR}|\sin\phi_{BR} \tag{2.34}$$

$$S = |V_{RN}||I_{RY}| + |V_{YN}||I_{YB}| + |V_{BN}||I_{BR}| \tag{2.35}$$

2.9.6 Three-Phase Unbalanced Four Wire Star-Connected Load

[May/June, 2014]

The circuit diagram for a three-phase four wire unbalanced star-connected load with phase sequence *RYB* is shown in Figure 2.19.

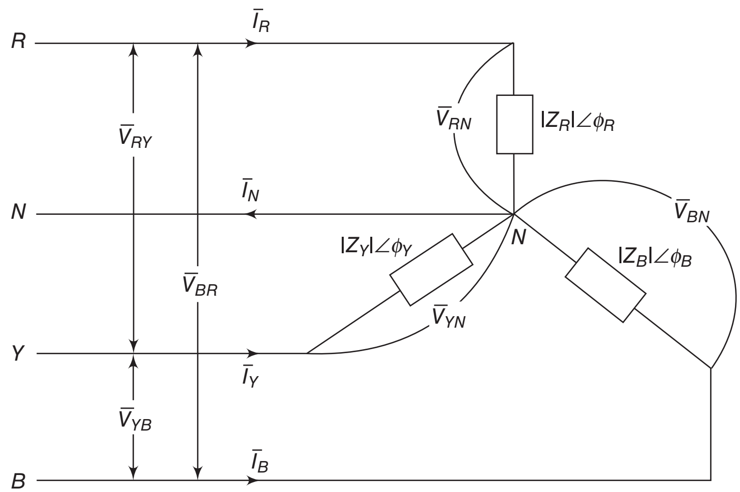


Figure 2.19 Three-phase four wire unbalanced star-connected load

The load impedance across *R-N*, *Y-N* and *B-N* terminals are given by $|Z_R|\angle\phi_R$, $|Z_Y|\angle\phi_Y$ and $|Z_B|\angle\phi_B$ respectively. In star-connected load, the line and phase currents are equal as given by

$$\bar{I}_R = \frac{\bar{V}_{RN}}{|Z_R|\angle\phi_R} = \frac{\bar{V}_{RN}}{Z_R},\ \bar{I}_Y = \frac{\bar{V}_{YN}}{|Z_Y|\angle\phi_Y} = \frac{\bar{V}_{YN}}{Z_Y} \text{ and } \bar{I}_B = \frac{\bar{V}_{BN}}{|Z_B|\angle\phi_B} = \frac{\bar{V}_{BN}}{Z_B} \tag{2.36}$$

The current flowing through the neutral point is obtained using Kirchhoff's current law as given by

$$\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B$$

The phase voltages in this system are given by,

$$|\bar{V}_{RN}| = \frac{|V_{RY}|}{\sqrt{3}}\angle\theta_R - 30^\text{o} \text{ or } \frac{|V_\text{ph}|}{\sqrt{3}}\angle\theta_R - 30^\text{o} \tag{2.37}$$

$$|\bar{V}_{YN}| = \frac{|V_{YB}|}{\sqrt{3}}\angle\theta_Y - 30^\text{o} \text{ or } \frac{|V_\text{ph}|}{\sqrt{3}}\angle\theta_R - 90^\text{o} \tag{2.38}$$

$$|\overline{V}_{BN}| = \frac{|V_{BR}|}{\sqrt{3}} \angle\theta_B - 30^\circ \text{ or } \frac{|V_{\text{ph}}|}{\sqrt{3}} \angle\theta_R - 210^\circ \tag{2.39}$$

Also, the total real power, reactive power and apparent power for the unbalanced delta-connected load are given by

$$P = |V_{RN}||I_R|\cos\phi_{RY} + |V_{YN}||I_Y|\cos\phi_{YB} + |V_{BN}||I_B|\cos\phi_{BR} \tag{2.40}$$

$$Q = |V_{RN}||I_R|\sin\phi_{RY} + |V_{YN}||I_Y|\sin\phi_{YB} + |V_{BN}||I_B|\sin\phi_{BR} \tag{2.41}$$

$$S = |V_{RN}||I_R| + |V_{YN}||I_Y| + |V_{BN}||I_B| \tag{2.42}$$

2.9.7 Three-Phase Unbalanced Three Wire Star-Connected Load

[May/June, 2014]

The circuit diagram for a three-phase three wire unbalanced star-connected load with phase sequence *RYB* is shown in Figure 2.20.

The potential of neutral point in this system is different from the potential of neutral point in balanced star-connected source. Such neutral points are called floating neutral point due to which the relation between the phase voltage, line voltage and supply voltage does not exist and the phase angle between any two phases will not be 120°. This creates difficulties in determining the line and phase voltages and currents of the load. The solution to these difficulties can be achieved by any one of the following three methods:

(i) Star to delta conversion
(ii) Mesh analysis
(iii) Millman's theorem

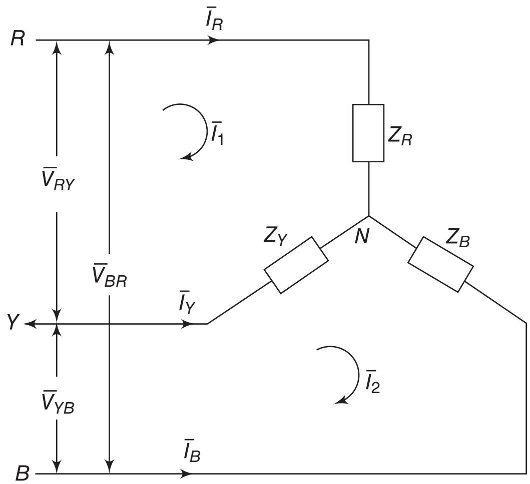


Figure 2.20 Three-phase three wire unbalanced star-connected load

(i) Star to Delta Conversion

In this method, the star-connected unbalanced load is converted into delta-connected unbalanced load which eliminates the problem of floating neutral point. Once the star to delta conversion is performed, the system starts to act as a three-phase unbalanced delta-connected load. The line and phase voltages and currents can be obtained using the equations indicated in section 2.9.5. The detailed description of star to delta conversion is discussed in section 2.11.

(ii) Mesh Analysis

The circuit diagram of three-phase unbalanced three wire star-connected load with phase sequence *RYB* is shown in Figure 2.20. Let $\overline{I}_1$ and $\overline{I}_2$ be the current flowing through loop 1 and loop 2.

Applying mesh analysis to Figure 2.20, we get

For loop 1,

$$\overline{V}_{RY} = \overline{I}_1 Z_R + (\overline{I}_1 - \overline{I}_2)Z_Y = \overline{I}_1 Z_R + \overline{I}_1 Z_Y - \overline{I}_2 Z_Y$$

$$\overline{V}_{RY} = \overline{I}_1(Z_R + Z_Y) - \overline{I}_2 Z_Y \tag{2.43}$$

For loop 2,

$$\overline{V}_{YB} = \overline{I}_2 Z_B + (\overline{I}_2 - \overline{I}_1) Z_Y = \overline{I}_2 Z_B + \overline{I}_2 Z_Y - \overline{I}_1 Z_Y$$

$$\overline{V}_{YB} = -\overline{I}_1 Z_B + \overline{I}_2 (Z_B + Z_Y) \tag{2.44}$$

Upon solving Equation (2.43) and Equation (2.44), the currents $\overline{I}_1$ and $\overline{I}_2$ can be determined.

From Figure 2.20, we get

$$\overline{I}_R = \overline{I}_1,\ \overline{I}_B = -\overline{I}_2 \text{ and } \overline{I}_Y = \overline{I}_1 - \overline{I}_2 \text{ (or) } \overline{I}_2 - \overline{I}_1$$

[Depending on the magnitude of higher value]

If $\overline{I}_R, \overline{I}_Y$ and $\overline{I}_B$ are determined, then the phase and line voltages can be calculated using Equations (2.45) and (2.46) respectively.

$$\overline{V}_{RN} = \overline{I}_R Z_R,\ \overline{V}_{YN} = \overline{I}_Y Z_Y \text{ and } \overline{V}_{BN} = \overline{I}_B Z_B \tag{2.45}$$

$$\overline{V}_{RY} = \overline{V}_{RN} - \overline{V}_{YN},\ \overline{V}_{YB} = \overline{V}_{YN} - \overline{V}_{BN} \text{ and } \overline{V}_{BR} = \overline{V}_{BN} - \overline{V}_{RN} \tag{2.46}$$

(iii) Millman's Theorem

According to Millman's theorem, if number of voltage sources $\overline{V}_1, \overline{V}_2, \overline{V}_3, \ldots, \overline{V}_n$ with internal impedances Z_1, Z_2, …, Z_n are in parallel as shown in Figure 2.21(a), then it can be replaced by an equivalent circuit consisting of a voltage source $\overline{V}_{\text{eq}}$ in series with impedance Z_{eq} as shown in Figure 2.21(b).

where
$$\overline{V}_{\text{eq}} = \frac{\dfrac{\overline{V}_1}{Z_1} + \dfrac{\overline{V}_2}{Z_2} + \cdots + \dfrac{\overline{V}_n}{Z_n}}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \cdots + \dfrac{1}{Z_n}} = \frac{\overline{V}_1 Y_1 + \overline{V}_2 Y_2 + \cdots + \overline{V}_n Y_n}{Y_1 + Y_2 + \cdots + Y_n}$$

and
$$Z_{\text{eq}} = \frac{1}{\dfrac{1}{Z_1} + \dfrac{1}{Z_2} + \cdots + \dfrac{1}{Z_n}} \quad \text{or} \quad \frac{1}{Y_{\text{eq}}} = \frac{1}{Y_1 + Y_2 + \cdots + Y_n}$$

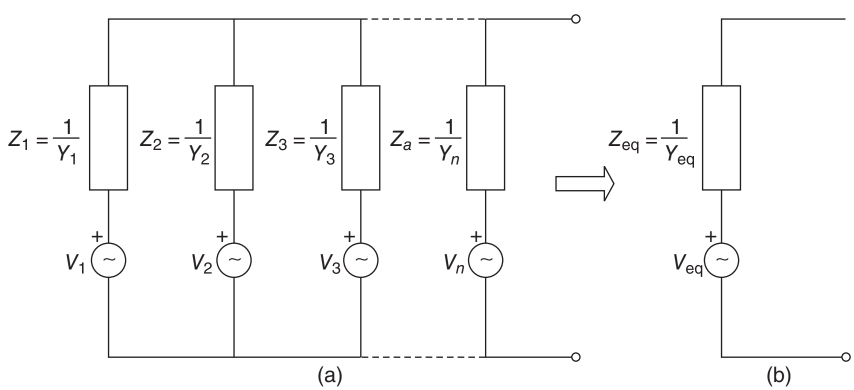


Figure 2.21 Milliman's Theorem representation

The unbalanced three wire star-connected load which is supplied by a balanced star-connected source to which Millman's Theorem is applied is shown in Figure 2.22.

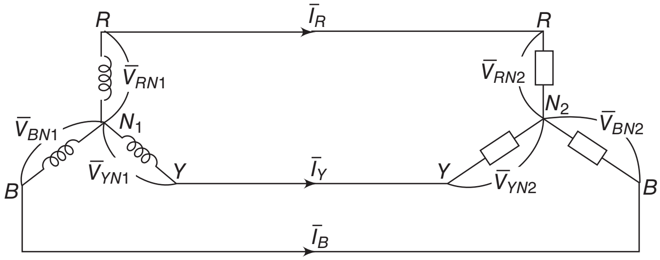


Figure 2.22 Application of Milliman's theorem to unbalanced three wire star-connected load

Applying Millman's theorem to Figure 2.22, we get

$$\bar{V}_{N1N2} = \frac{\bar{V}_{RN1}Y_R + \bar{V}_{YN1}Y_Y + \bar{V}_{BN1}Y_B}{Y_R + Y_Y + Y_B}$$

where Y_R, Y_Y and Y_B are admittances of unbalanced three wire star load connected such that

$$Y_R = \frac{1}{Z_R}; Y_Y = \frac{1}{Z_Y}; Y_B = \frac{1}{Z_B}$$

Therefore, $\bar{V}_{RN1} = \bar{V}_{RN2} + \bar{V}_{N1N2}$

and $\bar{V}_{RN2} = \bar{V}_{RN1} - \bar{V}_{N1N2}$

Similarly, $\bar{V}_{YN2} = \bar{V}_{YN1} - \bar{V}_{N1N2}$ and $\bar{V}_{BN2} = \bar{V}_{BN1} - \bar{V}_{N1N2}$

The line (or phase) currents are given by

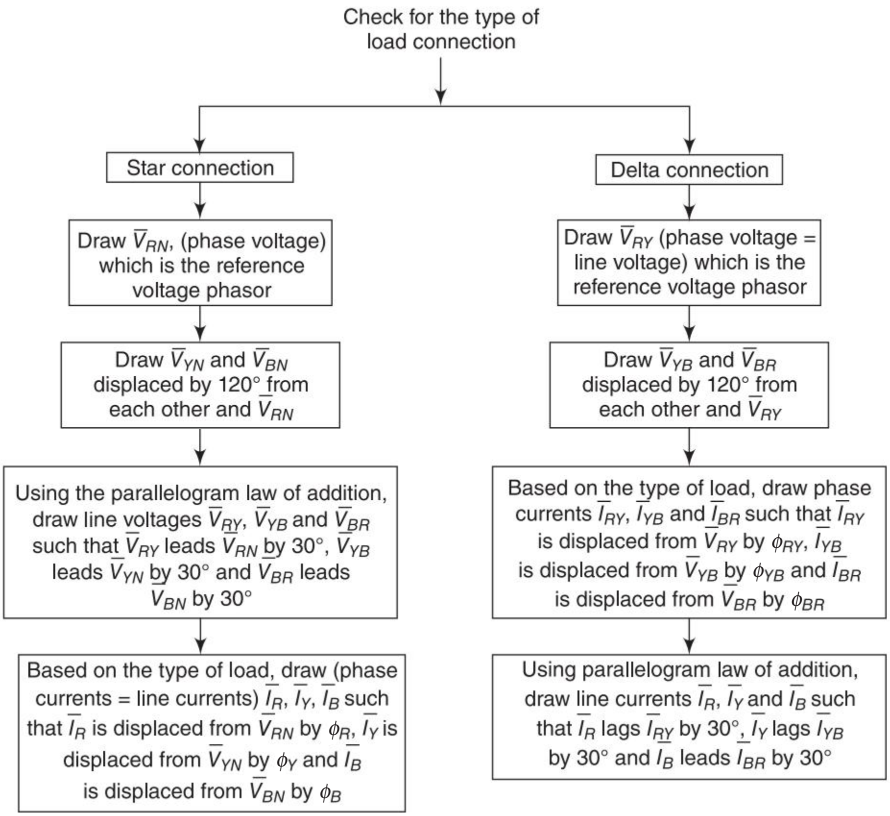
$$\bar{I}_R = \frac{\bar{V}_{RN2}}{Z_R} = \frac{\bar{V}_{RN2}}{|Z_R|\angle\theta_R}, \bar{I}_Y = \frac{\bar{V}_{YN2}}{Z_Y} = \frac{\bar{V}_{YN2}}{|Z_Y|\angle\theta_Y} \text{ and } \bar{I}_B = \frac{\bar{V}_{BN2}}{Z_B} = \frac{\bar{V}_{BN2}}{|Z_B|\angle\theta_B}$$

Different Types of Balanced Connection

The different types of connection which exist in three-phase systems are:

(i) Star-connected source - star-connected load
(ii) Star-connected source - delta-connected load
(iii) Delta-connected source - star-connected load
(iv) Delta-connected source - delta-connected load

2.10 Steps to Draw Phasor Diagram



2.11 Star-Delta Conversion **[May/June, 2013]**

The transformation or replacement of the three-phase star-connected load to a three-phase delta-connected load and vice versa is called star-delta conversion. This conversion is required in three-phase loads to simplify and analyse the complex circuits. For example, if a three-phase load is connected in delta, it can be transformed into an equivalent star-connected load and after analysis, the results are converted back into their original delta equivalent. The converted equivalent circuit will have the same current and voltage levels at its network terminals as appeared in the original circuit. The two different conversion existing in three-phase systems are: (i) Star to delta conversion and (ii) Delta to star conversion.

2.11.1 Star to Delta Conversion

In this conversion, star-connected three-phase load shown in Figure 2.23(a) is converted into delta-connected three-phase system as shown in Figure 2.23(b).

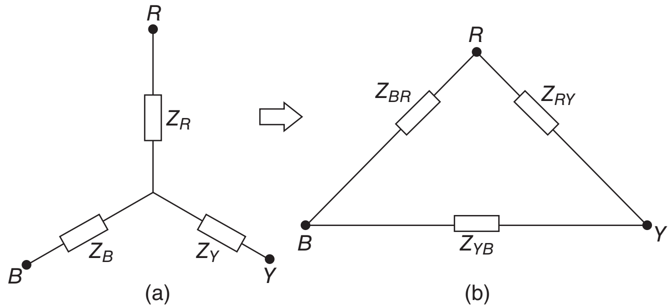


Figure 2.23 Star to delta conversion

The equivalent impedance in delta-connected load is given by

$$\left.\begin{aligned} Z_{RY} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B} \\ Z_{YB} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R} \\ Z_{BR} &= \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y} \end{aligned}\right\} \tag{2.47}$$

2.11.2 Delta to Star Conversion

In this conversion, star-connected three-phase load shown in Figure 2.24(a) is converted into delta-connected three-phase system as shown in Figure 2.24(b).

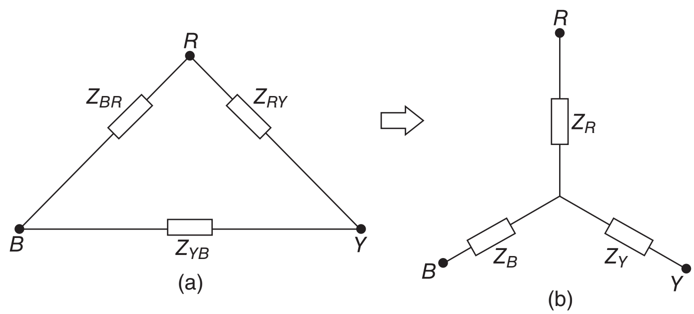


Figure 2.24 Delta to star conversion

The equivalent impedances in star-connected load are given by

$$\left.\begin{aligned} Z_R &= \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}} \\ Z_Y &= \frac{Z_{YB} Z_{RY}}{Z_{RY} + Z_{YB} + Z_{BR}} \\ Z_B &= \frac{Z_{YB} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}} \end{aligned}\right\} \tag{2.48}$$

Example 2.15

For the circuit shown in Figure E2.15, calculate the line current, power and power factor. The values of R, L and C in each phase are 10 Ω, 1 H and 100 μF respectively. **[AU Nov/Dec, 2012]**

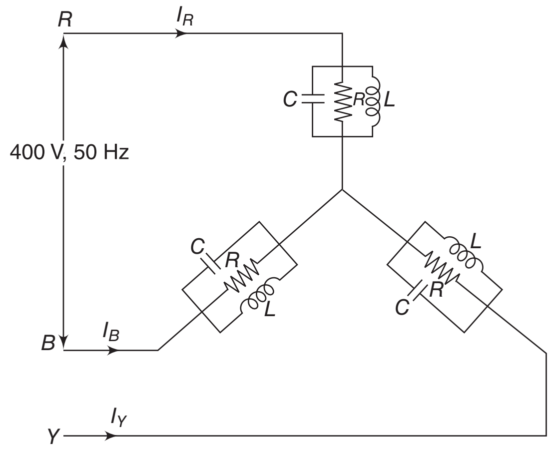


Figure E2.15

Solution

The load impedance in each phase is a parallel combination of R, L and C as given by

$$Z_R = Z_Y = Z_B = \cfrac{1}{\cfrac{1}{R} + \cfrac{1}{X_L} + \cfrac{1}{X_C}}$$
$$= 9.262 - j2.615$$
$$= 9.624\angle -15.77^\circ\ \Omega$$

For a star connected load, $|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}| = \dfrac{|\overline{V}_L|}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94\ \text{V}$

In a balanced star connected load, the line current or phase current is given by

$$\overline{I}_R = \overline{I}_Y = \overline{I}_B = \overline{I}_{\text{ph}} = \overline{I}_L = \frac{|\overline{V}_{\text{ph}}|}{Z_R} = \frac{230.94}{9.624\angle -15.77^\circ} = 24\angle 15.77^\circ\ \text{A}$$

The power factor of the system is given by

$$\cos\phi = \cos(\overline{V}_{\text{ph}} \wedge \overline{I}_{\text{ph}}) = \cos(15.77^\circ) = 0.9624$$

The power consumed by the load is

$$P = \sqrt{3} V_L I_L \cos\phi = \sqrt{3} \times 400 \times 24 \times 0.9624$$
$$= 16\ \text{kW}$$

Example 2.16

A 3ϕ, three wire 120 V, RYB system feeds a Δ-connected load whose phase impedance is 30∠45° Ω. Find the phase and line currents in the system and draw the phasor diagram. **[AU Nov/Dec, 2012]**

Solution

Given Load impedance, Z_{ph} = 30∠45° Ω and line voltage, $|\overline{V}_L|$ = 120 V

For a delta connected balanced load, $|\overline{V}_{\text{ph}}| = |\overline{V}_L| = 120\ \text{V}$

The phase current in a delta connected balanced load is given by

$$|\overline{I}_{RY}| = |\overline{I}_{YB}| = |\overline{I}_{BR}| = |\overline{I}_{\text{ph}}| = \frac{|\overline{V}_{\text{ph}}|}{Z_{\text{ph}}} = \frac{120}{30} = 4\ \text{A}$$

The line current in a delta connected balanced load is given by

$$|\bar{I}_R| = |\bar{I}_Y| = |\bar{I}_B| = |\bar{I}_L| = \sqrt{3}\,|\bar{I}_{\text{ph}}| = \sqrt{3} \times 4 = 6.92 \text{ A}$$

For drawing the phasor diagram, the angle between $\bar{V}_{\text{ph}}(\bar{V}_L)$ and $\bar{I}_{\text{ph}}$ and angle between $\bar{V}_{\text{ph}}(\bar{V}_L)$ and $\bar{I}_L$ are required.

Here, $$\bar{I}_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{\bar{Z}_{\text{ph}}} = \frac{120}{30\angle 45°} = 4\angle -45° \text{ A}$$

Therefore, the angle between $\bar{V}_{\text{ph}}(\bar{V}_L)$ and $\bar{I}_{\text{ph}}$ is 45° for any phase and since it is negative, it indicates that the phase current lags the phase (line) voltage.

Also, it is known that the angle between line and phase current is 30°. Hence, the angle between $\bar{V}_{\text{ph}}(\bar{V}_L)$ and $\bar{I}_L$ is 75° (i.e., 30° + 45°) for any phase. With the help of these details, the phasor diagram of the delta connected load is shown in Figure E2.16.

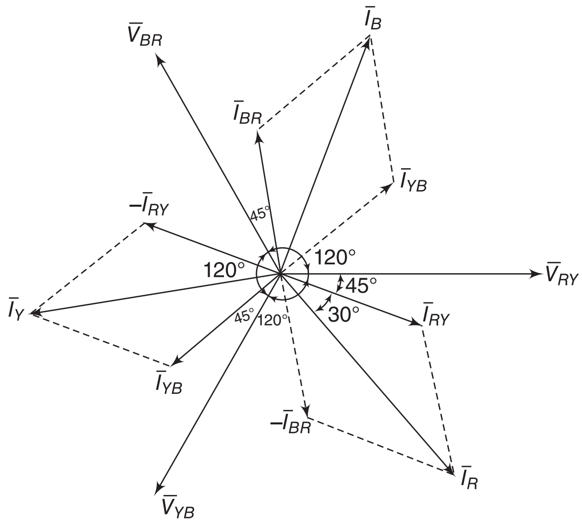


Figure E2.16 Phasor diagram of delta connected load

Example 2.17

Detremine the currents for the unbalanced delta connected load consisting of $Z_{RY} = 30 + j40\ \Omega$, $Z_{YB} = 8 - j4\ \Omega$ and $Z_{BR} = 15 + j12\ \Omega$. Assume the phase sequence to be RYB, $V = 200$ V.

Solution

Given load impedances are: $Z_{RY} = 30 + j40\ \Omega$, $Z_{YB} = 8 - j4\ \Omega$, and $Z_{BR} = 15 + j12\ \Omega$, line voltage, $\bar{V}_L = 200$ V.

For a delta connected load, $|\bar{V}_{\text{ph}}| = |\bar{V}_L| = 200$ V

Therefore, the phase currents in the delta connected load are given by

$$\bar{I}_{RY} = \frac{\bar{V}_{\text{ph}}}{Z_{RY}} = \frac{200\angle 0^{\circ}}{30 + j40} = 4\angle -53.13° \text{ A}$$

$$\bar{I}_{YB} = \frac{\bar{V}_{\text{ph}}}{Z_{YB}} = \frac{200\angle -120°}{8 - j4} = 22.36\angle -93.44° \text{ A}$$

and $$\bar{I}_{BR} = \frac{\bar{V}_{\text{ph}}}{Z_{BR}} = \frac{200\angle -240°}{15 + j12} = 10.41\angle 81.34° \text{ A}$$

The line currents of the system are

$$\bar{I}_R = \bar{I}_{RY} - \bar{I}_{BR} = 4\angle -53.13 - 10.41\angle 81.34° = 13.51\angle -86.46° \text{ A}$$

$$\bar{I}_Y = \bar{I}_{YB} - \bar{I}_{RY} = 22.36\angle -93.44° - 4\angle -53.13° = 19.48\angle -101.07° \text{ A}$$

and $$\bar{I}_B = \bar{I}_{BR} - \bar{I}_{YB} = 10.41\angle 81.34° - 22.36\angle -93.44° = 32.74\angle 84.90° \text{ A}$$

Example 2.18

A symmetrical 3ϕ, three wire 400 V supply is connected to a delta-connected load. Impedances in each branch are $Z_{RY} = 10\angle 30°\ \Omega$, $Z_{YB} = 10\angle 45°\ \Omega$ and $Z_{BR} = 2.5\angle 60°\ \Omega$. Find its equivalent star-connected load.

[May/Jun, 2014]

Solution

The equivalent impedances in star-connected load are given by

$$Z_R = \frac{Z_{RY} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$= \frac{10\angle 30° \times 2.5\angle 60°}{10\angle 30° + 10\angle 45° + 2.5\angle 60°}$$

$$= 0.724 + j0.864 = 1.127\angle 50.03°\ \Omega$$

$$Z_Y = \frac{Z_{YB} Z_{RY}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$= \frac{10\angle 30° \times 10\angle 45°}{10\angle 30° + 10\angle 45° + 2.5\angle 60°}$$

$$= 3.7 + j2.59 = 4.51\angle 34.99°\ \Omega$$

$$Z_B = \frac{Z_{YB} Z_{BR}}{Z_{RY} + Z_{YB} + Z_{BR}}$$

$$= \frac{10\angle 45° \times 2.5\angle 60°}{10\angle 30° + 10\angle 45° + 2.5\angle 60°}$$

$$= 0.48 + j1.022 = 1.129\angle 64.84°\ \Omega$$

Example 2.19

A symmetrical 3ϕ, 100 V, three wire supply feeds an unbalanced star connected load with impedances of the load as $Z_R = 5\angle 0°\ \Omega$, $Z_Y = 2\angle 90°\ \Omega$ and $Z_R = 4\angle -90°\ \Omega$. Find the line currents and voltage across the impedances.

[AU April/May, 2014]

Solution

Given load impedances are: $Z_R = 5\angle 0°\ \Omega$, $Z_Y = 2\angle 90°\ \Omega$ and $Z_B = 4\angle -90°\ \Omega$.

The line currents and phase voltages of unbalanced star connected three phase three wire system can be determined using any one of the following methods:

(i) Star to delta conversion
(ii) Mesh analysis
(iii) Milliman's theorem

(i) Star to Delta Conversion

The equivalent impedances for each branch in delta-connected load are

$$Z_{RY} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_B}$$

$$= \frac{(5\angle 0^\circ)(2\angle 90^\circ) + (2\angle 90^\circ)(4\angle -90^\circ) + (4\angle -90^\circ)(5\angle 0^\circ)}{4\angle -90^\circ}$$

$$= \frac{12.80\angle -51.34^\circ}{4\angle -90^\circ} = 3.201\angle 38.65^\circ\ \Omega$$

$$Z_{YB} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_R}$$

$$= \frac{(5\angle 0^\circ)(2\angle 90^\circ) + \left(2\angle 90^\circ\right)(4\angle -90^\circ) + (4\angle -90^\circ)(5\angle 0^\circ)}{5\angle 0^\circ}$$

$$= \frac{12.80\angle -51.34^\circ}{5\angle 0^\circ} = 2.56\angle -51.34^\circ\ \Omega$$

$$Z_{BR} = \frac{Z_R Z_Y + Z_Y Z_B + Z_B Z_R}{Z_Y}$$

$$= \frac{(5\angle 0^\circ)(2\angle 90^\circ) + (2\angle 90^\circ)(4\angle -90^\circ) + (4\angle -90^\circ)(5\angle 0^\circ)}{2\angle 90^\circ}$$

$$= \frac{12.80\angle -51.34^\circ}{2\angle 90^\circ} = 6.4\angle -141.34^\circ\ \Omega$$

For a delta connected load, $|\overline{V}_{\text{ph}}| = |\overline{V}_L| = 100$ V. Therefore, $|\overline{V}_{RN}| = 100\angle 0^\circ$ V, $|\overline{V}_{YN}| = 100\angle -120^\circ$ V and $|\overline{V}_{BN}| = 100\angle -240^\circ$ V.

In a delta connected load, the phase currents are obtained as

$$\overline{I}_{RY} = \frac{\overline{V}_{RN}}{Z_{RY}} = \frac{100\angle 0^\circ}{3.201\angle 38.65^\circ} = 31.24\angle -38.65^\circ\ \text{A}$$

$$\overline{I}_{YB} = \frac{\overline{V}_{YN}}{Z_{YB}} = \frac{100\angle -120^\circ}{2.56\angle -51.34^\circ} = 39.06\angle -68.66^\circ\ \text{A}$$

and $$\overline{I}_{BR} = \frac{\overline{V}_{BN}}{Z_{BR}} = \frac{100\angle -240^\circ}{6.4\angle -141.34^\circ} = 15.62\angle -98.66^\circ\ \text{A}$$

Therefore, the line currents are obtained as

$$\overline{I}_R = \overline{I}_{RY} - \overline{I}_{BR} = (31.24\angle -38.65^\circ) - (15.62\angle -98.66^\circ) = 27.057\angle -8.65^\circ\ \text{A}$$

$$\overline{I}_Y = \overline{I}_{YB} - \overline{I}_{RY} = \left(39.06\angle -68.66^\circ\right) - (31.24\angle -38.65^\circ) = 19.705\angle -121.11^\circ\ \text{A}$$

and $$\overline{I}_B = \overline{I}_{BR} - \overline{I}_{YB} = (15.62\angle -98.66^\circ) - (39.06\angle -68.66^\circ) = 26.70\angle 128.34^\circ\ \text{A}$$

(ii) Mesh Analysis

Applying mesh analysis to each loop in the system, we get

$$100\angle 0° = \overline{I}_1(5\angle 0^{o} + 2\angle 90°) - \overline{I}_2(2\angle 90°)$$
$$= \overline{I}_1(5.38\angle 21.80°) - \overline{I}_2(2\angle 90°) \tag{1}$$
$$100\angle -120° = -\overline{I}_1(2\angle 90°) + \overline{I}_2(2\angle 90° + 4\angle -90°)$$
$$= -\overline{I}_1(2\angle 90°) + \overline{I}_2(2\angle -90°) \tag{2}$$

Subtracting Equation (1) from Equation (2), we get

$$100\angle -120° - 100\angle 0° = -\overline{I}_1(2\angle 90°) - \overline{I}_1(5.38\angle 21.80°)$$

Therefore, $\overline{I}_1 = 27.057\angle -8.65°\,\text{A}$

Substituting $\overline{I}_1 = 27.057\angle -8.65°$ in Equation (1), we get

$$\overline{I}_2 = 26.70\angle -51.66°\,\text{A}$$

Therefore, the line currents are given by

$$\overline{I}_R = 27.057\angle -8.65°\,\text{A}$$
$$\overline{I}_B = -\overline{I}_B = 26.70\angle 128.34°\,\text{A}$$
$$\overline{I}_Y = 19.705\angle -121.11°\,\text{A}$$

(iii) Using Milliman's Theorem

Taking V_{RY} as the reference line voltage i.e., $\overline{V}_{RY} = 100\angle 0°$ V, we get

Phase voltage of source as

$$\overline{V}_{RN1} = \frac{100}{\sqrt{3}}\angle -30° = 57.73\angle -30°\text{ V}, \overline{V}_{YN1} = 57.73\angle -150°\text{ V and } \overline{V}_{BN1} = 57.73\angle -270°\text{ V}$$

The load admittances of the system are

$$Y_R = \frac{1}{Z_R} = \frac{1}{5\angle 0°} = 0.2\angle 0°\ ℧,\ Y_Y = \frac{1}{2\angle 90°} = 0.5\angle -90°\ ℧ \text{ and}$$
$$Y_B = \frac{1}{4\angle -90°} = 0.25\angle 90°\ ℧$$

We know that,

$$\overline{V}_{N1N2} = \frac{\overline{V}_{RN1}\overline{Y}_R + \overline{V}_{YN1}\overline{Y}_Y + \overline{V}_{BN1}\overline{Y}_B}{Y_R + Y_Y + Y_B}$$
$$= \frac{[57.73\angle -30 \times 0.2] + [57.73\angle -150° \times 0.5\angle -90°] + [57.73\angle -270° \times 0.25\angle 90°]}{0.2 + 0.5\angle -90° + 0.25\angle 90°}$$
$$\overline{V}_{N_1N_2} = 84.15\angle -174.19°\text{ V}$$

The voltage drops across the load are,

$$\bar{V}_{RN2} = \bar{V}_{RNI} - \bar{V}_{NIN2} = 135.26\angle -8.65°\,\text{V}$$

$$\bar{V}_{YN2} = \bar{V}_{YNI} - \bar{V}_{NIN2} = 39.39\angle -31.10°\,\text{V}$$

and $$\bar{V}_{BN2} = \bar{V}_{BN1} - \bar{V}_{NIN2} = 106.75\angle 38.35°\,\text{V}$$

The line currents are,

$$\bar{I}_R = \frac{\bar{V}_{RN2}}{Z_R} = 27.052\angle -8.65°\,\text{A}$$

$$\bar{I}_Y = \frac{\bar{V}_{YN2}}{Z_Y} = 19.705\angle -121.11°\,\text{A}$$

and $$\bar{I}_B = \frac{\bar{V}_{BN2}}{Z_B} = 26.7\angle 128.34°\,\text{A}$$

Example 2.20

An unbalanced star-connected load has balanced voltage of 100 V and *RBY* phase sequence. Calculate the line currents and the neutral current. Take $Z_R = 15\ \Omega$, $Z_B = (10 + j5)\Omega$, and $Z_Y = (6 - j8)\Omega$.

[AU April/May, 2013]

Solution

Given Three phase four wire unbalanced star connected load, Line voltage, $\bar{V}_L = 100$ V, Load impedances are: $Z_R = 15\ \Omega$, $Z_B = (10 + j5)\Omega$, and $Z_Y = (6 - j8)\Omega$ and *RBY* phase sequence.

For a star connected load, $|\bar{V}_{RN}| = |\bar{V}_{YN}| = |\bar{V}_{BN}| = |\bar{V}_{\text{ph}}| = \dfrac{|\bar{V}_L|}{\sqrt{3}} = \dfrac{100}{\sqrt{3}} = 57.74$ V

In a star connected load, the line or phase currents are given by

$$\bar{I}_R = \frac{\bar{V}_{\text{ph}}}{Z_R} = \frac{57.74}{15} = 3.849\ \text{A}$$

$$\bar{I}_B = \frac{\bar{V}_{\text{ph}}}{Z_B} = \frac{57.74\angle -120°}{10 + j5} = \frac{57.74\angle -120°}{11.180\angle 26.56} = 5.164\angle -146.56°\ \text{A}$$

and $$\bar{I}_Y = \frac{\bar{V}_{\text{ph}}}{Z_Y} = \frac{57.74\angle -240°}{6 - j8} = \frac{57.74\angle -240°}{10\angle -53.13} = 5.774\angle -186.87°\ \text{A}$$

The neutral current in the star connected unbalanced load is given by

$$\bar{I}_N = \bar{I}_R + \bar{I}_Y + \bar{I}_B$$

Substituting the known values in the above equation, we get

$$\begin{aligned}\bar{I}_N &= 3.849 + 5.164\angle -146.56° + 5.774\angle -186.87° \\ &= 3.849 - 4.309 - j2.845 - j5.732 + j0.6906 \\ &= -6.192 - j2.1544 \\ &= 6.55\angle -160.81° \text{ A}\end{aligned}$$

Example 2.21

An unbalanced star-connected load is supplied from a three-phase, 440 V symmetrical system. Determine the line currents and power input to the circuit shown in Figure E2.21. Assume *RYB* sequence.

[AU April/May, 2010]

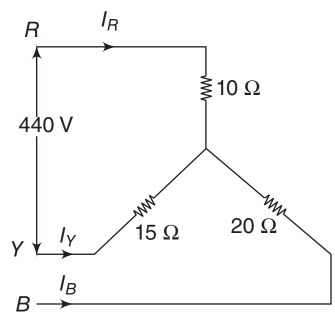


Figure E2.21

Solution

Given Line voltage, $|\bar{V}_{RY}|$ = 440 V, Load resistances are: R_R = 10 Ω, R_Y = 15 Ω and R_B = 20 Ω.

For star connected load, $|\bar{V}_{RY}| = |\bar{V}_{YB}| = |\bar{V}_{BR}| = |\bar{V}_L| = \sqrt{3}\,|\bar{V}_{\text{ph}}|$

Therefore, $|\bar{V}_{\text{ph}}| = \dfrac{440}{\sqrt{3}} = 254.03$ V

In star connected load, $|\bar{I}_L| = |\bar{I}_{\text{ph}}|$. Hence, the line or phase currents are obtained as

$$\bar{I}_R = \frac{\bar{V}_{\text{ph}}}{R_R} = \frac{254.03}{10} = 25.403 \text{ A}$$

$$\bar{I}_Y = \frac{\bar{V}_Y}{R_Y} = \frac{254.03\angle -120°}{15} = 16.93\angle -120° \text{ A}$$

and

$$\bar{I}_B = \frac{\bar{V}_B}{R_B} = \frac{254.03\angle -240°}{20} = 12.72\angle -240° \text{ A}$$

The total power consumed by the unbalanced star connected load is

$$P = P_R + P_Y + P_B = |\bar{V}_{\text{ph}}|\,(|\bar{I}_R| + |\bar{I}_Y| + |\bar{I}_B|)$$

Substituting the known values in the above equation, we get

$$P = 13.98 \text{ kW}$$

Assuming the system to be lossless system, the total power consumed by the load is equal to the total power supplied to the load. Therefore, the total power supplied to the load is 13.98 kW.

2.12 Introduction to Wiring

In general, electrical wiring refers to insulated conductor used to carry electricity and its associated devices. It is used to provide power to residential, commercial and industrial projects. Electrical wiring must be carefully designed, installed and maintained to ensure safety to personnel, equipment and property. In order to achieve this, the following things are needed.

(i) Requires good workmanship
(ii) Knowledge about electrical operating principle and circuit design
(iii) Familiar with recent electrical wiring accessories
(iv) Awareness and strict adherence to the Indian Standard specifications

Electrical wiring forms an integral part of the job of an engineer and it involves working with device boxes, raceways, conductors, and cables etc. Also, a proper coordination must exist between the electrical engineer and architect to ensure a good and safety wiring. In this section, housing wiring, types of electrical wiring, material and accessories used for electrical wiring are discussed.

2.12.1 Wiring Materials and Accessories

Switches

A switch is used to turn ON/OFF the electric circuit. Usually, the switch and the load to be controlled are connected in series. In general, 5 A and 15 A switches are used for light and heavy loads respectively and are fitted on phase conductors. Since it is to be operated continuously, it should be mechanically strong to withstand the current safely. In addition, it must be water and dust proof for longer operation. Different types of switches used in electrical wiring are explained below:

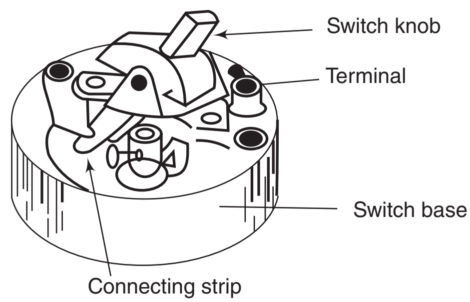


Figure 2.25 Tumbler switch

(i) Surface Switch or Tumbler Switch

These switches are mounted on the wooden block fixed directly on the surface of the wall. Such types of switches project out of the surface of the wall. These switches can be classified into one-way and two-way switches. The schematic diagram of tumbler switch is shown in Figure 2.25.

One-way switch: It is used to control the load at one point. The symbolic representation of one-way switch and its control circuit are shown in Figures 2.26(a) and (b) respectively.

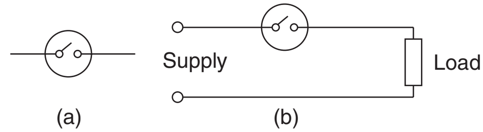


Figure 2.26 One-way switch (a) symbolic representation (b) control circuit

Two-way switch: It is used to control the load at two different points. The symbolic representation of two-way switch is shown in Figure 2.27.

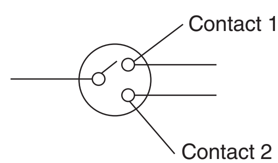


Figure 2.27 Symbolic representation of two-way switch

(ii) Flush Switch

The flush switch is fixed with the wall and it does not project out. These switches can also be classified into one-way and two-way switches.

(iii) Pull Switches or Ceiling Switches

The pull switches are fixed on the ceiling and all the live parts are out of reach of the operator. The switch has a strong mechanical action and it is usually operated with a single pull on the chord for ON/OFF.

(iv) Rotary Snap Switches

The rotary snap switch consists of an insulated handle to which the blades are fixed. These blades move in steps by the movement of the handle and make contact with the terminals of the electric circuit. The movement of the handle is controlled by cam or spring. As the handle is moved by a quarter turn, the blade is released and moves over quickly (with the help of spring) to make or break the circuit. These switches are available in one-or two-way patterns.

(v) Push Button Switch

This type of switch consists of one blade only. The blade is given a rocking action by press buttons and its movement is controlled by a cam and a spring. Thus, the blade opens or closes with quick motion.

(vi) Iron-Clad Water Tight Switches

Such switches are of cast iron and have robust construction. A cook gasket is fitted between the case and the cover which makes it watertight.

Lamp Holder

Lamp holder is used to hold the lamps for lighting purposes. The different types of lamp holders are:

(i) Pendent holder
(ii) Batten holder – for incandescent bulbs
(iii) Screw lamp holders – for bulbs rated 200 W and above
(iv) Fluorescent lamp holders – for fluorescent tubes
(v) Starter holders – for tube light starters

Lamp Holder Adopter

It is used for tapping temporary power for small portable electric appliances from lamp holders.

Ceiling Roses

These are used to provide a safe and efficient connection between load and circuit wiring. It is made up of moulded plastic type with back plate to provide terminals.

Mounting Blocks

These are nothing but wooden round blocks. They are used in conjunction with ceiling roses, batten lamp holders, surface switches, ceiling switches, etc.

Socket Outlets

The socket outlets have all insulated base with moulded or socket base having three terminals L, N and E. Generally, the two terminals, L and N, are connected to the load terminals and the third terminal E is used for a earth connection. The general diagram of socket is shown in Figure 2.28.

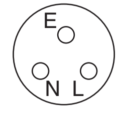


Figure 2.28 Socket

Plugs

These are used for tapping power from socket outlets.

Main Switch

The switch which is used to ON/OFF the entire load manually is called main switch. It is connected to the main supply and its rating depends on the load current. The different types of main switch are double poles and triple poles.

Distribution Fuse Boards

In industries or in very big buildings where a large number of circuits are to be wired, distribution fuse boards are used. They are usually iron clad designed with a large space for wiring and splitting the circuits. The fuse bank in the distribution board can easily be removed.

Accessories Used

The different accessories used for wiring are screw driver, side cutting plier, long nose plier, slip plier, pocket knife, hammer, wood saw, hacksaw, chisel, scratch awl, hand drill, auger hit, raw plug tool, centre punch, blow lamp, wire gauge, etc.

2.13 House Wiring

House wiring is to deal with the distribution system within the domestic premises. The wiring requirements may vary among the different consumers. House wiring is generally done for consumption of electrical energy at 230 V, single-phase or at 400 V, three phase. In the latter case, the total load in the house is expected to be divided among the three phases. An earth wire is used to connect all the power plugs from where large quantity of electrical energy is tapped by using various electrical appliances like heater, electric iron, hot plate, etc.

2.13.1 Systems of House Wiring

The systems of house wiring depend on various factors, like durability, safety, appearance, cost and consumer's budget. The different systems of house wiring are:

(i) Cleat wiring
(ii) Wooden casing capping wiring
(iii) Batten wiring
(iv) Conduit wiring

(i) Cleat Wiring

Generally, Vulcanised Indian Rubber (VIR) or Poly Vinyl Chloride (PVC) insulated wires are used in this wiring. These wires are braided, compounded and fixed on walls or ceilings using porcelain, plastic or wooden cleats. This cleat has two parts, namely base and cap. The cable is placed in the groove available in the base and cap is placed over it using the screws. The cost of this type of wiring is cheap when compared to other types of wiring. This wiring is a temporary wiring used in construction buildings or sites for lighting. The life time of this wiring is very minimum. The schematic representation of cleat wiring and the cross section of a cleat are shown in Figures 2.29(a) and (b) respectively.

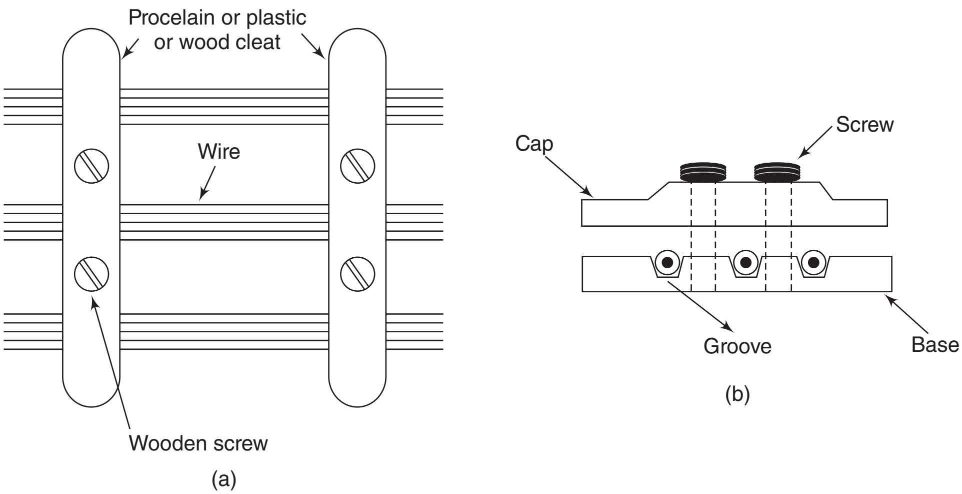


Figure 2.29 (a) schematic diagram of Cleat wiring (b) cross section of a cleat

(ii) Wooden Casing Capping

The commonly used wiring in residential buildings is the wooden casing wiring. The required materials for this wiring are casing, capping, VIR or PVC cables, screws, blocks, etc. The casing is rectangular in shape and it is made using seasoned teak wood or other defect less wood. Usually, grooves are provided on the surface of the casing to place the cables. The top of the casing is covered using capping. Similar to casing, the capping is a rectangular structure and it is made using seasoned teak wood or other defect-less wood. Once the casing is covered using capping, it is screwed to the walls. The schematic diagram of the wooden casing and capping is shown in Figure 2.30.

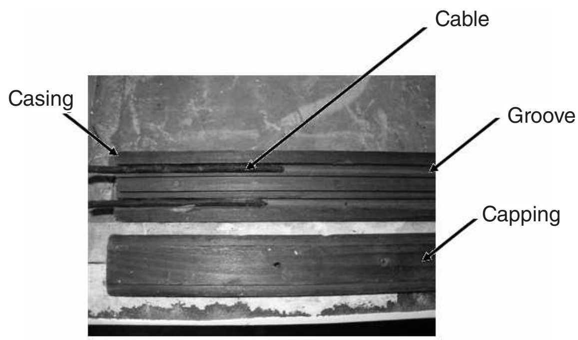


Figure 2.30 Wooden casing and capping

(iii) Batten Wiring

In general, batten is defined as a long squared timber or metal flat strip used to hold wire or fastening against a wall. Depending on the type of cable used, the batten wiring is also called PVC or cab tyre sheath (CTS) or tough rubber sheath (TRS) wiring. Here, the battens are joined using clips and then mounted to the walls using screws. The diagram to represent batten wiring is shown in Figure 2.31.

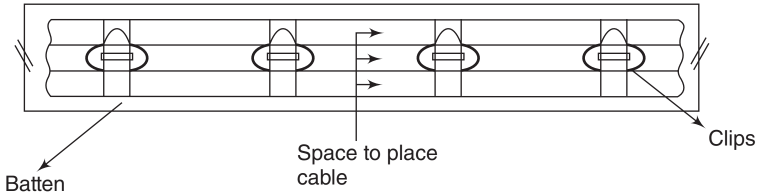


Figure 2.31 Batten wiring

(iv) Conduit Wiring

A tube or channel made up of metal or PVC used to protect the cable permanently from mechanical and electrical damages is called conduit and the wiring that used conduit is called conduit wiring. The schematic representation of conduit is shown in Figure 2.32.

The two different types of conduit wiring are: (i) Surface and (ii) Concealed conduit wirings. In surface conduit wiring, the conduit is placed on the wall surface using clamps or saddles. In concealed wiring, all the conduits are placed inside the walls and covered by plaster.

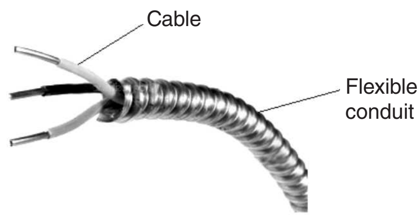


Figure 2.32 Schematic representation of conduit

2.13.2 Commonly used House Wiring

The commonly used house wirings are: (i) staircase wiring and (ii) fluorescent lamp wiring.

(i) Staircase Wiring

The house wiring in which the loads are controlled from two different places is called stair case wiring. The schematic representation of stair case wiring is shown in Figure 2.33.

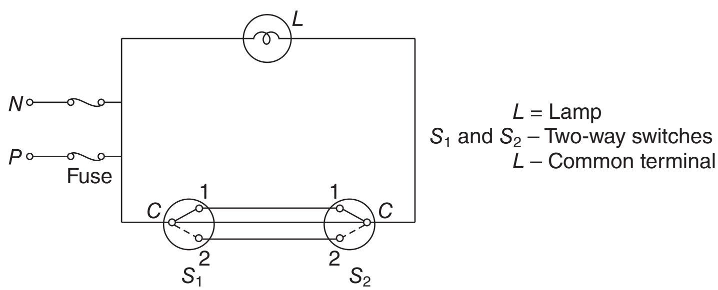


Figure 2.33 Schematic representation of stair case wiring

In this wiring, a single load is controlled using two different two-way switches. The inference that can be made from the wiring circuit shown in Figure 2.33 is listed in Table 2.2.

Table 2.2 *Inference from stair case wiring*

Position of Switch S_1	Position of Switch S_2	Condition of Load
1	1	ON
1	2	OFF
2	1	OFF
2	2	ON

(ii) Fluorescent Lamp Wiring

Another common house wiring is fluorescent lamp wiring as shown in Figure 2.34 Fluorescent tube (T) has filaments on either ends. They are coated with electron emitting material. The inside of the tube has a phosphorous coating used to convert ultraviolet radiation into visible light and give the required colour sensation. A ballast (B) is used to give a transient high voltage so as to initiate the electron movement. It is an iron cored coil having high inductance. G is a glow starter. A capacitance, C is used for improving the power factor of the circuit. Another capacitor, C_1, is used to suppress radio interference.

Working: With the switch (S) closed, the circuit gets closed. The current flows through the ballast and the starter. The glow switch suddenly breaks thereby breaking the circuit. Due to the high inductive property of the ballast, a transient high voltage is available across the filaments. Hence, electrons are emitted and travel through the tube. Such a continuous flow of electrons produces the sensation of light to human eyes.

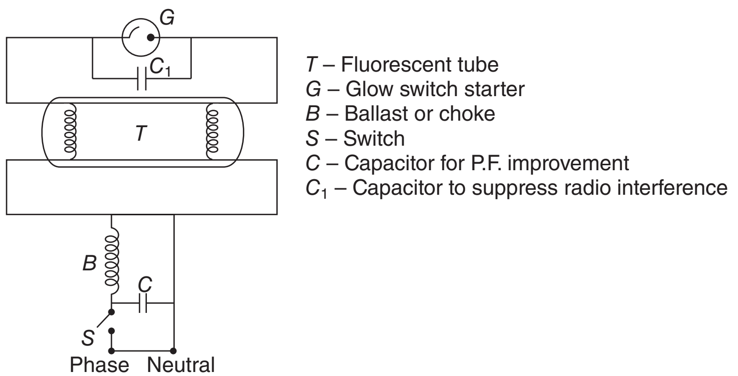


Figure 2.34 Fluorescent lamp wiring

2.14 Industrial Wiring

The industrial wiring depends on various factors like nature, size, layout, machinery, and process, etc., and will vary from industry to industry. However, it is possible to ensure an adequate, safe, reliable and economic installation, if the points listed below are kept in mind during industrial wiring. An industrial wiring must satisfy the standard "Code of Practice for Electrical Wiring Installations-System" where the voltage exceeds 650 V.

(i) Planning and Co-ordination: There should be proper co-ordination between the architect, building contractor and electrical engineer from the very beginning so that provision for substation, switchgear rooms, cable ducts, distribution boards etc. are made for future addition of loads during planning itself.

(ii) Independent Sub-Station: It is required to have an independent substation with a separate transformer and other protective devices for all industries with loads above 100 kVA. An independent sub-station has the following advantages:

(a) Reduction in tariff.
(b) More effective earth-fault protection.
(c) Use of large size motors, direct-on-line starters and welding equipment is possible without interference in supply to other consumers.
(d) Better-voltage control.

For industries with scattered heavy loads it is advisable to have more than one sub-station.

(iii) Selection of Voltage: A 400/440 V, three phase, 50 Hz voltage is quite suitable for motors and other individual loads up to 200 h.p. But for heavy loads, it is economical to go for higher operating voltages such as 1.1, 3.3, 6.6 and 11 KV.

(iv) Switch-Boards: In sub-station, feeding large plants, circuit-breakers with over-load, short-circuit and earth-leakage protection to control the outgoing feeders should be provided. In medium size plants, the outgoing feeders can be controlled by switch-fuses fitted with high rupturing capacity (H.R.C) fuses or main circuit breakers. The fuse units controlled by a circuit-breaker on the incoming side provide simple and economical arrangement. Apart from initial economy, there is less expense in maintaining and operating H.R.C fuses. Use of circuit-breakers is recommended where fuses are likely to blow frequently due to the nature of the process, because of high replacement cost of fuses. The switch boards should be located in clean and dry locations with easy access and ample space for maintenance works and also dust-tight enclosures should be used.

(v) Lighting Circuits: The wiring of light loads should be separated from motors, so that maintenance and repair can be easily carried out. Also during power shut down, the light loads alone can be operated using generators.

(vi) Load Shedding: Wiring for essential and non-essential loads is done separately, so that there is minimum inconvenience during load shedding.

(vii) Power-Factor Improvement: The desired power factor of 0.85–0.9 is to be maintained by using power factor improvement devices like shunt capacitor banks in order to avoid penalty charges from electricity board.

(viii) Selection of Conductor Size: The selection of conductor size is determined by the following factors:

(a) Total load connected on the feeder.
(b) Maximum anticipated ambient air temperature and number parallel lengths
(c) Distance of load from source supply
(d) As per I.S. : 732-1989, the permissible drop in consumer's premises shall not exceed ±5% and ±12.5 % for low /medium voltage and high voltage respectively
(e) Types of cable laying

Types of Cable

The following factors are considered while selecting the different types of cable.

(i) Based on Ambient Temperature: Vulcanised Indian Rubber (V.I.R) and Polyvinyl chloride (P.V.C) insulated cables are not suitable for ambient temperature exceeding 50°C and varnished-cambric, paper-insulated cables for ambient temperature exceeding 65°C. Metal-sheathed cables are better suited for damp locations.

(ii) Method of Laying:

(a) Underground cables in ground and trenches
(b) In cable trenches in outdoors switchyards
(c) In cable trays or cable ducts.
(d) Fixed with clamps in walls and ceilings.
(e) In conduits.

System of Wiring

The system of wiring to be followed depends on the nature and type of the industry.

(i) Main Connections:

(a) Bare conductors run on insulators, in metal ducts

(b) VIR or PVC-insulated cables installed in conduits or cable ducts.

(c) Paper-insulated, metal-sheathed, armoured cables laid directly in ground or supported on over headed catenary wire.

(d) PVC-insulated, armoured, and sheathed cables buried directly in the ground laid in trench, or catenary wire.

(e) Mineral-insulated cables.

(ii) Wiring of Sub-Mains and Sub-Circuits:

The wiring of sub-mains and sub-circuits can be done in one of the following ways:

(a) VIR or PVC insulated cables laid in steel conduit.

(b) Rubber/paper-insulated metal-sheathed or PVC-insulated and PVC-sheathed cables, run on brackets, racks, or in trenches.

(c) Same as "b" above but using armoured cables.

(d) Mineral-insulated cables.

The cables to be laid in floor should be placed in trenches covered with covers of non-combustible materials. The recommended methods used for fastening cables and conduits to steel columns in factories are shown in Figure 2.35.

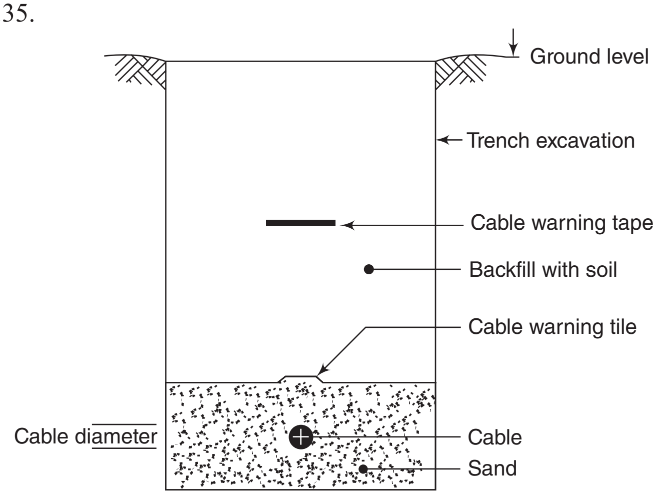


Figure 2.35 Underground cable laid in trench/ground

In industries involving fire risk, rubber/PVC-insulated cables installed in galvanised solid-drawn screwed conduit with flame proof fittings or mineral-insulated cables should be used. Wiring is generally done with radial feeders as shown in Figure 2.36. Distribution panels are placed near a group of loads. Large motors may be catered directly from the main panel.

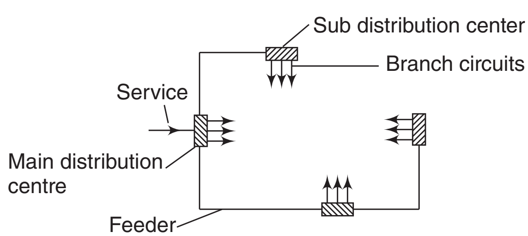


Figure 2.36 System of radial feeders

Figure 2.37 shows a power distribution in workshops.

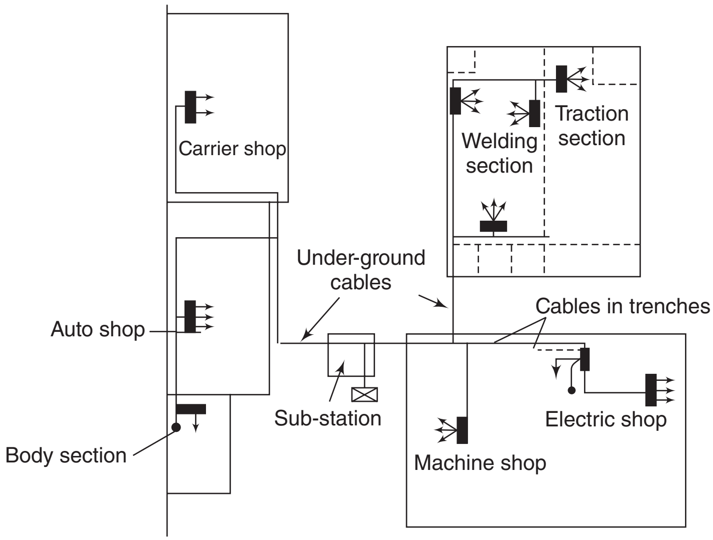


Figure 2.37 Power distribution in workshop

Over-Headed Bus-Bar System

The over-headed bus-bar system is shown in Figure 2.38. Copper or aluminium bus-bars run in metal trunking supported from the ceiling in this over-headed system. The individual machines are connected to the bus-bars using plug-in-boxes. This system can be used in places where laying of cables on floors is not permitted and also in places where frequent shifting of machines is required. This method suffers from high initial cost.

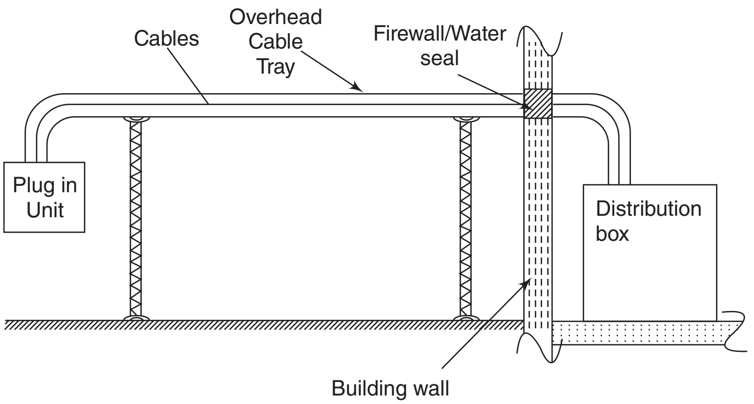


Figure 2.38 Overhead bus with Plug-in Unit

Two Marks Questions and Answers

1. State maximum power transfer theorem for DC networks. [AU Nov/Dec, 2014]

Maximum power transfer theorem for DC networks states that, "Maximum power will be transferred to the load resistance R_L by a circuit, if the resistance of R_L is the equal to the circuit resistance R_{eq} as viewed from the output terminals". This theorem is also known as "Jacobi's law".

2. Why do you short circuit the voltage source and open the current source when you find Thevenin's theorem of a network? [AU April/May, 2014]

The voltage and current sources are to be replaced by its internal resistances on applying Thevenin's theorem. When the voltage source has a zero internal, resistance, it is short circuited. When the current source has an infinite internal resistance, it is open circuited.

3. Distinguish between unbalanced source and unbalanced load. [AU April/May, 2014]

A three phase source is said to be an unbalanced source when the individual phase voltages/line voltages are not displaced from each other by 120°. Also, the algebraic sum of the phase voltages of a three phase system is not equal to zero.

A three phase load is said to be unbalanced when the individual impedances of the three phase load are not equal.

4. What is the condition for maximum power transfer in DC and AC circuits?
[AU April/May, 2016]

For DC circuits, maximum power will be transferred to the load resistance R_L by a circuit, if the resistance of R_L is the equal to the circuit resistance R_{eq} as viewed from the output terminals.

For AC circuits, maximum power will be transferred to the load impedance Z_L by a circuit, if the impedance of Z_L is the conjugate of the circuit impedance Z_S as viewed from the output terminals.

5. List the applications of Thevenin's theorem. [AU Nov/Dec, 2015]

(i) It is useful for circuit reduction.
(ii) It is used in transmission line drive calculation.
(iii) This theorem provides an effective way to simplify circuit allowing to search partial solution in the selected zone to be analysed.
(iv) It is especially useful in analysing power systems and other circuits where one particular resistor in the circuit (called the "load" resistor) is subject to change, and re-calculation of the circuit is necessary with each trial value of load resistance, to determine voltage across it and current through it.

It is used in source modelling and resistance measurement using the Wheatstone bridge.

6. State the Superposition theorem. [AU Nov/Dec, 2012]

Superposition theorem states that, "In a linear circuit consisting of more than one independent source, the total current in any part of that circuit equals the algebraic sum of the individual contributions of currents produced by each independent source separately". Superposition refers to the superposition of responses due to individual sources. It is to be noted that, this theorem is valid only for a linear circuit.

7. State the Thevenin's theorem. [AU Nov/Dec, 2012]

Thevenin's theorem states that, "Any two-terminal linear bilateral circuit comprising of energy sources and impedances can be replaced with an equivalent circuit comprising of a single Thevenin's equivalent voltage source $\bar{V}_{Th}$ in series with a Thevenin's equivalent impedance (Z_{Th}) connected to a load impedance (Z_L)". Thevenin's theorem is used to simplify complex circuits comprising various sources and impedances into Thevenin's equivalent circuit.

8. State maximum power transfer theorem. State the expression for the maximum power. [AU April/May, 2011]

Maximum power transfer theorem for DC networks states that, "Maximum power will be transferred to the load resistance R_L by a circuit, if the resistance of R_L is the equal to the circuit resistance R_{eq} as viewed from the output terminals." This theorem is also known as "Jacobi's law".

Maximum power transfer theorem for AC networks states that, "Maximum power will be transferred to the load impedance Z_L by a circuit, if the impedance of Z_L is the conjugate of the circuit impedance Z_S as viewed from the output terminals." This theorem is also known as "Jacobi's law".

The maximum power transferred to the load, $P_{max} = \dfrac{|\bar{V}_{Th}|^2}{4R_L}$.

9. What is three phase system? [AU April/May, 2013]

Three phase power system consists of three alternating currents of the same frequency are carried by three conductors. The instantaneous peak amplitudes of individual voltages are reached at different time instances as shown in Figure UQ2.9.

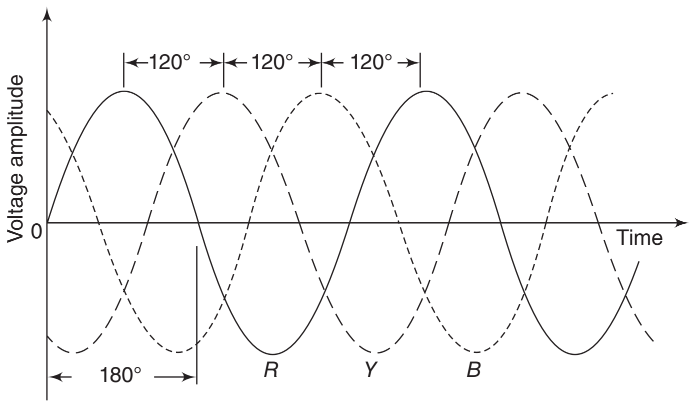


Figure UQ2.9 Three phase voltage waveform of a balanced system

Considering a voltage as reference, the other two voltages are delayed in time by one-third and two-thirds of one electrical current cycle, i.e., 120° apart. Also, it could be seen as a combination of three separate single-phase systems with a phase difference of 120° between every other voltage pair.

10. What is phase sequence of a three phase system? [AU April/May, 2013]

Phase Sequence is defined as the time order or the sequence in which the voltages in a polyphase system pass through their respective individual maximum values.

11. List the advantages of three-phase system over single-phase system. [AU April/May, 2016]

The advantages of three-phase system are:

(i) Three phase power system has higher efficiency.
(ii) It is reliable and economical.
(iii) Regulation of voltage is better.

(iv) For a given frame size, the Horse Power (HP) rating of three phase motors and kVA rating of three phase transformers is increased by 150%.
(v) Uniform torque production in motors occurs in a three phase system whereas pulsating torque is produced in the case of a single-phase system.
(vi) The amount of conductor material needed to transfer the same amount of power is reduced. The size of the conductors of a three phase balanced system is about 75% of the size of the conductor for a single-phase two wire system for the same kVA rating.
(vii) It is also possible to get single and two phase power supplies from three phase power supply. Therefore, three phase system can be used both in domestic and industrial applications.

12. When a three phase supply system is called balanced supply system?
[AU Nov/Dec, 2015]

A three phase system is called balanced three phase system, if the phase voltage of each phase has same magnitude and frequency and the phase difference between the phase voltages is 120°

13. A delta connected load has $(30 - j40)\Omega$ impedance per phase. Determine the phase current if it is connected to a 415 V, three phase, and 50 Hz supply. [AU April/May, 2013]

Solution

For delta connected load, $V_L = V_{\text{ph}}$.

Therefore, $I_{\text{ph}} = \dfrac{V_{\text{ph}}}{Z_{\text{ph}}} = \dfrac{415}{30 - j40} = 8.3\angle -306.87°$

14. In the circuit shown in Figure UQ2.14, find the value of the load impedance Z_L for maximum power to be transferred to the load. [AU Nov/Dec, 2012]

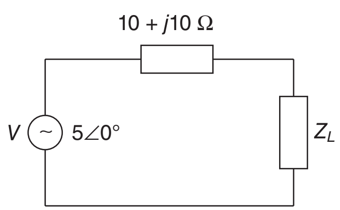


Figure UQ2.14

Using maximum power transfer theorem, we get

$$Z = 10 \; - j10 \; \Omega$$

for the maximum power to be transferred to the load.

15. Three inductive coils each having resistance of 16 Ω and reactance of $j12$ Ω are connected in star across a 400 V, 3 phase, 50 Hz supply. Calculate phase voltage. [AU Nov/Dec, 2012]

Solution

In star connected load,

$$V_L = \sqrt{3}V_{\text{ph}}$$

Therefore, $V_{\text{ph}} = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.94 \text{ V}$.

16. A three phase motor can be regarded as a balanced Y load. A three phase motor draws 5.6 kW when the line voltage is 220 V and the line current is 18.2 A. Determine the power factor of a motor. [AU Nov/Dec, 2012]

The power drawn by a three phase motor or load is given by

$$P = \sqrt{3} V_L I_L \cos\phi = 5.6 \times 10^3$$

Therefore, $$\cos\phi = \frac{5.6 \times 10^3}{\sqrt{3} \times 220 \times 18.2} = 0.807$$

REVIEW QUESTIONS

1. State and explain Thevenin's and Norton's theorems with suitable examples.
2. Explain the procedures to be followed to apply Thevenin's and Norton's theorems for solving a circuit.
3. Explain the methods to determine Z_{Th} when dependent sources are present in a circuit.
4. A Thevenin's equivalent contains V_{Th} = 20 V and R_{Th} = 8 Ω . Find the Norton's equivalent for the same.
5. A Norton's equivalent circuit contains I_N = 20 A and R_N = 10 Ω. Find the Thevenin's equivalent for the same.
6. With suitable examples state and explain the following theorems:
   (i) Superposition theorem
   (ii) Maximum power transfer theorem
7. State and prove the maximum power transfer theorem. Derive the expression for the maximum power delivered to the load.
8. What are the advantages of three-phase system?
9. What is a symmetrical system?
10. Describe the generation of three-phase voltages.
11. What are the equations of three-phase voltages?
12. For a three-phase balanced, star-connected source, obtain
    (i) Current relationship
    (ii) Voltage relationship
    (iii) Power relationship
    (iv) Phase diagram
13. What is a balanced load?
14. State the voltage and current relations for balanced, star-connected load and balanced, delta-connected load.
15. Discuss the different types of wiring bringing out the merits and demerits of each.
16. Draw a neat wiring diagram for staircase lighting and explain its working.
17. Draw the wiring diagram of fluorescent lamp. Describe its working.
18. Write a short note on wiring for residential building.
19. Give an account of the various wiring materials used.

20. Write a short note on industrial wiring.
21. Why is the neutral of the supply earthed?
22. Discuss the different types of wiring bringing out the merits and demerits of each.
23. Draw a neat wiring diagram for staircase lighting and explain its working.
24. Draw the wiring diagram of fluorescent lamp. Describe its working.
25. Write a short note on wiring for residential building.
26. Give an account of the various wiring materials used.
27. What is the necessity of earthing in domestic wiring? Discuss in brief the various methods of earthing employed.
28. Find Thevenin's equivalent resistance seen by the 4 kΩ resistor in the circuit of Figure Q2.28.

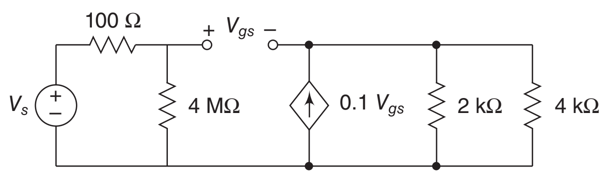


Figure Q2.28

29. Find Thevenin's equivalent circuit for the network external to the resistor *R* for the network in Figure Q2.29. Calculate the maximum power delivered to *R*?

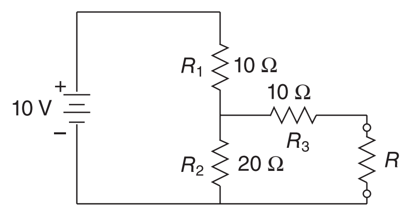


Figure Q2.29

30. Find Norton's equivalent of the circuit in Figure Q2.30.

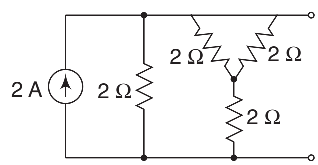


Figure Q2.30

31. Find Norton's equivalent circuit external to points *a* and *b* in Figure Q2.31.

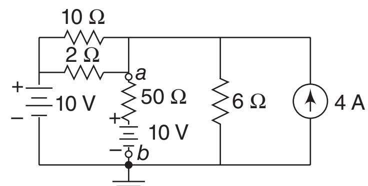


Figure Q2.31

32. Using the superposition theorem, determine the current through the 14 Ω resistor of Figure Q2.32.

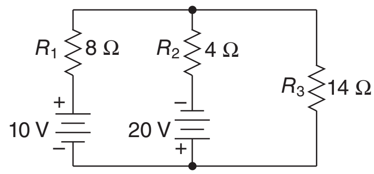


Figure Q2.32

33. Using the superposition theorem, determine the current through the 30 Ω resistor of Figure Q2.33.

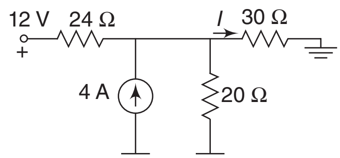


Figure Q2.33

34. For the network in Figure Q2.34, determine the level of R that will ensure maximum power to the 500 kΩ resistor. Find the maximum power to R_L.

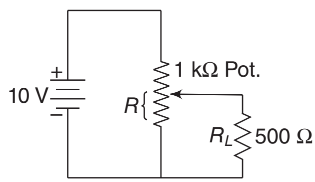


Figure Q2.34

35. Find the load impedance Z_L for the network of Figure Q2.35 for maximum power to the load, and find the maximum power to the load.

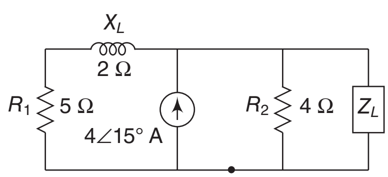


Figure Q2.35

36. A three-phase 400 V power is supplied to a balanced, star-connected load of impedance $8 + j6$ Ω in each branch. Find the line current.
37. Three inductive coils each having resistance of 16 Ω and reactance of $j12$ Ω are connected in star across a 400 V, three phase, and 50 Hz supply. Calculate the phase voltage.
38. An unbalanced four wire star connected load has balanced supply voltage of 400 V. The load impedances are $Z_R = 4 + j8$ Ω, $Z_Y = 3 + j4$ Ω and $Z_B = 15 + j10$ Ω. Determine: (i) $j4$ line currents, (ii) Neutral current and (iii) Total power. Draw the phasor diagram.
39. In a three-phase, balanced, delta system, the voltage across R and Y is $400\angle 0°$ V. What will be the voltage across Y and B? Assume RYB phase sequence.
40. In a three-phase balanced star system, the voltage across R and Y is $400\angle 0°$ V. What will be the voltage across Y and B? Assume RYB phase sequence.

CHAPTER 3

Electrical Machines

3.1 Introduction

Electrical energy system accounts for generation, transmission, distribution and utilisation of electrical energy. At every stage, different electrical machineries (static and rotating machines) serve specific purpose. An electrical machine is used to convert the electrical energy into mechanical energy and vice versa. The rotating machine which converts mechanical energy into electrical energy is called generator. Based on the type of emf generated, it is classified as alternating current (AC) generator and a direct current (DC) generator. Transformer is a static electrical machine which changes the value of the AC voltage without changing frequency for transmission and distribution systems. The rotating machine which converts electrical energy into mechanical energy is called electrical motor. Based on nature of supply voltage, it is classified as AC motors or DC motors. The various motors such as DC motor, induction motor, synchronous motor, stepper motor and brushless DC motor find their applications in domestic and industrial areas. In this chapter, working principle, construction and operation of various electrical machines are discussed in detail.

3.2 DC Machines

A highly versatile machine that operates with DC supply to generate unidirectional torque or current is known as DC machine. It works on the principle of Faraday's law of electromagnetic induction. Fleming's left and right-hand rules are used to determine the direction of torque or current developed in the DC machine. In general, the DC machine can be constructed in many forms to use for various purposes. The size of a DC machine varies from very small machine used in a quartz crystal watch, to a giant, 75,000 kW or more rolling-mill machine. It is also easily adaptable for drives with a wide range of speed control and fast reversal. Depending on the generation of torque or emf generated, a DC machine is classified as DC motor and DC generator. If the DC machine is used to generate unidirectional torque, then it is called DC motor, whereas if the DC machine generates unidirectional current, it is called DC generator. In this chapter, the working principle, construction, working, equations governing DC machines (DC generators and DC motors), their types and characteristics are discussed. In addition, a special type of motor that can run on single-phase AC or DC power supply is discussed.

3.3 DC Generator **[AU Nov/Dec, 2016]**

3.3.1 Construction

The schematic diagram of a DC generator with necessary parts is shown in Figure 3.1.

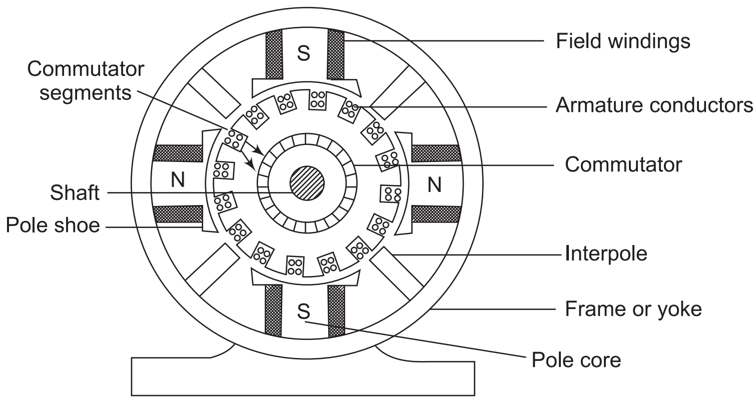


Figure 3.1 Schematic Diagram of a DC Generator

The components of a DC generator are:

- Magnetic frame or yoke
- Pole cores and pole shoes
- Field or exciting windings
- Armature core and windings
- Commutators
- Brushes
- End housings
- Bearings
- Shaft

The above nine parts can be grouped into four major components, namely magnetic field system, armature, commutator and brush assembly.

Magnetic Field System

The magnetic field system is a stationary or fixed part of the DC machine where the main magnetic flux is generated. Mainframe or yoke, pole core and pole shoes and field or exciting coils are included in this system, as described below:

Magnetic Frame or Yoke

The outer frame of a DC generator, to which pole core and pole shoes are fixed, is known as yoke. In large DC generators, it is made up of cast steel and in small DC generators it is made up of cast iron. The important functions that the yoke performs are:

- It supports pole core and pole shoes.
- It provides a low reluctance path for the magnetic flux produced by the field winding.
- It protects inner parts of the DC generator.

Pole Core and Pole Shoes

The curved pole core and pole shoes are fixed to the magnetic frame or yoke with the help of bolts. The poles in the DC generator are called salient poles, since they are projected inwards. Thin cast steel or iron with or without lamination is used to make pole core and pole shoes. Using lamination eddy current losses can be reduced. The important functions that the pole core and pole shoes perform are:

- Field or exciting coils are wound around the pole core.
- Helps in uniform distribution of magnetic flux to the armature.
- Helps in increasing the cross-sectional area of the magnetic circuit, which in turn reduces the reluctance of the magnetic path.

Field Coils or Exciting Coils

The enamelled copper wire wound over each pole core to produce the required magnetic field is called *field or exciting coils*. It always requires a relatively small DC power to produce the required strong magnetic field. These coils are connected in such a way that when DC current is used for excitation, the poles on which they are wound attain opposite polarity i.e., the poles get magnetised to produce the required flux.

Armature

The rotating component of the DC generator is called armature and it consists of a laminated cylinder called armature core, placed over the shaft.

Armature Core

The drum or cylindrical component fixed to the rotating shaft in a DC generator is the armature core. It accommodates the armature winding in the grooves or slots provided at its outer periphery. The armature core serves the following purposes:

- Provides space for conductors in the slots.
- Provides a low reluctance path for the magnetic flux.

In armature core, by using silicon steel material, the hysteresis loss produced due to the reversal of flux is reduced. Also, in order to reduce the eddy current loss produced due to the induced emf in the armature, lamination with 0.3 to 0.5 mm thickness stamping is used in the armature core.

Armature Windings

The insulated conductors made up of bands of steel wire are placed in the armature slots. These conductors are suitably connected by winding around the armature core, which forms the armature winding. Since the mechanical power to electrical power conversion takes place here, armature winding is called the heart of DC generator. Based on the way the armature conductors are connected, armature windings are classified as:

- Lap winding
- Wave winding

Lap Winding

If the armature conductors are connected in such a way that the number of poles and number of parallel paths are equal, then it is called a *lap winding*. If a DC generator has P number of poles and Z number of armature conductors, then there will be P parallel paths and Z/P conductors will be connected in series per parallel path.

Wave Winding

The armature conductors are connected in such a way that if the number of parallel paths is two, irrespective of the number of poles, then it is called *wave winding*. If a DC generator has Z number of armature conductors, then there will be two parallel paths and in each parallel path $Z/2$ number of armature conductors will be connected in series. The comparison between lap and wave windings is listed in Table 3.1.

Table 3.1 *Comparison between Lap Winding and Wave Winding*

S. No.	Lap Winding	Wave Winding
1	The number of parallel path is equal to the total of number poles i.e., $A = P$	The number of parallel paths is equal to two i.e., $A = 2$
2	It is known as parallel or multiple winding.	It is known as two-circuit or series winding.
3	EMF induced in the machine is less.	EMF induced in the machine is more.
4	Number of brushes required is equal to the number of parallel paths.	Number of brushes required is two
5	Different types of lap winding are: simplex and duplex lap winding.	Different types of wave winding are: progressive and retrogressive wave winding.
6	Efficiency of the machine using this winding is less.	Efficiency of the machine using this winding is high.
7	Equaliser ring is the additional component while using this winding.	No such component is required.
8	Winding cost is high since more number of conductors is required.	Winding cost is low since less number of conductors is required.

Commutators

The cylindrical wedge-shaped hard-drawn copper bars or segments, which rotate along with the armature, are called commutators. A ring shape is formed around the armature shaft using these commutator segments. Each commutator segment is insulated from each other and also from the rotating shaft. Ends of each armature coil are connected to the commutator segment. The functions that the commutator of a DC generator serves are:

- Through the brushes, it provides a connection between the rotating armature conductors and the stationary external circuit.
- The alternating current induced in the armature conductor is converted into unidirectional current in a DC generator.

Brushes

A set of carbon or graphite components attached to the rotating armature gently via commutator, connecting the external circuit to the DC generator, are called *brushes*. The main purpose of brushes is to tap the electrical power generated in the rotating armature. Brush box or holder is a metal box supporting the brushes. Springs are used to adjust the pressure exerted on the commutator by the brushes.

End Housings

The components attached to the yoke ends that provide support to the bearings is called end housings. Both bearing and brushes get support from front housings, whereas the rear housings support only the bearings.

Bearings

Fitting a high carbon steel ball or roller bearing in the machine can reduce the friction existing between the rotating and stationary parts of the DC generator.

Shaft

The mechanical power transfer to the machine is done with the help of a mild steel shaft having maximum breaking strength. Rotating parts of the DC generator like armature core, commutator etc., are attached to the shaft.

3.3.2 Working

A DC generator is a dynamic DC machine which generates electrical energy from mechanical energy. The emf induced is called dynamically induced emf. It operates on the principle of Faraday's law of electromagnetic induction, which states that whenever a current carrying conductor cuts the magnetic flux, a dynamically induced emf is generated. Its magnitude depends on the rate of change of magnetic flux linked with the conductor. If the conductor is connected to a closed circuit, then a current will flow through it. The basic elements required in a DC generator to generate a dynamically induced emf are:

- A steady magnetic field
- A current carrying conductor and
- Relative motion between conductor and magnetic field

Working of a DC Generator

The working of a DC generator is illustrated in Figure 3.2.

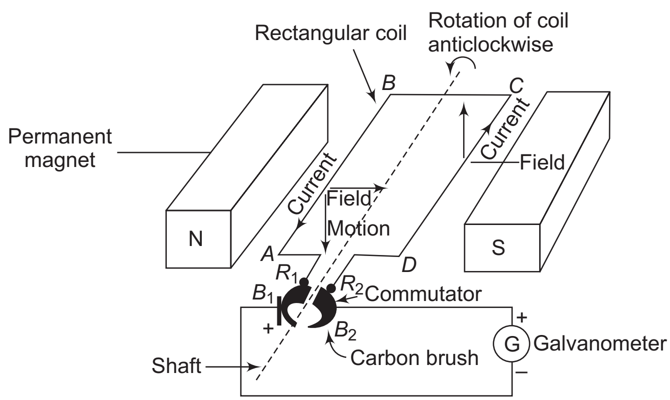


Figure 3.2 Working of a DC Generator

The basic working of a DC generator can be explained with the help of a single rectangular coil *ABCD*, as shown in Figure 3.2. This rectangular coil is placed between two opposite poles of the magnet. When the field coil wound over the magnet is excited by the DC source, the magnets get energised and magnetic flux is generated between these opposite poles. The rectangular coil connected to the shaft gets rotated in a specific direction driven by the prime mover. Therefore, the

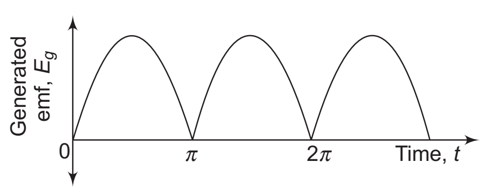


Figure 3.3 Unidirectional emf Generated in a DC Generator

rectangular coil, when it rotates about its axis in the magnetic field i.e., when it moves from horizontal to vertical position, it cuts the magnetic field generated by the field windings. Hence, an emf is induced in both the sides of the coil i.e., on both *AB* and *CD*. Since the loop is closed, a current will be circulated through the loop. Using Fleming's right-hand rule, the direction of current can be determined. The emf generated in a DC generator is shown in Figure 3.3.

Requirement of a Split-ring Commutator

Consider an armature core, rotating in clockwise direction, which makes the current in the left conductor i.e., *AB* to move upwards and the current in the right conductor i.e., *CD* to move downwards, as shown in Figure 3.4(a). It is clear that in the load, the current is flowing from right to left, as shown in Figure 3.4(a). The direction of current in both the conductors remains same till the armature core completes a half rotation. After half-rotation of the armature core, the current through the conductors gets reversed since it is placed in the armature core i.e., conductor *AB* carries current in the downward direction and conductor *CD* carries current in the upward direction. This reversal of current in the armature conductor will make the current to flow from left to right in the load, which results in alternating current. But the output of DC generator should be unidirectional. Hence, some mechanism is required to make the current to flow through the load in the same direction, irrespective of the rotation of armature core. This can be achieved by using a split-ring commutator. When it is attached to the conductor, it rotates, thereby making the direction of current unidirectional in the load. This mechanism is shown in Figure 3.4(b).

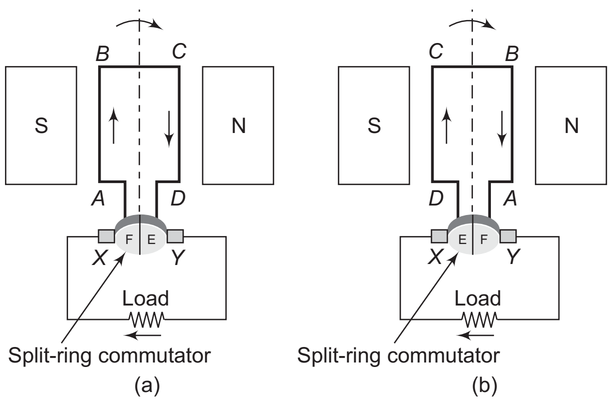


Figure 3.4 Working of a Split-ring Commutator

3.3.3 EMF Equation **[AU Nov/Dec, 2012]**

In a DC generator, when the armature core is rotated using the prime mover in the magnetic field, an induced emf is generated in the armature windings. This induced emf in the armature windings is called generated emf, denoted as E_g. An expression for E_g is obtained as follows:

Let P be the total number of poles of the DC generator, ϕ be the flux produced per pole in Webers, Z be the total number of armature conductors, N be the armature speed in rpm, A be the number of parallel paths existing in the armature winding.

In one revolution of armature core, the total flux cut by one conductor of the armature is given by

$$d\phi_T = P\phi \tag{3.1}$$

The time taken by the armature core to complete one revolution is given by

$$dt = \frac{60}{N} \tag{3.2}$$

According to Faraday's law, the average emf induced in one armature conductor is given by

$$e_g = \frac{d\phi_T}{dt} \tag{3.3}$$

Substituting Eqns. (3.1) and (3.2) in the above equation, we get

$$e_g = \frac{P\phi N}{60} \tag{3.4}$$

Since the total number of conductors connected in series, per parallel path, is given by Z/A, the average emf induced in the armature is given by

$$E_g = e_g \times \frac{Z}{A}$$

Substituting Eqn. (3.4) in the above equation, we get the generated emf in the armature of DC generator as

$$E_g = \frac{\phi ZNP}{60A} \tag{3.5}$$

Therefore, from the above equation, it is clear that the induced emf in the DC generator is directly proportional to the speed and flux per pole. Hence, changing the direction of the magnetic field or the direction of the rotating armature core can change the polarity of the induced emf. But if both the magnetic field and armature core rotation are reversed, then the polarity of the induced emf remains the same.

Case (i) If the armature windings are lap wound, then $A = P$. Therefore, from Eqn. (3.5), the induced emf in the DC generator becomes

$$E_g = \frac{\phi ZN}{60}$$

Case (ii) If the armature windings are wave wound, then $A = 2$. Therefore, from Eqn. (3.5), the induced emf in the DC generator becomes

$$E_g = \frac{\phi ZNP}{120}$$

Example 3.1

A four-pole generator with wave wound armature has 51 slots, each having 24 conductors per slot. The flux per pole is 0.01 Wb. At what speed must the armature rotate to give an induced emf of 250 V? What will be the voltage developed, if the winding is lap-connected and the armature rotates at the same speed?

[AU April/May, 2008]

Solution

Given $P = 4$, Number of slots = 51, Number of conductors per slot = 24, $\phi = 0.01$ Wb and $E_g = 250$ V

The total number of armature conductors in DC generator is

$$Z = 51 \times 24 = 1224$$

Case (i) When the DC generator has wave-connected armature winding i.e., $A = 2$.
It is known that the emf generated in a DC generator is given by

$$E_g = \frac{\phi ZNP}{60A}$$

Substituting the known values in the above equation, we get

$$250 = \frac{0.01 \times 4 \times N \times 1224}{60 \times 2}$$

Therefore, speed at which armature rotates is given by $N = 613$ rpm.

Case (ii) When the wave-connected armature winding is replaced by lap-connected winding, $A = P = 4$. It is given that the armature rotates at same speed as rotated in wave winding. Therefore, $N = 613$ rpm.

Hence, the emf generated in a DC generator with lap-connected winding is

$$E_g = \frac{0.01 \times 4 \times 613 \times 1224}{60 \times 4} = 125.05 \text{ V}$$

3.4 Types of DC Generator and its Equivalent Circuit

3.4.1 Types of DC Generator **[Nov/Dec, 2012]**

The DC generators are classified as shown in Figure 3.5, based on the excitation given to the field windings as:

(i) **Separately excited DC generator:** The required power for exciting the field windings is obtained from a separate DC source.

(ii) **Self excited DC generator:** The required power for exciting the field windings is obtained from the power developed in the armature of the DC generator.

The self excited DC generator is further classified based on the connection between the field winding and armature winding as:

(i) **DC shunt generator:** In this, field winding is in parallel to the armature winding.

(ii) **DC series generator:** In this, field winding is in series to the armature winding.

(iii) **DC Compound generator:** It consists of two windings; one is connected in series and other is connected in parallel to the armature windings. Based on these winding connections, it is further classified as:

   (a) DC long-shunt compound generator
   (b) DC short-shunt compound generator

If the magnetic flux generated by the series field winding aids the magnetic flux generated by the shunt field winding, the DC compound generator is said to be cumulatively compound. Conversely, if the magnetic flux generated by the series field winding opposes the magnetic flux generated by the shunt field winding, the DC compound generator is said to be differentially compound. It is noted that both long-shunt and short-shunt compound generators can be either cumulative or differentially compounded generator.

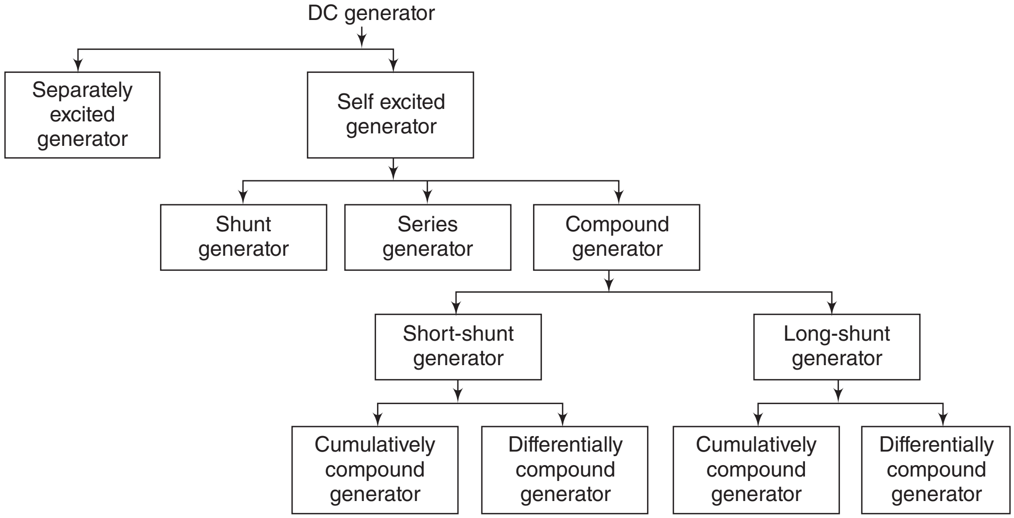


Figure 3.5 Types of DC generator

3.4.2 Electrical Equivalent Circuits, Current and Voltage Equations of DC Generator

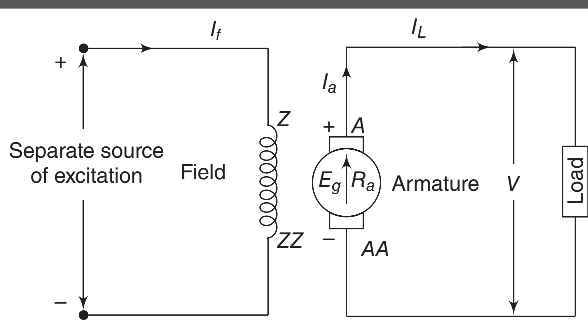
Let R_a, R_{sh} and R_{se} be the resistances of armature, shunt field and series field respectively, I_a, I_{sh} and I_{se} be the winding currents through the armature, shunt field and series field respectively, I_L be the load current, V and V_{sh} be the terminal voltage or voltage across the load and voltage across the shunt field winding respectively, V_b and V_{ar} be the voltage drop across the brush contact resistance and voltage drop due to armature reaction respectively and E_g is the emf generated at the armature of the DC generator.

Terminal Markings

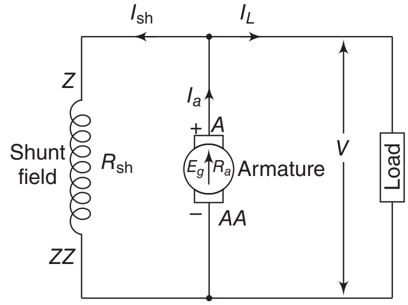
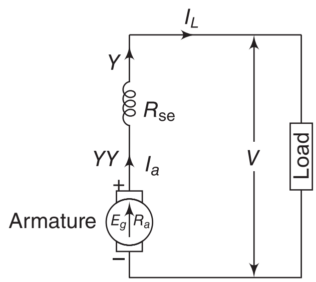
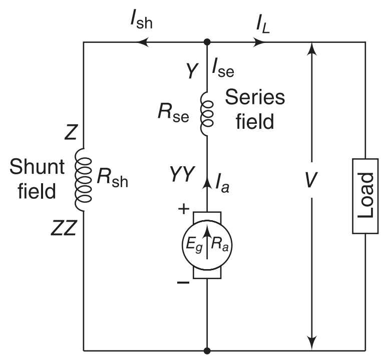
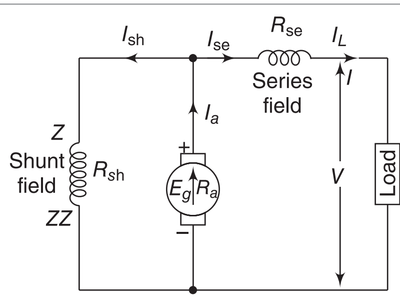
In DC generator, the symbols used to indicate shunt field windings, series field windings and armature terminals are Z and ZZ, Y and YY and A and AA respectively.

The equivalent circuit, current and voltage equations of different types of DC generator are given in Table 3.2.

Table 3.2 *Current and Voltage equations of DC generator*

Type of Generator	Equivalent Circuit	Armature Current	EMF Equation	Field Current
Separately excited		$I_L = I_a$	$E_g = V + I_aR_a$	I_f

(Contd.)

DC shunt		$I_a = I_L + I_{sh}$	$E_g = V + I_a R_a$	$I_{sh} = \dfrac{V}{R_{sh}}$
DC series		$I_a = I_L = I_{se}$	$E_g = V + I_a (R_a + R_{se})$	$I_{se} = I_L$
DC long-shunt		$I_a = I_L + I_{sh}$	$E_g = V + I_a(R_a + R_{se})$	$I_{sh} = \dfrac{V}{R_{sh}}$, $I_{se} = I_a$
DC short-shunt		$I_a = I_L + I_{sh}$ $= I_{se} + I_{sh}$	$E_g = V + I_a R_a + I_L R_{se}$	$I_{sh} = \dfrac{V_{sh}}{R_{sh}}$, $V_{sh} = V + I_L R_{se}$

Example 3.2

A four-pole lap-wound DC shunt generator has a useful flux/pole of 0.6 Wb. The armature winding consists of 200 turns, each turn having a resistance of 0.003 Ω. Calculate the terminal voltage when running at 1000 rpm, if armature current is 45 A.

Solution

Given $P = 4$, $N = 1000$ rpm, $I_a = 45$ A, $\phi = 0.6$ Wb, Number of turns = 200, Resistance per turn = 0.003 Ω
Here, Armature resistance, $R_a = 0.003 \times 200 = 0.6\ \Omega$
and the total number of armature conductors, $Z = 2 \times$ Number of turns $= 2 \times 200 = 400$.

In DC generator, the emf generated, $E_g = \dfrac{\phi PNZ}{60A}$.

$$E_g = \frac{0.6 \times 4 \times 1000 \times 400}{60 \times 4} = 4000\ \text{V}$$

The voltage equation for a DC shunt generator is given by

$$E_g = V + I_a R_a$$
$$4000 = V + 45 \times 0.6$$

Therefore, the terminal voltage of the DC shunt generator is $V = 3973$ V

Example 3.3

A long-shunt compound generator delivers a load current of 50 A at 500 V and has armature, series field and shunt field resistances of 0.05 Ω, 0.03 Ω and 250 Ω respectively. Calculate the generated voltage and the armature current. Allow 1 V per brush for contact drop. **[AU Nov/Dec, 2006]**

Solution

Given $R_a = 0.05\ \Omega$, $R_{se} = 0.03\ \Omega$, $R_{sh} = 250\ \Omega$, $I_L = 50$ A, $V = 500$ V, V_{brush}/brush = 1 V.
The long-shunt compound DC generator with the given details is shown in Figure E3.3.

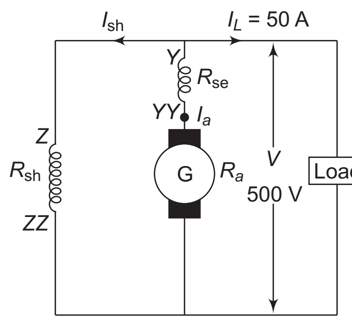


Figure E3.3

Here, $I_{sh} = \dfrac{V}{R_{sh}} = \dfrac{500}{250} = 2$ A

The armature current is given by

$$I_a = I_L + I_{sh} = 50 + 2 = 52\ \text{A}$$

Considering the brush drop in a long-shunt DC compound generator, its voltage equation is given by

$$E_g = V + I_a R_{se} + I_a R_a + \text{Brush drop}$$

Since the number of brushes existing in the machine is 2, we get the generated voltage as

$$E_g = 500 + 52 \times 0.03 + 52 \times 0.05 + 1 \times 2 = 506.16\ \text{V}$$

3.5 Characteristics of DC Generator **[AU April/May, 2015]**

The explanation of relation between the loads, excitation and terminal voltages through graphs for different DC generators is known as the characteristics of a DC generator. The important characteristics of DC generator are:

(i) **No Load Characteristic or Open Circuit Characteristic (OCC)**
The characteristic obtained by drawing the graph between the emf generated at no load condition, E_0 and field current, I_f at a constant prime mover speed is known as no load characteristic or open circuit characteristic (OCC) or magnetisation characteristics. It gives the magnetisation curve of the material used in the magnets. In practice, the shape of the curve is same for all DC generators.

(ii) **Internal or Total Characteristic**
The characteristic obtained by drawing the graph between the emf generated in the armature E_g and armature current, I_a is known as internal or total characteristic.

(iii) **External Characteristic**
The characteristic obtained between the terminal voltage, V and the load current I_L is known as external characteristic. It is also known as performance characteristic or voltage regulating curve. In general, this characteristic curve lies below the internal characteristic.

3.5.1 Separately Excited DC Generator

(i) No Load Characteristic

The circuit diagram for obtaining no load characteristic is shown in Figure 3.6(a). The potentiometer is used to vary the field current in the DC generator. The open circuit characteristics of a separately excited DC generator are shown in Figure 3.6(b). The emf generated in the DC generator is given by Eqn. (3.5). Therefore, for a given speed of the generator, $E_g \alpha \phi$, since the other terms are constant in Eqn. (3.5).

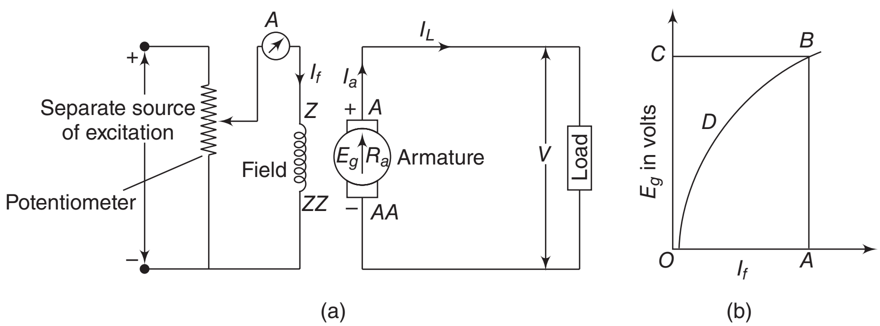


Figure 3.6 (a) Circuit diagram (b) No load characteristic curve

Therefore, it is clear that if I_f increases from its initial value, the magnetic flux ϕ increases and hence induced emf E_g also increases till the poles get saturated. This is represented as the straight line OD in Figure 3.6(b). Once the poles get saturated, a large increase in I_f is required to increase the induced emf as represented by the curve ODB.

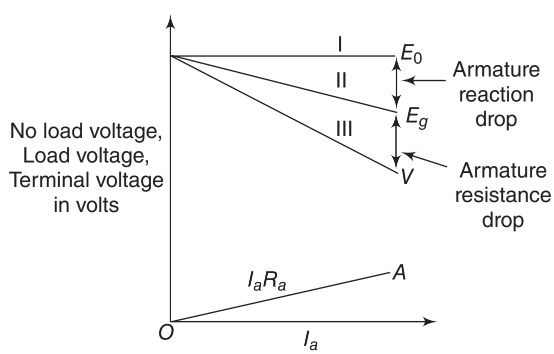


Figure 3.7 Internal and external characteristics of separately excited DC generator

(ii) Internal Characteristics

At no load condition, the graph of the generated emf E_o versus armature current I_a is almost constant due to the absence of armature reaction and armature resistance drop as shown in Graph I of Figure 3.7. But when the load is increased step by step in a DC generator, there will be a drop in the generated emf due to the armature reaction. Therefore, if the armature reaction drop is subtracted from each point in Graph I for each load, the internal characteristics can be obtained as shown in Graph II of Figure 3.7.

(iii) External Characteristics

The external characteristics of a separately excited DC generator are obtained by subtracting the armature resistance drop from internal characteristics at each and every point and are shown as Graph III in Figure 3.7.

3.5.2 DC Shunt Generator

(i) No Load Characteristic

The no load characteristic of any self excited DC generator is obtained in a similar way. The field winding is disconnected from the circuit and it is excited by the separate DC source as shown in Figure 3.8(a).

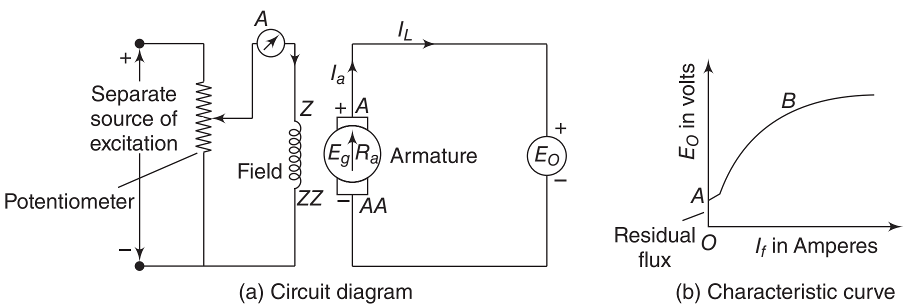


Figure 3.8 No load characteristic for DC shunt generator

The armature of the machine is allowed to rotate at a constant speed, *N*. If the field current applied to the field winding is increased with the help of potentiometer, the no load emf generated, E_o, starts increasing. If these are plotted, the no load characteristic of a DC shunt generator can be obtained as shown in Figure 3.8(b). As there exists residual magnetism in the poles, a small emf (i.e., OA) is generated even when there is no excitation given to the field windings. Hence, the no load characteristic of a DC shunt generator does not start from zero. It can be observed that the major portion of this characteristic is linear.

Voltage Build-Up in a DC Shunt Generator

The loading in a DC shunt generator should be done only after the generator reaches a no load voltage, E_o. The process by which the DC shunt generator is able to generate E_o is known as voltage build up process as shown in Figure 3.9.

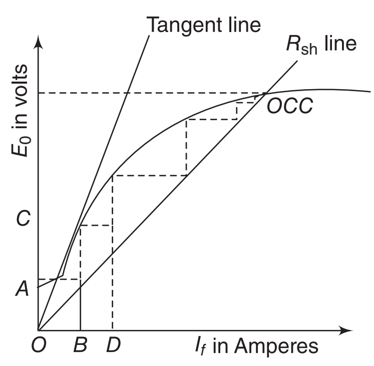


Figure 3.9 Voltage Build-Up in DC shunt generator

Conditions for Voltage Build-Up in a DC Shunt Generator

The factors which affect the voltage build up in a DC shunt generator are:

(a) **Residual flux:** If there is no residual flux, no emf is induced. Hence, there is no further increase in field flux and the induced emf remains zero.

(b) **Reversal connection of shunt field coil:** Shunt field winding should be properly connected with respect to armature windings so that the flux produced by it aids the residual flux.
(c) **Shunt field circuit resistance:** The shunt field winding resistance R_{sh} should be equal to or less than the critical resistance, R_c for the voltage to build up.
(d) **Speed of armature:** The speed at which the armature is getting rotated must be equal to or greater than the critical speed, N_c for the voltage to build up.

Critical Field Resistance, R_c

The maximum value of shunt field resistance with which the DC shunt generator will just build up voltage is known as critical field resistance, R_c. If the shunt field resistance is increased beyond R_c, then the DC shunt generator will fail to build up the voltage.

Critical speed, N_c

The minimum speed at which the armature must be rotated so that the DC shunt generator will just build up the voltage is known as critical speed, N_c. It is also defined as the speed at which the shunt field resistance, R_{sh} is equal to the critical field resistance i.e., $R_{sh} = R_c$. The DC shunt generator fails to build up the voltage if the armature is rotated below this critical speed. The determination of R_c and N_c is shown in Figure 3.10.

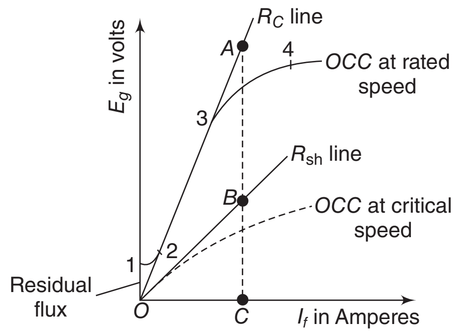


Figure 3.10 Determination of R_c and N_c

(ii) External Characteristics

The DC shunt generator is loaded once the voltage has been built up in it. During loading, the terminal voltage V drops when there is an increase in load current. The reasons for this terminal voltage drop are:
(a) **Armature resistance drop:** Since the armature resistance consumes more voltage when the load current increases, the terminal voltage gets reduced.
(b) **Armature reaction drop:** Since the demagnetising effect of armature reaction reduces the field flux, there is a reduction in the emf generated and the terminal voltage.
(c) The reduction in terminal voltage due to armature resistance drop and armature reaction drop reduces the field current which further reduces the emf generated and terminal voltage.

The circuit diagram to obtain the external characteristics is shown in Figure 3.11. Ammeters A_1 and A_2 are used to denote the load and field currents for different load conditions. The terminal voltage across the load is given by the voltmeter reading. Therefore, the ammeter and voltmeter readings for different load conditions are noted and graph between the terminal voltage V and load current I_L is drawn to obtain the external characteristics.

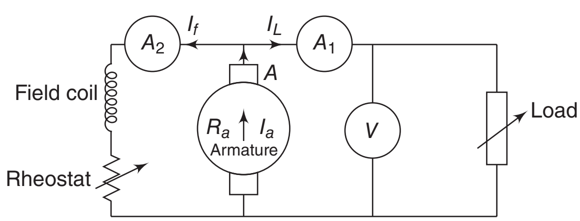


Figure 3.11 Circuit Diagram to Obtain External Characteristics

The external characteristic of the DC shunt generator is shown in Figure 3.12.

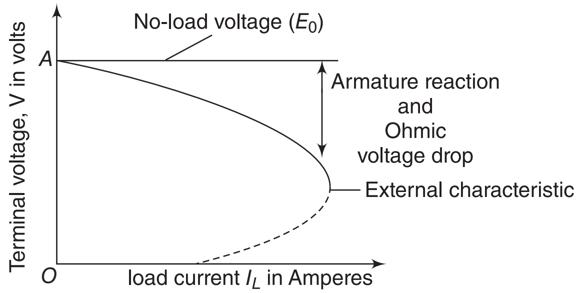


Figure 3.12 External and Internal Characteristics of a DC Shunt Generator

During a normal running condition, when load increases, there will be increase in load current and hence there will be a decrease in terminal voltage. But if any effort is made to increase the load current beyond certain limit, there will be drastic decrease in terminal voltage due to excessive armature reaction and Ohmic losses. Hence, beyond this limit, any further increase in the load results in decreasing the load current. Consequently, the external characteristic curve turns back as shown by dotted line and it cuts the current axis. The terminal voltage V at that particular point is zero even when there exists a residual emf in the DC shunt generator.

(iii) Internal Characteristics

In DC shunt generator, $I_a = I_L + I_{sh}$ and $E_g = V + I_aR_a$. Therefore, the internal characteristics curve can be obtained from external characteristic curve as shown in Figure 3.13.

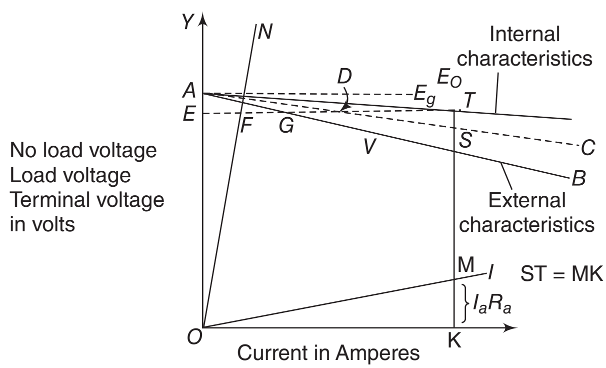


Figure 3.13 Internal characteristic of a DC Shunt Generator

In Figure 3.13, the line AB represents the external characteristic of DC shunt generator and ON represents R_{sh} line. The field currents for different values of V are obtained by calculating the horizontal distance between Y axis and ON line. The armature current line AC is obtained by adding these field currents horizontally to the existing external characteristic line, such that GD = EF. The armature resistance drop line, i.e., OI for different armature current is drawn. For an armature current OK, the armature resistance drop is MK. Now, if this armature resistance drop is added to the external characteristic curve such that ST = MK, the internal characteristic curve can be obtained.

3.5.3 DC Series Generator

(i) No-Load Characteristics

The no-load characteristics of the DC series generator can be obtained with the help of the circuit shown in Figure 3.14. The procedure to obtain these characteristics is same as it is available in the DC shunt generator. The no-load characteristics of the DC series generator are shown in Figure 3.14.

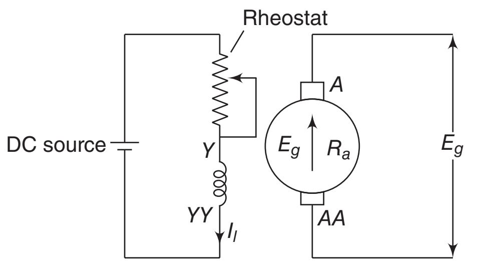


Figure 3.14 Circuit to Obtain No-Load Characteristics of a DC Series Generator

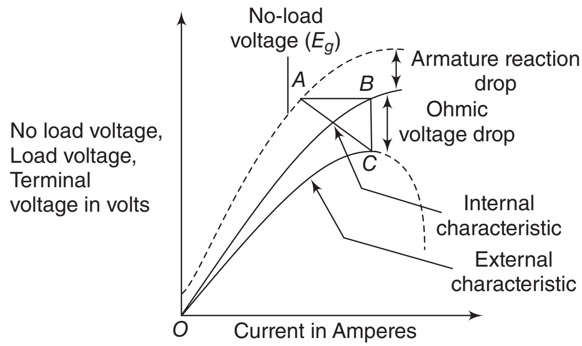


Figure 3.15 Characteristics of DC series generator

(ii) Internal Characteristics

Since the field winding is connected in series, the relation among the currents is $I_L = I_a = I_{se}$. When the DC series generator is loaded, there will be a drop in the generated voltage due to the demagnetizing effect of armature reaction. Therefore, the additional exciting current required to generate the same emf is AB. Therefore, the point B lies on the internal characteristics of the DC series generator. In a similar way, all other points in the internal characteristics can be obtained as shown in Figure 3.15.

(iii) External Characteristics

If the armature voltage drop is subtracted from the internal characteristics, the external characteristics of DC series generator can be obtained as shown in Figure 3.15.

3.5.4 DC Compound Generator

The external characteristics of DC compound generator for both cumulative and differential compounded generator are shown in Figure 3.16.

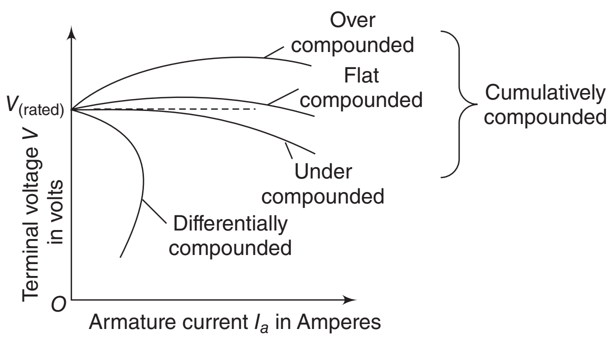


Figure 3.16 External characteristics of DC compound generator

Cumulative Compound Generator

If the series field ampere-turns are adjusted such that they induce the same voltage at both rated load and at no-load, then the generator is said to be flat compounded. If the series field ampere-turns are adjusted such that rated voltage at load condition is greater than voltage at no-load condition, then the generator is said to be over compounded. If the series field ampere-turns are adjusted such that rated voltage at load condition is less than the no-load voltage, then the generator is said to be under compounded.

If the terminal voltage of the DC generator drops very rapidly with increasing armature current, it is called differential compound generator.

3.6 Applications of DC Generator

Separately Excited DC Generator

- Since it has a very wide range of output voltage, it is used in laboratories.
- For DC motors, it is used as a supply source.

DC Shunt Generator

- It is used for excitation in AC generators or alternator.
- Since the voltage drop is very small, it is used as a source for loads, which need constant voltage.
- It acts as a source for battery charging, electroplating and electrolytic purposes.

DC Series Generator

- It is used in series arc lighting and incandescent lighting.
- It is used as a booster to compensate the voltage drop in the line during loading.
- It is used for regenerative braking of DC locomotives.

Compound Generator

- It is used to provide constant voltage at the line by proper compounding.
- Differently compounded generator is used for arc-welding purposes.
- Cumulative compounded generator is used to supply power to railway circuits, incandescent lamps, elevator motors, lighting, heavy power supply, offices, hotels, homes, schools etc.

3.7 DC Motor

3.7.1 Construction

The construction of a DC motor is exactly similar to a DC generator. The important parts of a DC motor are: yoke or frame, main field system, brushes, armatures and commutator. The functions of certain components vary with respect to a DC motor. In a DC motor, the commutator is used to convert the alternating torque produced in the armature into a unidirectional torque. A separate supply is given to armature winding to produce the required torque in the armature. The electrical power is converted into a mechanical power in the armature winding.

3.7.2 Working **[AU Nov/Dec, 2016]**

A DC motor is a machine which converts electrical energy into mechanical energy. It is generally used in locations where it gets exposed to various environmental conditions and damages. Hence, a DC motor needs to be drip-proof, fire-proof, etc., according to the requirement. The principle of a DC motor is that when a current carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force and its direction is determined by using Fleming's left-hand rule.

Working Principle

When the field windings wound on the poles are excited by a DC source, the poles get magnetised and a strong magnetic flux is produced. In such a magnetic field, consider a single armature coil, as shown in

Figure 3.17 (a). When a DC supply is used to excite this armature coil, the current flows through the coil. Due to this, the armature coil creates its own magnetic field, as shown in Figure 3.17 (b). Here, due to the interaction between these two fields, a resultant field, F is developed, as shown in Figure 3.17 (c).

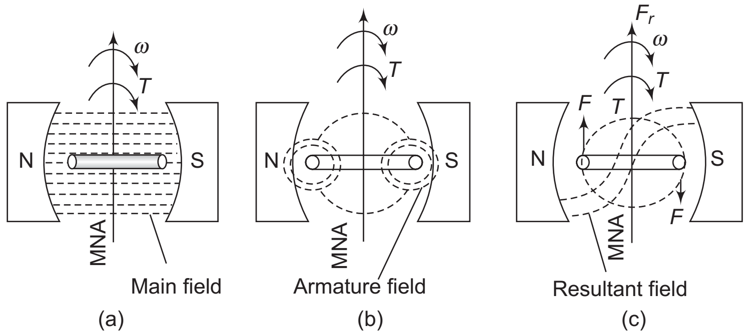


Figure 3.17 Working of a DC Motor (a) Main Field (b) Armature Field and (c) Resultant Field

This resultant field has a tendency to align with the main field position i.e., it tries to align itself along a straight line and hence a force is exerted on the armature coil and a torque is developed, which helps in rotating the armature coil. The working of a DC motor can also be explained using the mmf developed due to these two fluxes, as shown in Figure 3.18.

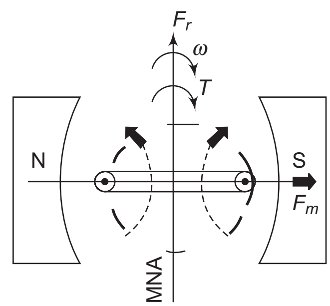


Figure 3.18 Working of a DC Motor Based on MMF

The mmf developed due to the main field is given by F_m, whose direction is shown in Figure 3.18. The mmf is also developed due to the field produced in the armature when the armature windings are excited by a DC source. It is denoted as F_r, which is perpendicular to F_m. F_r tries to align with F_m and thus an electromagnetic torque, T is developed in clockwise direction and hence the armature starts rotating in the same direction with an angular speed of ω. In practice, there are Z number of armature conductors in a DC motor. Hence, when the armature windings are excited by a DC source, the armature conductors placed under the influence of a pole carry current in a particular direction, while the armature conductors placed under the influence of another pole carry current in the opposite direction indicated as '•' and 'x' respectively, in Figure 3.19.

The electromagnetic torque developed in the armature conductors will be continuous if the direction of current in each conductor or coil side changes when it crosses the magnetic neutral axis (MNA). This reversal of direction of current can be achieved using a commutator and it helps in developing a continuous torque.

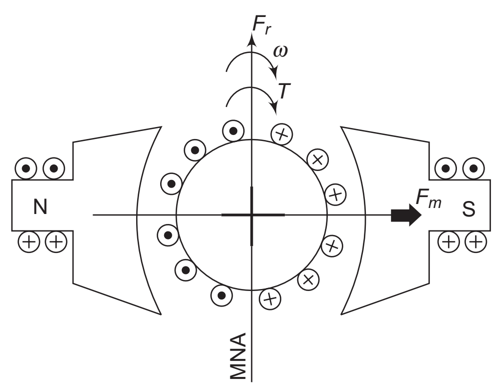


Figure 3.19 DC Motor With 'Z' Number of Armature Conductors

3.7.3 Back emf **[AU Nov/Dec, 2012]**

When a DC source is used to excite the armature conductor, which is placed in the main magnetic field, an electromagnetic torque is developed and hence, the armature of the DC motor starts rotating. Due to the rotation of armature, the armature conductors cut the magnetic flux of the main magnetic field and hence an induced emf is developed in the armature conductor. Since this induced emf developed in the armature conductors opposes the cause that produces it, this induced emf is known as back emf, E_b. Since this emf is induced in the armature due to the generator action, its magnitude is given by the same expression as that of the generated emf in a DC generator. Therefore, the expression for back emf, E_b is given by

$$E_b = \frac{\phi ZNP}{60\ \text{A}}$$

It is noted that the supply voltage, V, applied to the armature windings will be greater than E_b.

The significance of back emf is to regulate the armature current, according to the load connected to the motor.

3.7.4 Voltage and Power Equation

The equivalent circuit of a DC motor armature is shown in Figure 3.20.

In a DC motor, the current flows from the line into the armature against the voltage generated in the armature. Using KVL in the circuit shown in Figure 3.20, we get

$$V = E_b + I_aR_a + V_b \tag{3.6}$$

If the drop across the brushes is negligible, we get

$$V = E_b + I_aR_a \tag{3.7}$$

The Eqns. (3.6) and (3.7) represent the fundamental voltage equation of a DC motor.

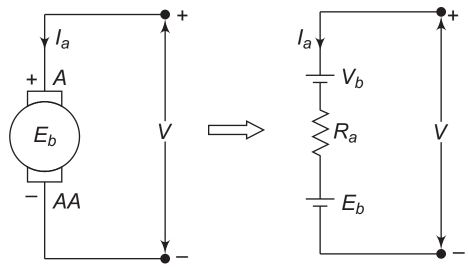


Figure 3.20 Equivalent Circuit of a DC Motor Armature

Multiplying Eqn. (3.7) by I_a, we get

$$VI_a = E_bI_a + I_a^2R_a \tag{3.8}$$

i.e., $P_i = P_m + P_c$

where $P_i = VI_a$ is the input power supplied to the armature, $P_m = E_bI_a$ is the mechanical power developed in the armature and $P_c = I_a^2R_a$ is the copper loss in the armature. Eqn. (3.8) represents the power equation of a DC motor.

Condition for Maximum Power Developed in Armature

The mechanical power developed in the armature is given by

$$P_m = VI_a - I_a^2R_a$$

Differentiating the above equation with respect to I_a and equating it to zero, we get

$$I_aR_a = \frac{V}{2} \tag{3.9}$$

Substituting Eqn. (3.9) in Eqn. (3.7) and solving, we get

$$E_b = \frac{V}{2}$$

Therefore, the condition to develop the maximum power in the armature is given by the above equation.

Motor Efficiency

The motor efficiency, η_m is given by the ratio of mechanical power developed in the armature to the electrical power supplied to the armature. Therefore,

$$\eta_m = \frac{P_m}{P_i} = \frac{E_b I_a}{V I_a} = \frac{E_b}{V}$$

3.7.5 Torque and Speed Equations **[AU Nov/Dec, 2009]**

Torque Equation

In a DC motor, when a current-carrying armature conductor is placed in the magnetic field developed by the poles, an electromechanical torque is developed in the armature, T_a. It is also known as armature torque and is given by the product of the force and the radius at which this force acts.

i.e., $$T_a = F \times r \tag{3.10}$$

where F is the force acting on the armature conductor and r is the radius of the armature.

In one revolution, the work done by the force, F is given by

$$W_D = F \times 2\pi r$$

where $2\pi r$ is the circumference of the armature.

If the armature is rotating at a speed of N rpm, the time taken to complete one revolution is $\frac{60}{N}$. Therefore, the net mechanical power developed in the armature, P_m is given by

$$P_m = \frac{W_D}{\text{time}} = \frac{F \times 2\pi r \times N}{60}$$

Using Eqn. (3.10) in the above equation, we get

$$P_m = T_a \times \frac{2\pi N}{60} = T_a \times \omega \tag{3.11}$$

where $\omega = \frac{2\pi N}{60}$ is the angular velocity of the armature in radians per second.

But the mechanical power developed in the armature is given by

$$P_m = E_b I_a \tag{3.12}$$

Equating Eqns. (3.11) and (3.12), we get

$$T_a \times \frac{2\pi N}{60} = E_b I_a$$

i.e., $$T_a = 9.55 \times \frac{E_b I_a}{N} \tag{3.13}$$

Substituting $E_b = \dfrac{\phi ZNP}{60\text{ A}}$ in the above equation and rearranging, we get

$$T_a = \frac{\phi ZPI_a}{2\pi A} = 0.159 \times \frac{\phi ZPI_a}{A} \text{ N-m} \tag{3.14}$$

It is clear from the above equation that the armature torque is directly proportional to the product of the flux and the armature current i.e., $T_a \; \alpha \; \phi I_a$ since the other terms $0.159 \times \dfrac{ZP}{A}$ are constants. Therefore, Eqns. (3.13) and (3.14) represent the armature torque of a DC motor.

Shaft Torque (Tsh)

In a DC motor, the electromagnetic torque developed in the armature, T_a will not be available at the shaft, since a part of the torque is lost due to iron and mechanical losses. Therefore, the actual torque available at the shaft for doing useful mechanical work is known as shaft torque, T_{sh}. It is also defined as the torque available at the motor shaft and it is always less than T_a. If the speed of the motor is N rpm, then the shaft torque of the motor is given by

$$T_{sh} = \frac{\text{Output power of motor}}{\dfrac{2\pi N}{60}} = 9.55 \times \frac{\text{Output power of motor}}{N}$$

If the speed of the motor is N rps, then the shaft torque of the motor is given by

$$T_{sh} = \frac{\text{Output power of motor}}{2\pi N} = 0.159 \times \frac{\text{Output power of motor}}{N}$$

Speed Equation

Using the voltage equation and back emf equation of a DC motor, we get

$$E_b = V - I_a R_a = \frac{\phi ZNP}{60\text{ A}}$$

Therefore, the speed at which the DC motor rotates is given by

$$N = \frac{V - I_a R_a}{\phi} \times \frac{60A}{ZP} \text{ or}$$

$$N = \frac{E_b}{\phi} \times \frac{60A}{ZP}$$

It is clear from the above equation that the speed of a DC motor is directly proportional to the back emf, E_b and inversely proportional to the flux per pole, ϕ.

Speed Regulation of a DC Motor

It is defined as the change in speed when the load on the motor is reduced from rated value to zero, expressed as a percentage of rated load speed. It can also be defined as the ratio of the difference between no load and full load with respect to full load.

$$\%\text{ speed regulation} = \frac{N_{NL} - N_{FL}}{N_{FL}} \times 100$$

where N_{NL} is the no-load speed of a DC motor and N_{FL} is the full-load speed of a DC motor.

3.8 Types and Equivalent Circuit of DC Motor [AU Nov/Dec, 2014]

The DC motors are classified, as shown in Figure 3.21, based on the excitation given to the field windings as:

- **Separately excited DC motor:** The required power for exciting the field windings is obtained from a separate DC source.
- **Self-excited DC motor:** The required power for exciting the field windings is obtained from the power supplies to the armature of the DC motor.

Further, the self-excited DC motor is classified based on the connection between the field winding and armature winding as:

- ***DC shunt motor:*** Field winding is parallel to the armature winding
- ***DC series motor:*** Field winding is in series with the armature winding
- ***DC compound motor:*** Consists of two windings; one is connected in series and other is connected in parallel to the armature windings. Based on these winding connections, it is further classified as:
  - Long-Shunt DC compound motor
  - Short-Shunt DC compound motor

If the magnetic flux generated by the series field winding aids the magnetic flux generated by the shunt-field winding, the DC compound motor is said to be cumulative compounded. Conversely, if the magnetic flux generated by the series-field winding opposes the magnetic flux generated by the shunt-field winding, the DC compound motor is known as differentially compounded. It is noted that both long-shunt and short-shunt compound motors can be either cumulative or differentially compounded.

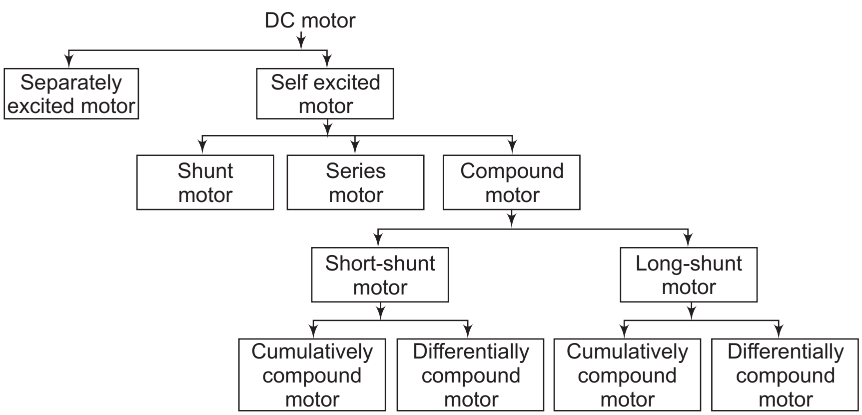
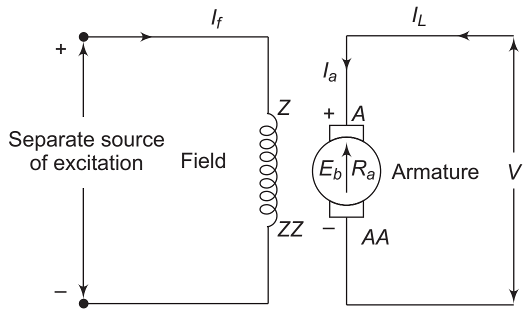
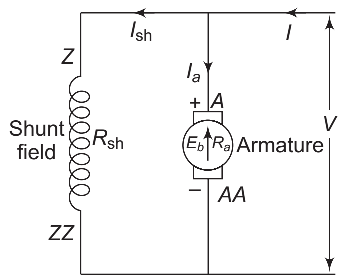
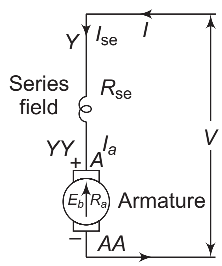
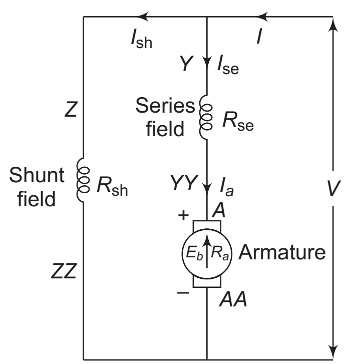


Figure 3.21 Types of DC Motors

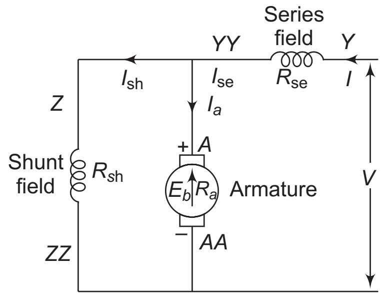
Table 3.3 lists the equivalent circuit, voltage equation, current equation and flux relation for each type of DC motor.

Let ϕ_{se} and ϕ_{sh} be the fluxes produced by the series and shunt field windings respectively. Let ϕ_f and I_f be the fluxes produced by the field winding and current through the field winding in the separately excited DC motor.

Table 3.3 *Equivalent Circuit, Voltage and Current Equations and Flux Relation for Each Type of DC Motor*

Type of DC Motors	Equivalent Circuit	Voltage Equation	Current Equation	Flux Relation
Separately excited DC motor		$V = E_b + I_aR_a$	$I_a = I_L$	$\phi_f \,\alpha\, I_f$
DC shunt motor		$V = E_b + I_aR_a$	$I_{sh} = \dfrac{V}{R_{sh}}$	$\phi_{sh} \,\alpha\, I_{sh}$
DC series motor		$V = E_b +$ $I(R_a + R_{se})$	$I_a = I_L = I_{se}$	$\phi_{se} \,\alpha\, I_{se}$ $\alpha\, I_a$
Long-Shunt DC Compound Motor		$V = E_b +$ $I_a(R_a + R_{se})$	$I_L = I_a + I_{sh}$ $I_{se} = I_a$ $I_{sh} = \dfrac{V}{R_{sh}}$	$\phi_{se} \,\alpha\, I_{se}$ $\alpha\, I_a$ ϕ_{sh} $\alpha\, I_{sh}$

(*Contd.*)

Short-Shunt DC Compound Motor		$V = E_b + I_aR_a + I_{se}R_{se}$	$I_L = I_a + I_{sh}$ $I_{se} = I_L$ $I_{sh} = \dfrac{V - I_{se}R_{se}}{R_{sh}}$	$\phi_{se}\ \alpha\ I_{se}$ $\alpha\ I_L$ ϕ_{sh} $\alpha\ I_{sh}$

Example 3.4

A 250 V, four-pole wave-wound DC series motor has 782 conductors on its armature. It has armature and series field resistance of 0.75 Ω. The motor takes a current of 40 A. Determine its speed and gross torque developed, if it has a flux per pole of 25 mWb. **[AU April/May, 2006]**

Solution

Given $V = 250$ V, $P = 4$, $Z = 782$, $R_a + R_{se} = 0.75\ \Omega$, $I_a = 40$ A, $\phi = 25$ mWb

The back emf equation of a DC series motor is

$$E_b = V - I_a(R_a + R_{se})$$

Substituting the known values, we get

$$E_b = 220 \text{ V}$$

The other expression for back emf of DC motor is $E_b = \dfrac{\phi PNZ}{60\ A}$. Since the DC series motor uses wave-wound armature winding, $A = 2$. Therefore,

$$220 = \frac{25 \times 10^{-3} \times 4 \times N \times 782}{60 \times 2}$$

Therefore, $$N = \frac{220 \times 60 \times 2}{25 \times 10^{-3} \times 4 \times 782} = 338 \text{ rpm}$$

The armature torque developed in the armature is given by

$$T_a = \frac{E_b I_a}{\dfrac{2\pi N}{60}} = \frac{220 \times 40}{\dfrac{2\pi \times 338}{60}} = 248.62 \text{ Nm}$$

3.9 Characteristics of DC Motor **[AU April/May, 2015]**

The different characteristics of DC motors are:

- ***Electrical characteristics:*** Give the relation between T_a and I_a and also known as T_a/I_a characteristics
- ***Speed vs. armature current characteristics:*** Give the relation between N and I_a
- ***Mechanical characteristics:*** Give the relation between N and T_a and also known as N/T_a characteristics

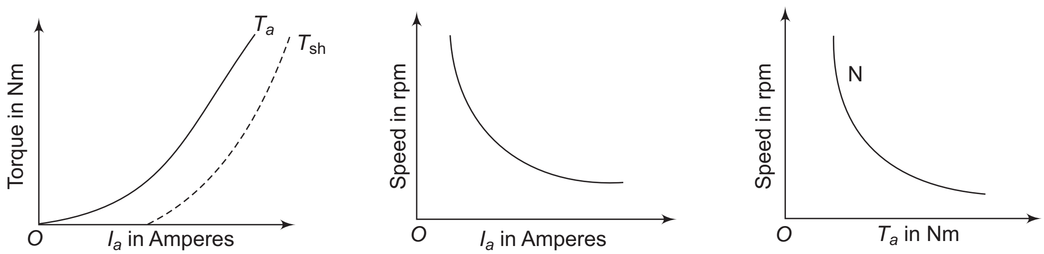
These characteristics are obtained for different DC motors, based on the back emf and armature torque equations of a DC motor. It is known that $T_a = \frac{\phi Z P I_a}{2\pi A} = 0.159 \times \frac{\phi Z P I_a}{A}$, $T_a = 9.55 \times \frac{E_b I_a}{N}$ and $E_b = \frac{\phi Z N P}{60\ \text{A}}$.

The relations that we get using these equations are: $T_a \alpha \frac{E_b I_a}{N}$, $T_a \alpha \phi I_a$ and $N \alpha \frac{E_b}{\phi}$. Using these relations, the characteristics of different DC motors are studied. It is to be noted that ϕ denotes the flux produced by the field windings.

3.9.1 DC Series Motor

T_a/I_a Characteristics

It is known that, in a DC series motor, $I_a = I_L = I_{se}$. Here, the T_a/I_a characteristics are divided into two parts: (i) before the magnetic saturation of field flux and (ii) after saturation of field flux. In the first part, the field flux, ϕ, varies and is directly proportional to I_a i.e., $\phi \alpha I_a$. Therefore, the armature torque, $T_a \alpha I_a^2$. Hence, the T_a/I_a characteristics curve in the first-half is parabolic in nature. In the second part, once the saturation of field flux occurs, ϕ does not vary even when I_a is varied. Therefore, armature torque varies linearly with armature current i.e., $T_a \alpha I_a$. Hence, T_a/I_a characteristic curve is linear in the second part, as shown in Figure 3.22 (a). The shaft torque, T_{sh}, which is less than T_a due to losses in a DC motor, is shown as a dotted line in Figure 3.22 (a).



(a) T_a/I_a characteristics (b) N/I_a characteristic (c) N/T_a characteristic

Figure 3.22 Characteristics of a DC Series Motor

N/I_a Characteristics

Since $\phi \alpha I_a$ in a DC series motor, the relation between speed and ϕ is $N \alpha \frac{E_b}{I_a}$. If the back emf, E_b is neglected, then the speed varies inversely with the armature current, I_a. The N/I_a characteristics using these relations are shown in Figure 3.22 (b). The following inference can be made from the N/I_a characteristics shown in Figure 3.22 (b).

(a) When heavy load is connected to a DC motor, I_a becomes large and hence the speed is low, which further decreases E_b and allows more I_a to flow.
(b) Similarly, when I_L falls to a small value and since $I_a = I_L$ in a DC series motor, I_a falls to a small value and hence the speed N reaches a very high value. This is the reason why a DC series motor should not be started at no-load condition.
(c) Also, from N/I_a characteristics, we can infer that a DC series motor is a variable speed motor.

N/T_a Characteristics

The relation used to obtain this characteristic is $T_a \, \alpha \, \frac{E_b I_a}{N}$. It is clear from the equation that, when speed of the DC motor is high, the torque developed in the armature is low and vice-versa. The N/T_a characteristics curve a of DC series motor is shown in Figure 3.22 (c).

3.9.2 DC Shunt Motor

T_a/I_a Characteristics

Since the field winding in a DC shunt motor is excited by the constant supply voltage, V, the current through the field winding, I_{sh} is constant. Since the field current in the DC shunt motor is constant, the flux produced by the field is also constant i.e., ϕ is a constant value. Hence, $T_a \, \alpha \, I_a$. Therefore, the T_a/I_a characteristics curve of a DC shunt motor is linear and is shown in Figure 3.23 (a) along with T_{sh}, which is less than T_a due to losses in the DC motor.

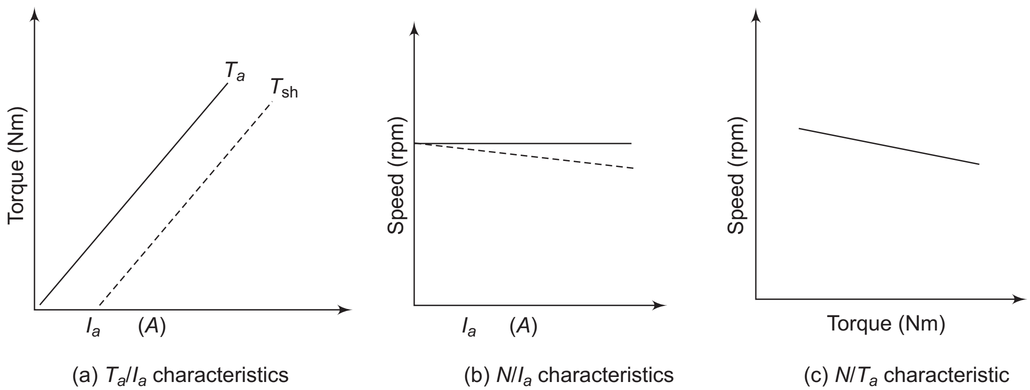


Figure 3.23 Characteristics of a DC Shunt Motor

N/I_a Characteristics

Since the field flux is constant, $N \, \alpha \, E_b$ in a DC series motor. At normal condition, E_b is almost constant but due to armature reaction and armature resistance drop, there will be a slight drop in the back emf, E_b. Hence, there will be a slight drop in the N/I_a characteristics, as shown in Figure 3.23 (b). In general, the speed of a DC shunt motor drops only to 5–15 % of the full load speed. Hence, the DC shunt motor is referred as constant speed motor.

N/T_a Characteristics

Using the above two characteristics, it is possible to get N/T_a characteristics for different values of armature current I_a, as shown in Figure 3.23 (c). It is clear that the speed of DC shunt motor decreases when the armature torque increases.

3.9.3 DC Compound Motor

The characteristics of a DC compound motor are studied based on the connection between the shunt and the series field windings. If the field windings are connected in such a way that the flux produced by the

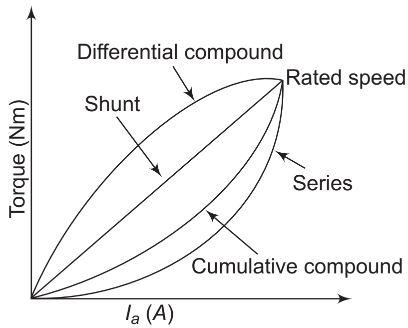


Figure 3.24 T/I_a Characteristics Curve of a DC Compound Motor

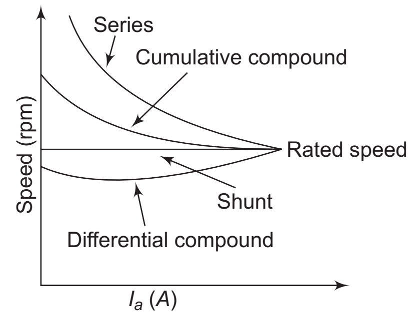


Figure 3.25 N/I_a Characteristics of a DC Compound Motor

series field winding is in the direction of flux produced by the shunt field winding, then it is called a DC cumulative compound motor. But if the fluxes produced by the field windings are in the opposite direction, it is called a DC differential compound motor. The characteristics of these two compound motors are shown in Figure 3.24 and Figure 3.25.

In a DC cumulative compound motor, at heavy load, the series field winding plays a major role, while the shunt field winding prevents this motor from running at dangerously high speed at light load. In a DC differential compound motor, since the two field fluxes oppose each other, the speed of this motor remains almost constant and it increases slightly when there is an increase in load.

3.10 Speed Control of DC Motor **[AU Nov/Dec, 2014]**

The speed equation of a DC motor is given by

$$N = \frac{V - I_a R_a}{\phi} \times \frac{60A}{ZP}$$

i.e.,

$$N \ \alpha \ \frac{V - I_a R_a}{\phi} \quad \text{(since all other terms are constant)}$$

From the above equation, it is concluded that the speed of a DC motor can be controlled by the following methods:

- ***Armature resistance control method:*** Here, the armature resistance of DC motor is varied to control the speed.
- ***Flux control method:*** Here, varying the flux produced by the poles controls the speed of the DC motor.
- ***Applied voltage:*** Here, varying the excitation voltage given to the armature and field windings controls the speed of the DC motor.

3.10.1 DC Series Motor

Flux Control Method

The different methods in which the flux produced by the field windings can be varied, thereby controlling the speed of DC series motor, are:

- Field diverter method
- Armature diverter method

- Tapped-field control method and
- Paralleling field coils

Field Diverter Method

The circuit diagram for this method is shown in Figure 3.26. In this method, a field diverter or a variable resistance is connected in parallel to the series field windings of a DC series motor. This field diverter is used to reduce the line current flowing through the series field windings. Since the field current is getting decreased, the flux developed by the poles gets lowered, which increases the speed of the motor. Using this method, the speed of a DC motor above rated speed can be obtained.

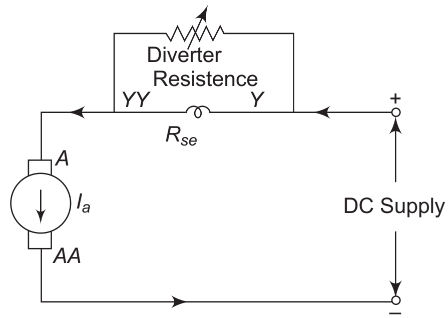


Figure 3.26 Field Diverter Method

Armature Diverter Method

Using this method, the speed of a DC series motor below the rated speed can be obtained. This method uses armature diverter to reduce the current flowing through the armature, as shown in Figure 3.27. By varying this armature diverter, the speed control of a DC series motor can be achieved.

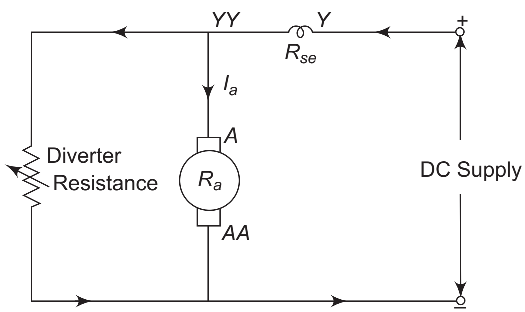


Figure 3.27 Armature Diverter Method

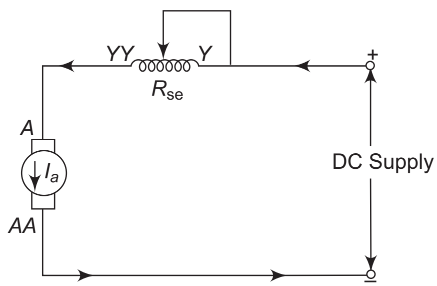


Figure 3.28 Tapped-Field Control Method

Tapped-field Control Method

The difference between the field diverter method and the tapped-field control method is that tapped-field windings are used in the tapped-field control method, whereas a field diverter is used in the field diverter method. The circuit diagram of tapped-field control method is shown in Figure 3.28. The switch *S* is used to create a connection to the desired point in the tapped-field winding. Here, by reducing the number of turns in the field winding reduces the field flux, which in turn increases the speed of the DC series motor. Similar to field diverter method, this method can be used to control the speed above the rated speed of the DC series motor.

Paralleling Field Coils

In this method, a single series field winding is divided into number of parts and these sub-coils are arranged in any one way, as shown in Figure 3.29, to reduce the flux produced by the field winding, thereby increasing the speed of the DC series motor.

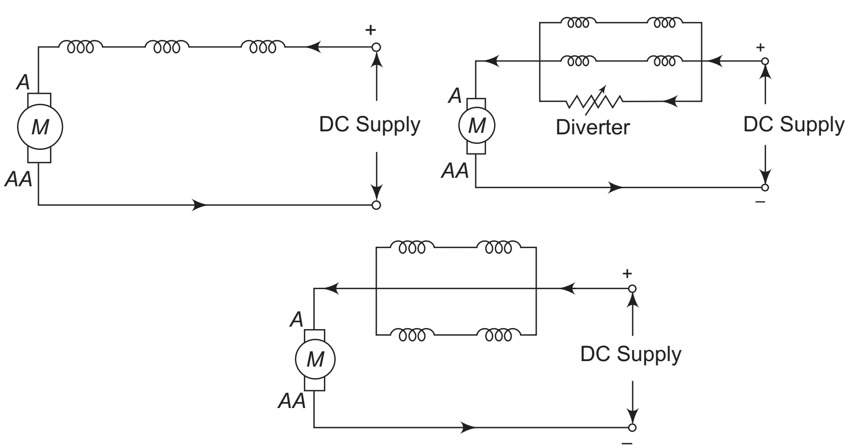


Figure 3.29 Paralleling-Field Coils

Armature Resistance Control Method

The circuit diagram for the speed control of a DC series motor, by varying armature resistance, is shown in Figure 3.30. In this method, an adjustable resistor is connected in series with the source voltage to control the speed of a DC series motor below the rated speed.

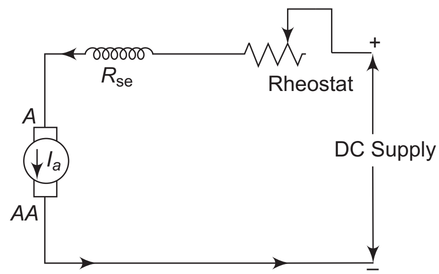
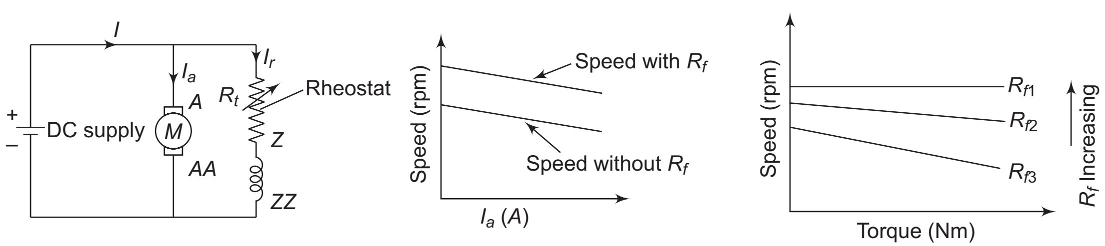


Figure 3.30 Armature Resistance Control Method

3.10.2 DC Shunt Motor

Flux control Method

The circuit diagram of flux control method to control the speed of a DC shunt motor is shown in Figure 3.31 (a). This method is used to control the speed of a DC shunt motor above the rated speed, by reducing the field flux, since $N \;\alpha\; \dfrac{1}{\phi}$. In this method, a variable resistance element called shunt field rheostat is connected in series with the shunt-field winding, which helps in reducing the flux and the field current. Since the field Ohmic loss is very small, this method is frequently used. Also, it is very simple and economic. The N/I_a characteristics and N/T_a characteristics are shown in Figures 3.36 (b) and (c) respectively.



(a) Circuit Diagram (b) N/I_a Characteristics Curve (c) N/T_a Characteristics Curve

Figure 3.31 Flux Control Method of a DC Shunt Motor

Armature Resistance Control Method

In this method, including a resistance in series with R_a controls the speed of a DC shunt motor, as shown in Figure 3.32 (a). This method is used to vary the speed of a DC shunt motor below the rated speed since $N \alpha V - I_a R_a$ i.e., the inclusion of resistance in series with R_a reduces the armature current, which further reduces the speed of a DC shunt motor. The N/I_a characteristics of a DC shunt motor with and without the extra resistance are shown in Figure 3.32 (b).

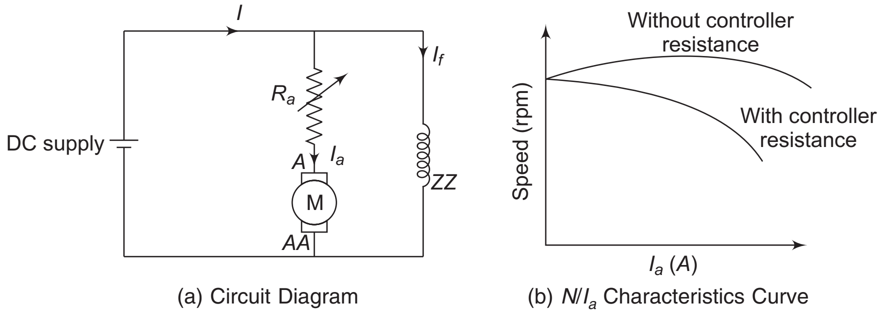


Figure 3.32 Armature Control Method of a DC Shunt Motor

Voltage Control Method

The different voltage control methods used for controlling the speed of a DC shunt motor are:

- Multiple voltage control
- Ward–Leonard system

Multiple Voltage Control

In this method, a suitable arrangement is made in such a way that the field winding gets a constant supply voltage and armature winding gets a variable voltage. The different voltages that can be used for exciting armature winding are obtained using a switch-gear. Since the speed of the motor is directly proportional to the supply voltage applied to the armature, varying the voltage applied to the armature can control the speed.

Ward–Leonard System **[AU Nov/Dec, 2010]**

The arrangement of Ward–Leonard system to control the speed of a DC shunt motor is shown in Figure 3.33. Here, M_2 is the DC shunt motor whose speed has to be controlled, M_1 can be either AC or DC motor with constant speed, which is coupled directly to generator G. As shown in Figure 3.33, the output of G_1 is fed as the input to the armature of motor M_2. The field winding of M_2 is given by a constant DC supply voltage. Therefore, varying the generator output can vary the supply voltage applied

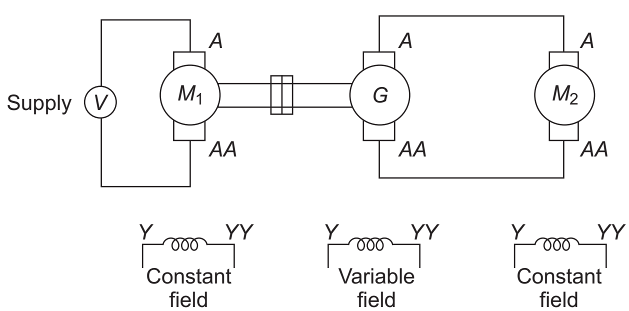


Figure 3.33 Ward–Leonard System of a DC Shunt Motor

to armature of M_2. The generator output can be varied using a field regulator. Thereby, a very smooth speed control of DC shunt motor can be obtained using this method. This method is used in applications where very sensitive speed control is required. For example, elevators, electric excavators etc.

3.11 Applications of DC Motor

DC Shunt Motor

1. Used in applications where constant speed is required.
2. Used in applications where adjustable speed in the range of 2 : 1, along with a medium starting-torque is required.
3. Used in lathe machine, centrifugal pumps, fans and blowers, machine tools like wood working machines, reciprocating pumps, spinning and weaving machines etc.

DC Series Motor

1. Used in applications where variable speed and high starting-torque are required.
2. Used in traction work, electric locomotives, rapid transit systems, trolleys, cars, cranes, conveyors etc.

DC Compound Motor

1. Used in applications where high starting-torque and moderately constant speed are required.
2. Used in devices like elevators, conveyors, heavy planers, rolling mills, ice machines, printing presses and air compressors.

3.12 Stepper Motor

The stepper motor has gained more importance in recent years because it can be easily interfaced with digital circuits. A special type of synchronous motor designed to rotate through a specific angle for each applied electrical pulse is known as a stepper motor. The specific angle through which the stepper motor rotates is called step. The electrical pulses are received from the control unit of the stepper motor. Stepper motor is used along with electronic switching devices to switch the control windings according to the command received. The number of steps per revolution and the rate at which the pulses are applied determine the rotational rate.

The stepper motor completes a full rotation by sequencing through a series of discrete rotational steps (stepwise rotation). Each step position is an equilibrium position, so that without further excitation, the rotor position stays at the latest step. Thus, a train of input pulses, each of which causes an advance of one step, achieves continuous rotation.

Classification of Stepper Motor

The stepper motor can be classified depending on:

(i) The type of rotor:
   - (a) Variable-reluctance stepper motor
   - (b) Permanent-magnet stepper motor
   - (c) Hybrid stepper motor

(ii) The windings on the stator
   - (a) Two-phase stepper motor
   - (b) Three-phase stepper motor
   - (c) Four-phase stepper motor

3.12.1 Variable-Reluctance Stepper Motor

The variable-reluctance stepper motor has a single or several stacks of stators and rotors. The stators have a common frame, while the rotor has a common shaft.

The stator of this stepper motor has six laminated poles with exciting windings wound for three-phases. Slotted steel lamination is used in making the rotor. The number of poles on the stator and the rotor are different and it gives the variable-reluctance motor the ability to rotate in both directions and for self-starting. The stator and the rotor of this type of stepper motor have toothed structures. The longitudinal cross-sectional view of a three-stack variable-reluctance stepper motor is shown in Figure 3.34.

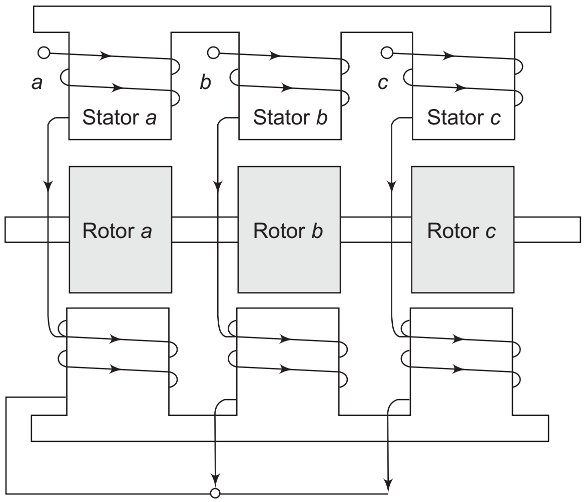


Figure 3.34 Three-stack Variable-Reluctance Stepper Motor

The difference in angular displacement of the stator and the rotor, when the teeth of the rotor are perfectly aligned, is given by

$$\alpha = \frac{360^\circ}{nT}$$

Where n is the number of stacks and T is the number of rotor teeth.

The schematic diagram explaining the concept of separation between the stator and the rotor is shown in Figure 3.35.

The schematic diagram of a variable-reluctance stepper motor and its driving circuit are shown in Figures 3.36 (a) and (b) respectively.

The working principle of this stepper motor is based on the different reluctance positions of the rotor with respect to the stator. When any one stator-phase is excited, a magnetic field whose axis lies along the poles is produced. Then the rotor rotates in a particular direction so that the reluctance position between the stator and the rotor is minimum and in such position, the magnetic field axis of the stator passes through any two rotor-poles. The working of a variable-reluctance stepper motor when the phases A, B and C are energised in this sequence, using switches S_1, S_2 and S_3, is shown in Figures 3.37 (a) to (c) respectively.

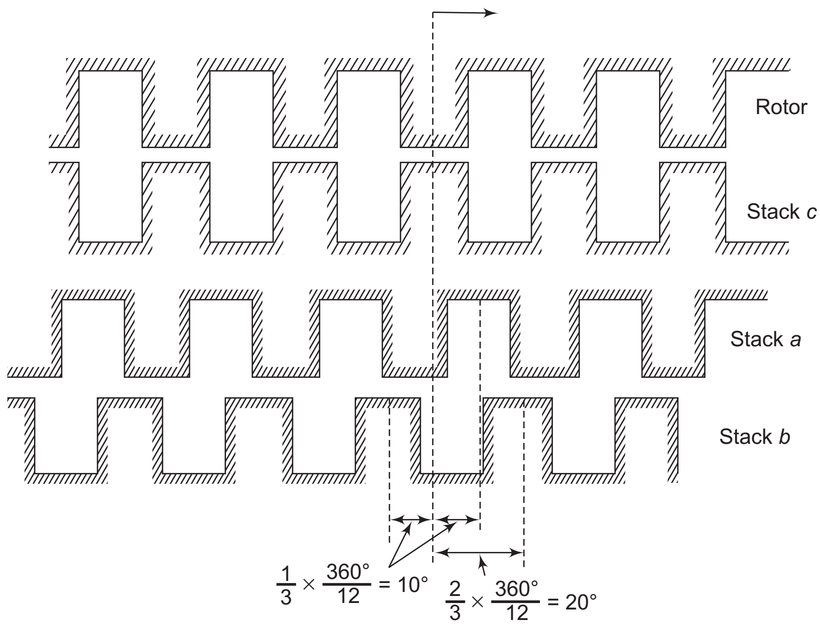


Figure 3.35 Separation Between Stator and Rotor

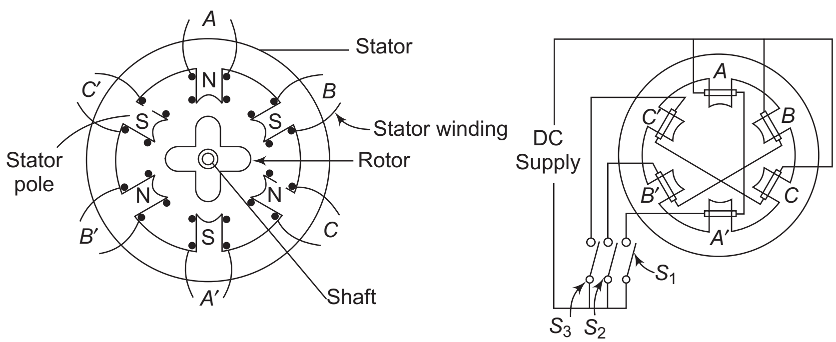


Figure 3.36 Variable-Reluctance Stepper Motor (a) Schematic Diagram (b) Driving Circuit

When phase A is excited using switch S_1, a vertical magnetic axis is formed along the poles of phase A. Then the rotor rotates and adjusts itself to a minimum reluctance position i.e., rotor axis gets matched with stator magnetic axis through any two poles and this position is shown in Figure 3.37 (a). Similarly, Figures 3.37 (b) and (c) indicate the rotor position and the stator magnetic axis, when the phases B and C are energised respectively.

The torque acting on the rotor, when the current i flows through the stator and any one of the phases is energised, is given by

$$T_m = \frac{1}{2} i^2 \frac{dL}{d\theta}$$

where L is the inductance of a phase at an angle θ.

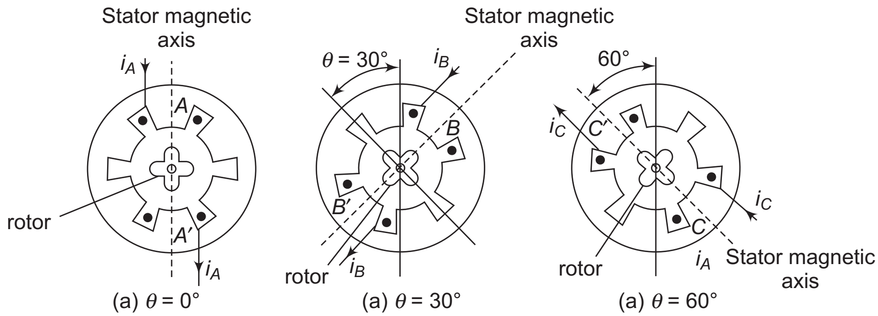


Figure 3.37 Working of a Variable-Reluctance Stepper Motor

Since the torque is proportional to the square of the phase current, the rotor rotation is independent of the direction of the current, *i*. But changing the sequence of the phase, which is energised using switches, can change the direction of rotation.

3.12.2 Permanent-Magnet Stepper Motor

In this type of stepper motor, the stator has salient poles, which carry control windings. A phase is created when the two control windings are connected in series. In this type, the rotor is made in the form of a spider cast integral permanent magnet or assembled permanent magnets. The schematic diagram of a permanent-magnet stepper motor and its driving circuit are shown in Figures 3.38 (a) and (b) respectively.

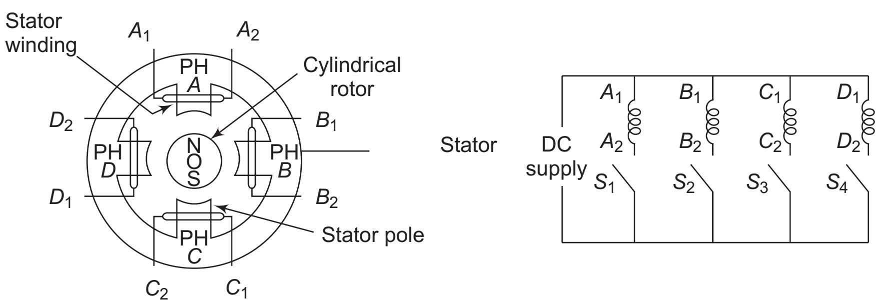


Figure 3.38 Permanent-Magnet Stepper Motor (a) Schematic Diagram (b) Driving Circuit

The working of this stepper motor is explained when the phases *A*, *B*, *C* and *D* are energised using switches S_1, S_2, S_3 and S_4, as shown in Figures 3.39 (a) to (d) respectively.

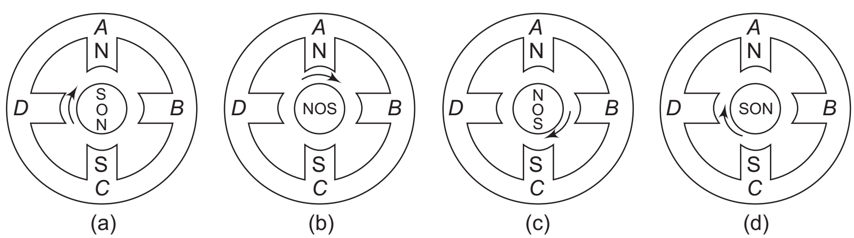


Figure 3.39 Working of a Permanent-Magnet Stepper Motor

The permanent magnet stepper motor operates at larger steps of up to 90°, at a maximum response rate of 300 pps.

3.12.3 Hybrid Stepper Motor

The hybrid stepper motor is similar to the permanent-magnet stepper motor with the constructional features of the rotor adopted from variable-reluctance stepper motor. The teeth on the stack of the rotor at both ends are of different polarities. Thus, the two sets of teeth in the rotor are displaced from each other by one-half of the tooth pitch (pole pitch). The constructional features of this type of stepper motor are shown in Figures 3.40 (a) and (b).

3.12.4 Operation of Stepper Motor

Depending on the step angle by which the stepper motor rotates, the operation of stepper motor can be classified into:

- Full-step operation
- Half-step operation
- Micro-step operation

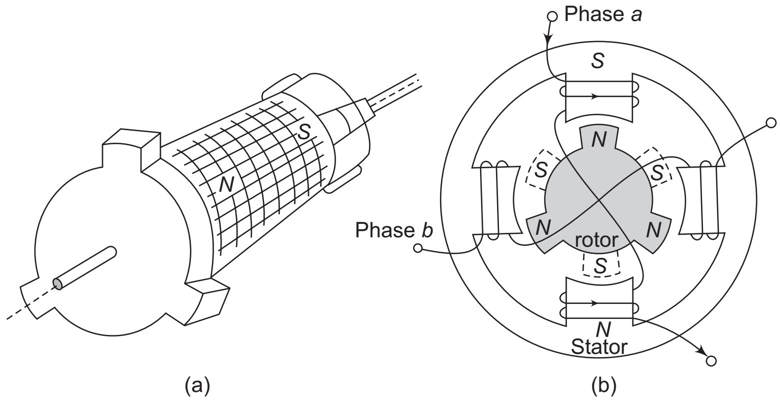


Figure 3.40 Constructional Details of a Hybrid Stepper Motor

Full-step Operation

In this type of operation, the stepper motor moves one full-step for each of the input pulses applied.

$$\text{Full step} = \frac{360°}{N_R \times N_S}$$

where N_R is the number of rotor poles
N_S is the number of stator pole-pairs

Half-step Operation

In this type, the stepper motor moves one-half of the full step for each of the input pulses applied. In the full-step operation, the motor rotates at $X°$ for each input pulse, and in the half-step operation, the motor will rotate at $\frac{X°}{2}$ for each input pulse.

Micro-step Operation

In this type of operation, the stepper motor moves through angles of 1/10, 1/16, 1/32 and 1/125 of a full step. The major advantage of this type of operation is that it provides a much finer resolution.

3.13 Brushless Direct Current (BLDC) Motor

The Brushless Direct Current (BLDC) motor is a derivative of the DC motor and it shares the same torque and speed performance characteristics. The major difference between BLDC and DC motor is the use of brushes for commutation. The DC motor assembly contains a physical commutator, which changes the motor phase at the appropriate times to produce the required torque, whereas a BLDC motor does not have brushes and electrical current powers a permanent magnet of the motor through an electronically commutated system to produce the required torque. It is highly reliable since it does not have any brushes to wear out and replace and it has longer life-expectancy, when operated at its rated conditions. Moreover, it has tremendous benefits for long-term applications.

3.13.1 Construction

The construction of a BLDC motor is similar to that of a three-phase induction motor and a conventional DC motor and it is shown in Figures 3.41(a) and (b). The motor consists of four primary parts, namely: Permanent-magnet rotor; Stator; Stator windings; and Hall-Effect sensors.

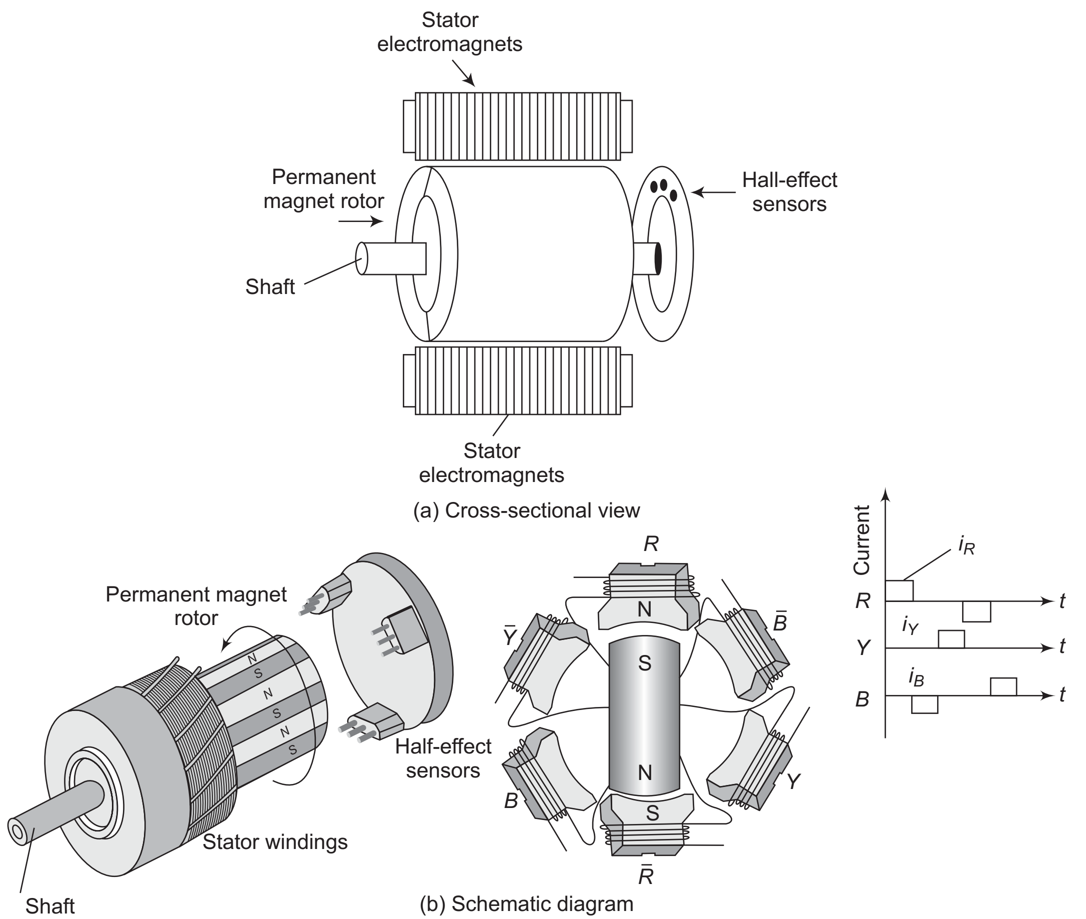


Figure 3.41 Construction of a BLDC Motor

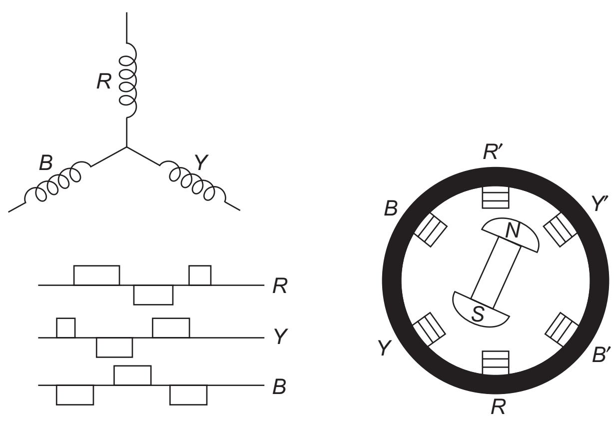
Stator electromagnets of a BLDC motor consist of stacked steel laminations to carry the stator windings. The windings are placed in slots in such a way that it axially cuts along the inner periphery of the stator. The windings are arranged in either star or delta-connected forms. However, most of the BLDC motors have three-phase star-connected stator windings. Each winding in the stator is constructed with numerous interconnected coils, where one or more coils are placed in each slot. In order to form an even number of poles, each of these stator windings is distributed over the entire stator periphery of the motor.

In a BLDC motor, the rotor is made up of a permanent magnet. Based on core configurations, the rotor is constructed as a circular core with the permanent magnet on the periphery, a circular core with rectangular magnets, etc. Depending on the application, the number of poles in the rotor can vary from 2 to 8 pole-pairs and the magnets are placed with alternate north and south poles. In order to achieve maximum torque, the flux density of the material should be high in the rotor. Rare-earth alloy magnets such as Samarium-Cobalt (SmCo), Neodymium (Nd), and Ferrite and Boron (NdFeB) are commonly used for new designs. Normally, Ferrite magnets are inexpensive but suffer from low flux-density for a given volume.

In order to synchronise the stator armature excitation with the rotor position, a Hall sensor is used. This sensor is embedded in the stator and senses the position of the rotor. Before energising a particular stator winding, acknowledgment of rotor position is necessary in this motor. Normally, three Hall sensors are embedded into the stator. Whenever the rotor poles pass near this sensor, it will generate Low and High signals. Based on the combination of this sensor's response, the excitation sequence to the stator winding is determined and energised.

3.13.2 Working

The principle and operation of a BLDC motor is similar to that of a brushed DC motor. According to the Lorentz force law, whenever a current carrying conductor is placed in a magnetic field, it experiences a force. As a result of this force, the magnet will experience an equal and opposite force. In case of stationary current-carrying conductors, the permanent magnet moves in the motor. The stator coil becomes an electromagnet, when it is switched electrically by a supply source and starts producing a uniform magnetic field in the air gap. Even though the supply is a DC source, by electrical switching, it generates an AC voltage waveform with trapezoidal shape and in turn starts producing a uniform field in the air gap. Due to the interaction between the stator electromagnets and the permanent-magnet rotor, the rotor continues to rotate.



(a) (b)

Figure 3.42 Working Principle of a BLDC Motor (a) Switching Sequence (b) Stator Electromagnets and Permanent Magnet Rotor

The working principle of a BLDC motor can be understood with the help of diagrams shown in Figures 3.42(a) and (b).

The stator windings of the motor are excited based on different switching sequences. Due to the switching of windings as high and low states, the corresponding winding is energised as north and south poles. The north and south poles of the permanent-magnet rotor will align with the stator poles, causing the motor to rotate. The motor produces torque because of the development of attraction forces when north–south or south–north alignment occurs. Similarly, it produces repulsion forces when north–north or south–south alignment occurs. By this way, the motor rotates in a clockwise direction based on the switching sequence applied to the coils.

3.13.3 Classification

The BLDC motors are classified based on two different parameters.

Depending on the Arrangement of the Permanent Magnet Rotor and Stator Electromagnets

Outer Rotor Design

In an outer rotor BLDC design, the windings are located in the core of the motor and the rotor permanent magnets surround the stator windings, as shown in Figure 3.43.

Here, the rotor permanent magnet acts as an insulator and thereby reduces the rate of heat dissipation from the motor. Therefore, outer rotor design motor is mainly used in lower duty cycles or lower-rated current applications. The main advantage of an outer rotor design is its motor, which offers relatively low cogging torque.

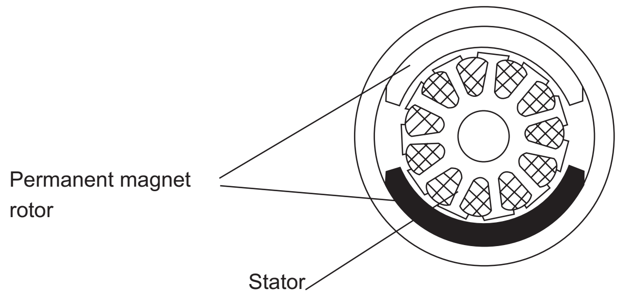


Figure 3.43 Outer Rotor Design

Inner Rotor Design

In an inner rotor BLDC design, the stator windings surround the permanent-magnet rotor and are affixed to the motor's housing, as shown Figure 3.44.

The main advantage of this type of rotor construction is that it is capable of dissipating heat easily. For this reason, the majority of BLDC motors use inner rotor design. Moreover, an inner rotor design offers lower rotor inertia.

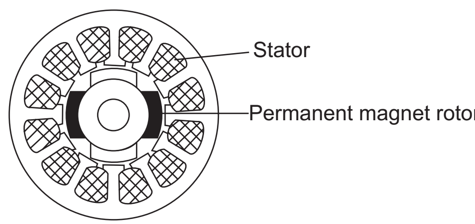


Figure 3.44 Inner Rotor Design

Based on Physical Configurations of Stator Windings, as:

(i) Single-phase motor
(ii) Two-phase motor and
(iii) Three-phase motor

3.13.4 Advantages, Disadvantages and Applications

Advantages

1. Effective operation over a wide range of speeds with its rated load current.
2. Has high efficiency due to the presence of a permanent-magnet rotor.
3. Due to the absence of brushes, it operates at high speed even in loaded and unloaded conditions.
4. It is small in size and less in weight. It has high output power-to-size ratio.
5. It has reduced size with far superior thermal characteristics.

6. Higher dynamic response due to low inertia.
7. Less electromagnetic interference.
8. Higher speed operation is possible with low electric noise.
9. Less maintenance cost due to the absence of brushes.

Disadvantages

1. Requires complex drive circuitry for its operation.
2. Needs additional Hall sensors.
3. Control of BLDC motor requires expensive electronic controllers.

Applications

The BLDC motor is used for a wide variety of applications, such as:

1. Computer hard-disk drives
2. Electric vehicles, hybrid vehicles, and electric bicycles
3. Industrial robots
4. Domestic appliances, such as washing machines, fans, and dryers
5. Compressors, pumps and blowers etc.

3.14 Transformers

The AC system is generally used instead of a DC system for the generation, transmission and distribution of electrical power because the AC voltage can be increased or decreased according to the requirement. Single-phase transformer is one such device used in power systems. A transformer is a static device used for coupling two or more electric circuits. It works on the principle of mutual induction and transfers the electric energy from one circuit to another when there is no electrical connection between the two circuits. When compared to the input voltage, the output voltage of transformer can be increased or decreased with a proportional decrease or increase in the current ratings. The size of the single-phase transformer can vary from very small to large size. But irrespective of its shape or size, a transformer is used only to transfer electric energy from one voltage level to another with same frequency. The analysis of a single-phase transformer, including its working principle and different tests are explained in this chapter. Also, the construction and operation of a three-phase transformer and an auto-transformer are discussed.

3.15 Single-Phase Transformer

A transformer is a stationary apparatus by which electric power in one circuit is transformed to another circuit with the same frequency. It transforms electrical power without any direct electrical connection between input and output, but with the help of mutual induction between the two windings. The working principle and construction of single-phase transformer is explained below.

3.15.1 Construction

The essential components of the transformer are:

1. Magnetic core
2. Two windings, namely primary and secondary windings
3. A time varying magnetic flux

The constructional diagram of a single-phase transformer is shown in Figure 3.45.

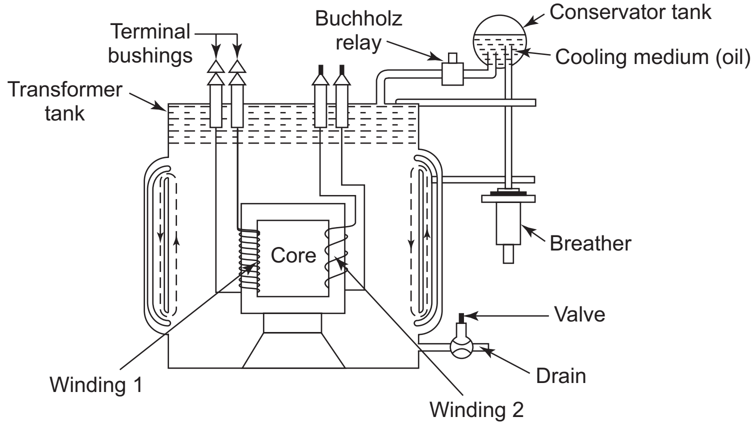


Figure 3.45 Constructional Diagram of a Single-phase Transformer

The different components of a single-phase transformer are:
1. Core
2. Limb
3. Yoke
4. Windings
5. Conservator
6. Cooling medium
7. Breather
8. Explosion vent
9. Buchholz relay

These components can be grouped as magnetic circuit, electrical circuit, dielectric circuit, tanks and accessories.

Magnetic Circuit

The core, yoke and limb of the transformer form the magnetic circuit. The core of the transformer is made of high-grade silicon steel or sheet steel lamination, which has low hysteresis loss and provides a continuous magnetic path, when it is assembled. The vertical position of the core on which the coil is wound is called limb, while the horizontal position of the core is known as yoke. The main functions of the magnetic circuit are:
1. Provides low reluctance path for carrying the flux.
2. Carries the windings required for electric power transfer.

Electrical Circuit

In the transformer, there are two windings, namely primary and secondary windings that form the electrical circuit. The winding connected to the AC source is called the *primary winding,* normally indicated using ‘1’. Similarly, the winding connected to the load is called the *secondary winding,* normally indicated using ‘2’. These windings are made of copper and its cross-section can be either rectangular or circular, depending on the voltage level. The rectangular cross-section is used for both low and high-voltage windings in large transformers, whereas, the circular cross-section is used for high-voltage windings in small transformers.

According to the core construction and the manner in which these windings are wound, transformers are classified as core-type and shell-type transformers. In the core-type transformer, the windings surround a considerable part of the core, whereas in the shell-type transformer, the core surrounds a considerable portion of the windings.

Core-type Transformer **[AU April/May, 2015]**

In core-type transformer, rectangular frame laminations are formed to build the core of the transformer. The laminations are pressed or punched out from larger steel sheets and arranged into thin steel strips to assemble the letters "E", "I", "L" and "U" as shown Figure 3.46.

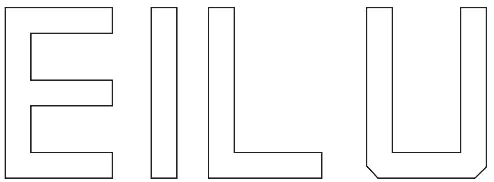


Figure 3.46 Thin Steel-strip Laminations

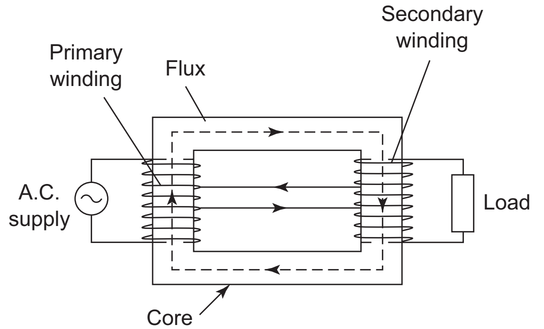


Figure 3.47 Core-type Transformer With Windings

For simplicity, the primary and secondary windings are located on the separate limbs of the core as shown in Figure 3.47.

Shell-type Transformer

The shell-type transformer can be obtained using the laminations "E", and "I". This type of transformer has three limbs or legs in which the width of the central limb is twice the width of the outer limbs. The centre limb carries the whole flux generated due to input voltage and the side limb carries half of the flux. The two windings of the transformer are wound on the centre limb are shown in Figure 3.48.

The comparison between core and shell-type transformers is given in Table 3.4.

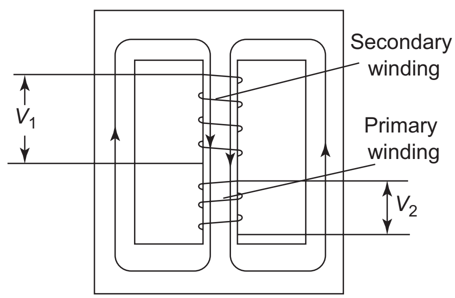


Figure 3.48 Shell-type Transformer with Windings

Table 3.4 *Core-type vs. Shell-type Transformer* **[AU April/May, 2015]**

Core-type Transformer	Shell-type Transformer
The windings surround the core and are placed on the side limbs.	The core surrounds a considerable portion of the windings and is placed on the central limb
Lamination is cut in the form of L, I or U strips.	Lamination is cut in the form of long strips that are arranged in the shape of E and I.

(*Contd.*)

Requires more copper material	Requires less copper material
Two limbs exist	Three limbs exist
Requires more insulation	Requires less insulation
Equal distribution of flux on the limbs	Unequal distribution of flux in the limbs
Two magnetic circuits are formed by the windings.	One magnetic circuit is formed by the windings.
More loss exists in terms of leakage flux	Less loss exists in the system
Easy to maintain	Difficult to maintain
High output can be achieved	Less output can be achieved
Natural cooling is possible	Cooling mechanism exists

Dielectric Circuit

In a transformer, to insulate the conducting parts from each other, insulations are used. These insulations comprise of the dielectric circuit and are used in various places to reduce eddy current losses. A light coating of varnish or any oxide is used to insulate the lamination whose thickness varies from 0.35 mm to 0.5 mm for a normal AC operation.

Tanks and Accessories

The essential protective devices attached to the transformer that increase the life span of a transformer are as follows:

Conservator

A cylindrical tank that is placed at the top or roof of the transformer main tank is called a conservator. It is provided with a large cover for proper maintenance and cleaning of the transformer. In addition to acting as the transformer-cooling medium, it acts like a reservoir. An adequate space is provided in the conservator since the volume of the cooling medium might increase due to rise in transformer temperature, when it is fully loaded.

Cooling Medium

When the transformer is loaded, some losses occur within. These losses appear in the form of heat, which increases the transformer temperature. Hence, a proper provision should be made in the transformer to dissipate this heat and to maintain the transformer temperature within its limits. Therefore, a cooling medium in the form of air or oil is required to remove the heat generated during loading.

Breather

The heart of the transformer is the breather, which is similar to the human heart. The breather transports fresh air in and out of the transformer. This component is required to maintain the cooling-medium level in the conservator. In addition, the breather is provided with silica gel to eliminate moisture content in the cooling medium and to maintain the quality of cooling medium.

Explosion Vent

A thin aluminium pipe that is placed at the ends of the transformer to prevent it from damage is called an explosion vent. It helps in maintaining the pressure inside the transformer, which drastically increases when there is an increase in temperature of the transformer.

Buchholz Relay

A gas-actuated relay placed in the large-size transformer to protect it from internal fault is called Buchholz relay. It is used in the transformer with a rating greater than 500 kVA. Its working principle is that, when an internal fault takes place, evaporation of oil in the form of gas occurs due to increase in temperature. The evaporated gas activates the Buchholz relay and alarms the personnel, which helps in disconnecting the transformer from the supply.

3.15.2 Working **[AU April/May, 2014]**

The working principle of a two-winding transformer can be understood with the help of the diagram shown in Figure 3.49. Based on the principle of Faraday's law of electromagnetic induction and mutual induction, the transformer transfers electric power from one circuit to another. Faraday's law of electromagnetic induction states that whenever a current-carrying conductor cuts the magnetic flux, an emf gets induced in the conductor. When another coil is brought near to the former coil, another emf gets induced in the latter coil due to mutual induction. This induced emf in the stationary conductor is called statically induced emf. Therefore, when the latter coil forms a closed loop, the induced emf produces a current that flows through the loop and load.

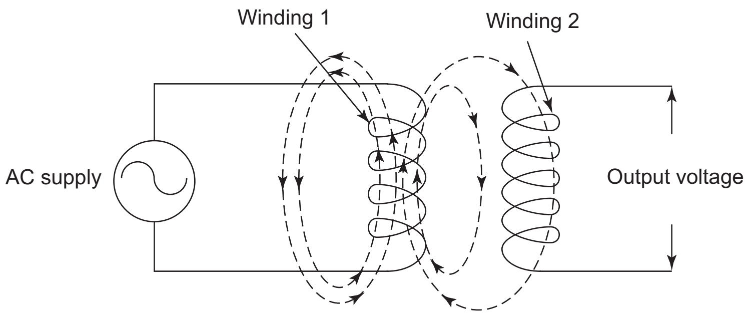


Figure 3.49 Two-winding Transformer

Working of a Transformer

The schematic diagram to explain the working of a transformer is shown in Figure 3.50. It consists of two inductive coils, which are electrically separated but are wound on a high-permeability, laminated-steel magnetic core to maximise the coupling. These coils have high mutual inductance and are used to transfer electric energy from one voltage level to another. One of the two coils connected to a source of alternating voltage, V_1, with frequency, f, is called primary winding. The second coil connected to a load is called the secondary winding. The load draws out the electrical energy transformed to this winding. The primary winding has N_1 turns while the secondary winding has N_2 turns. When an alternating voltage excites the primary winding, it circulates an alternating current. This current produces an alternating flux, ϕ, which completes its path through the common magnetic core. Thus, an alternating flux links with the secondary winding and a mutually induced emf develops in the secondary winding. If the load is connected to the secondary winding, this emf drives a current through it.

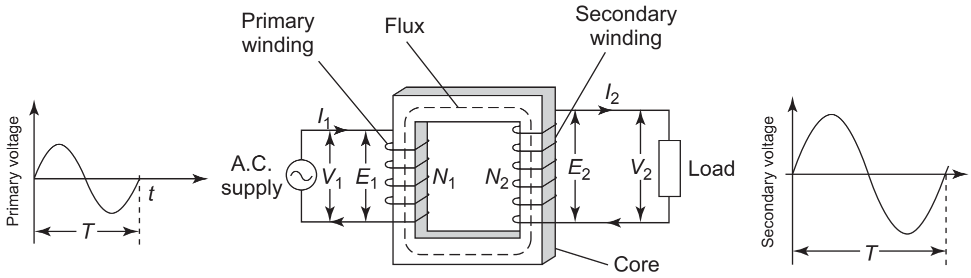


Figure 3.50 Working of a Transformer

Here, the rms values of the induced voltages in the primary and the secondary windings are E_1 and E_2 volts respectively. These voltages will have sinusoidal waveform with same frequency as that of the applied voltage V_1. The currents which flow in the closed primary and the secondary circuits are I_1 and I_2 respectively. Hence, the electrical energy is transferred from the primary circuit to the secondary circuit without changing the frequency of the input voltage.

***Turns ratio*:** It is the ratio of the number of turns in primary to the number of turns in secondary. It is given by

$$\text{Turns ratio } = \frac{N_1}{N_2}$$

If $N_2 > N_1$, the transformer is called a step-up transformer and if $N_2 < N_1$, the transformer is called a step-down transformer.

3.15.3 EMF Equation of the Single Phase Transformer **[AU April/May, 2016]**

Consider that N_1 and N_2 are the number of turns in the primary and secondary windings respectively, ϕ is the alternating magnetic flux generated in the primary winding with its maximum value as ϕ_m in Weber and f is the frequency of the supply voltage in Hertz. When a supply voltage V_1, with frequency f, is applied to the primary winding, an alternating magnetic flux ϕ with same frequency is generated, as shown in Figure 3.51.

Using Faraday's law, the average emf induced per turn in the primary winding, e_1, is proportional to the average rate of change of flux.

i.e.,
$$e_1 = \frac{d\phi}{dt} = \frac{\phi_m - 0}{\frac{1}{4f} - 0} = 4f\phi_m \text{ Wb/sec}$$

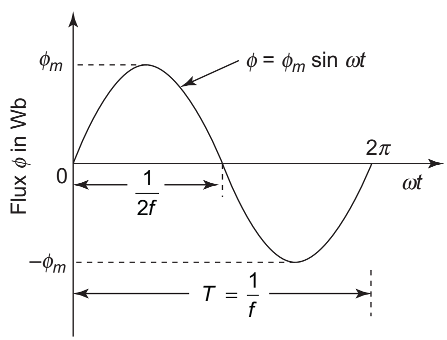


Figure 3.51 Alternating Flux Generated at the Primary Winding

Here, the rate of change of flux per turn is the induced emf in volts. Therefore, the average emf/turn = $4f\phi_m$ V.

If the flux ϕ variation is sinusoidal, then rms value of the induced emf is obtained by multiplying the average value with form factor.

$$\text{Form factor} = \frac{\text{rms value}}{\text{average value}}$$

For a sinusoidal waveform, the form factor is 1.11.

Therefore, the rms value of the induced emf per turn, in the primary winding is

$$e_{1r} = 1.11 \times 4 f \phi_m = 4.44 f \phi_m \text{ V}$$

Hence, for N_1, the rms value of the induced emf in the primary winding is given by

$$E_1 = 4.44 f \phi_m N_1 \tag{3.15}$$

Similarly, the rms value of the induced emf in the secondary winding is given by

$$E_2 = 4.44 f \phi_m N_2 \tag{3.16}$$

In an ideal transformer on no load, the relation between the input and output voltages and the induced emfs in the windings are given by:

$$V_1 = E_1 \text{ and } V_2 = E_2$$

where V_2 is the terminal voltage.

Using the above equations, we get the transformation ratio as

$$K = \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{V_2}{V_1}$$

The transformation ratio (K) is defined as the ratio of the secondary induced voltage (E_2) to the primary induced voltage (E_1).

Also, the input and output powers of the transformer in lossless condition is given by V_1I_1 and V_2I_2 respectively. Assuming both the powers are equal, we get

$$V_1I_1 = V_2I_2$$

$$\frac{V_2}{V_1} = \frac{I_1}{I_2} = K$$

Here, the currents are in the inverse ratio of the (voltage) transformation ratio.

3.15.4 Reasons for not Using DC Supply as a Source **[AU Nov/Dec, 2013]**

The following are the reasons for not connecting a transformer to a DC source or supply:

1. A constant flux is produced in the primary winding when a DC source is applied to the primary winding of the transformer. Due to this constant flux, no emf is induced in the secondary winding. Hence, no current will be delivered to the load connected to the secondary winding.
2. When AC source is used, due to the alternating flux produced in the primary winding, a self-induced emf is generated in it, which opposes the applied voltage and limits the primary current. But, when a DC source is used, there will be no induced emf and hence heavy current flows in the primary winding, which results in the failure of the transformer.

3.15.5 Types of Transformers

1. **Based on transformation ratio, K, or number of turns in the windings**
   - ***Step-up transformer:*** It transfers a high current, low AC voltage into a low current, high AC voltage. Here, $N_2 > N_1$ and $V_2 > V_1$. Therefore, the transformation ratio is greater than 1, i.e., $K > 1$.

- ***Step-down transformer:*** It is the opposite of a step-up transformer i.e., it transfers a low current, high AC voltage into a high current, low AC voltage. Here, $N_2 < N_1$ and $V_2 < V_1$. Therefore, the transformation ratio is less than 1, i.e., $K < 1$.

2. **Based on the service it provides**
   - ***Power transformer***
   - ***Distribution transformer***
   - ***Instrument transformer:***
     - Current transformer
     - Potential transformer
     - Auto-transformer
3. **On the basis of the supply**
   - Single-phase transformer
   - Three-phase transformer
4. **On the basis of cooling medium**
   - Air-cooled transformer
   - Oil-cooled transformer

3.16 Ideal Transformer **[AU April/May, 2016]**

In an ideal transformer, there is no dissipation loss, as the primary and secondary windings have zero resistance, and the core has infinite permeability. The coils should be tightly coupled so that there is no leakage flux and the coefficient of coupling should be equal to one i.e., $k = 1$. As compared to the load connected, the inductive reactances of primary and secondary windings should be extremely large. Self-inductance of primary or secondary winding is proportional to the square of the number of turns of the coil.

An ideal transformer has the following properties:

- Absence of winding resistance
- Very high or infinite permeability of the core
- No leakage flux
- 100% efficiency

3.17 Characteristics of Single-Phase Transformer

3.17.1 Ideal Transformer on No-load **[AU Nov/Dec, 2015]**

The ideal transformer with no load is shown in Figure 3.52. Here, V_1 is the voltage supplied to the primary winding, with N_1 turns. Since the windings used in an ideal transformer are free from losses and the core has finite permeability, the current drawn from the source is used to produce the magnetic flux by magnetising the iron core. Hence, the current drawn from the primary winding is called magnetising current and is denoted by I_m. As the winding is purely inductive, the magnetising current I_m lags the supply voltage by 90°. Due to I_m, an alternating flux ϕ in phase with I_m is produced. This alternating flux ϕ induces an emf in the primary winding, E_1, which opposes the supply voltage i.e., the phase angle between E_1 and V_1 is 180°.

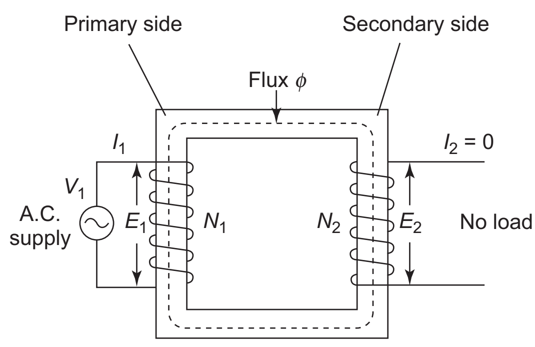


Figure 3.52 Ideal Transformer on No Load

In an ideal transformer, there is no leakage flux. Hence, all the flux produced in the primary winding ϕ links the secondary winding. Since an alternating flux ϕ links the secondary winding, an emf, E_2, is induced in the secondary winding. Here, E_2 is in phase with E_1 and in anti-phase with V_1. Though E_1 and E_2 are in phase with each other, the magnitude of induced emf in secondary winding, E_2, depends on the number of turns, N_2, in the secondary winding. Since the secondary circuit is not closed, the secondary current, $I_2 = 0$. The phasor diagram for the ideal transformer on no load with $N_2 > N_1$ is shown in Figure 3.53.

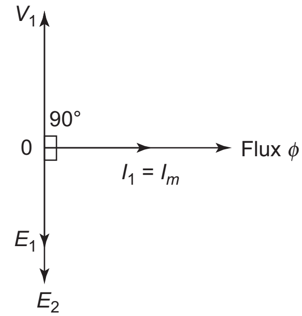


Figure 3.53 Phasor Diagram of an Ideal Transformer on No Load

3.17.2 Practical Transformer on No-load

[AU Nov/Dec, 2016]

In practice, there exists no ideal transformer i.e., any transformer has losses in it. The hysteresis and eddy current losses exist in the transformer core due to the alternating flux produced by the AC source. These losses can be reduced by using: (i) high-grade materials like silicon steel and (ii) laminations or stacks of thin laminations for building the core. No winding in the transformer is purely inductive and it has small resistance, which contributes to copper loss when the current starts flowing through the winding. The primary current drawn from the source that contributes to both core loss and copper loss is called no-load current, which is denoted as I_0. Therefore, the two components of no-load current are:

- ***Wattless component:*** The magnetizing current, I_m, which is a purely reactive component of I_o that lags V_1 by 90°, is required to magnetise the core and to produce the magnetic flux.
- ***Wattful component:*** The power component, I_c, supplies the total losses of the transformer under no-load condition. It is the active or core loss component of I_o, which is in phase with V_1.

Hence, the total no-load current of a transformer, I_o is the vector addition of I_m and I_c and is given by

$$\overline{I}_o = \overline{I}_m + \overline{I}_c$$

Also, the phase angle between I_o and V_1 is ϕ_0 and the no-load power factor of the transformer is cos ϕ_0. The phasor diagram of practical transformer on no-load is shown in Figure 3.54.

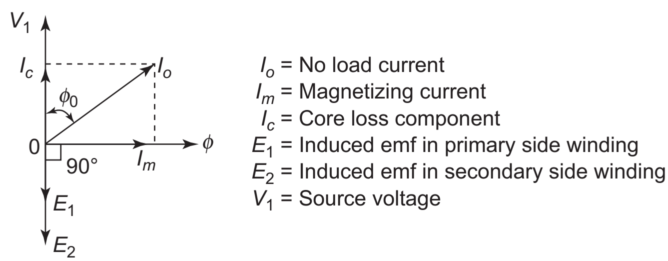


Figure 3.54 Phasor Diagram of Practical Transformer on No Load

Using Figure 3.54, the two components of I_o are given by

$$I_c = I_o \cos \phi_0 \text{ and } I_m = I_o \sin \phi_0$$

The magnitude of no-load current is given by

$$I_o = \sqrt{I_m^2 + I_c^2}$$

The input power on no-load, W_o, supplied to the primary winding of the transformer is given by

$$W_o = V_1 I_o \cos\phi_o = V_1 I_c$$

Since I_c is very small, approximately 0.04 times the full-load rated current, it contributes more to core loss or iron loss when compared to primary copper loss. Hence, the input power, W_o, supplies the iron loss of the transformer, P_i, which is constant for different load conditions.

i.e., $W_o = V_1 I_o \cos\phi_o = P_i =$ iron loss

Since the secondary circuit is not closed, the secondary current, $I_2 = 0$ and hence, the output power developed in the transformer is zero.

3.17.3 Transformer on Load [AU Nov/Dec, 2016]

The practical transformer connected with load is shown in Figure 3.55. When the primary winding of the transformer is connected to the AC source, V_1 an emf E_2 is induced in the secondary winding due to transformer action. Since the load is connected to the secondary winding, the current I_2 flows through the winding and an output voltage V_2 is obtained across the load. The phase difference between the output voltage V_2 and I_2 depends on the load i.e., if load is inductive, I_2 lags V_2 by ϕ_2, if load is capacitive, I_2 leads V_2 by ϕ_2 and if load is resistive, I_2 is in phase with V_2.

The current I_2 flowing through secondary winding produces its own magnetic flux ϕ_2, which opposes the magnetic flux ϕ i.e., the direction of ϕ_2 is opposite to ϕ, as shown in Figure 3.55(a). Since ϕ_2 opposes ϕ, there is a momentary reduction in the magnetic flux ϕ, which further decreases the induced emf E_1. This decrease in E_1 increases the difference between V_1 and E_1 that causes the primary winding to draw extra current I_2', thereby producing an additional flux ϕ_2' opposite to ϕ_2, as shown in Figure 3.55(b). This extra current drawn by the primary winding is called load component of I_1. Since this current I_2' neutralizes ϕ_2, the flux is maintained constant in the transformer and hence, it is also known as constant flux machine.

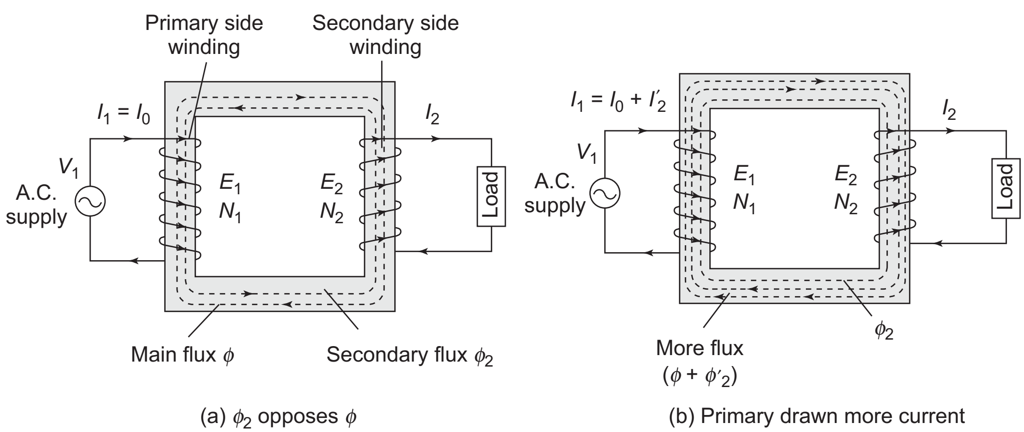


Figure 3.55 Transformer on Load

The ampere-turns which produce the flux ϕ_2 is N_2I_2, and the ampere-turns which produce the flux, ϕ_2' is N_1I_2'. As these ampere-turns get balanced, the flux in the transformer is maintained constant for any load condition.

i.e., $$N_1I_2' = N_2I_2$$

Therefore, $$I_2' = \frac{N_2}{N_1}I_2 = KI_2$$

where K is the transformation ratio.

Example 3.5

The required no-load ratio in a single-phase, 50 Hz, core type transformer is 6600/250 V. Find the number of turns in each winding, if the flux is 0.06 Wb. **[AU April/May, 2016]**

Solution

Given $f = 50$ Hz and $\phi_m = 0.06$ Wb

At no load, $E_1 = V_1 = 6600$ V and $E_2 = V_2 = 250$ V

The induced emf in the primary winding is given by

$$E_1 = 4.44\phi_m f N_1$$

Substituting the given values, we get

$$6600 = 4.44 \times 0.06 \times 50 \times N_1$$

Therefore, the number of turns in primary winding, $N_1 = 495.495 \approx 495$

The transformation ratio is given by

$$K = \frac{E_2}{E_1} = \frac{250}{6600} = 0.03787$$

The number of turns in secondary winding is given by

$$N_2 = KN_1 \left(\text{since } K = \frac{N_2}{N_1} \right)$$

Therefore, $$N_2 = 0.03787 \times 496 = 18.78 \approx 19$$

Example 3.6

The no-load current of a transformer is 15 A at a power factor of 0.2, when connected to a 460 V, 50 Hz supply. If the primary winding has 550 turns, calculate (i) the magnetising component of no-load current, (ii) the iron loss and (iii) the maximum value of the flux in the core. **[AU Nov/Dec, 2015]**

Solution

Given $I_o = 15$ A, $\cos\phi_o = 0.2$, $V_o = V_1 = 460$ V, $f = 50$ Hz and $N_1 = 550$

(i) The magnetising component of no-load current is given by

$$I_m = I_o \sin\phi_o$$
$$= 15 \times \sin(\cos^{-1}(0.2)) = 14.6969 \text{ A}$$

(ii) The iron loss of the transformer is given by

$$P_i = V_o I_o \cos\phi_o$$
$$P_i = 460 \times 15 \times 0.2 = 1380 \text{ W}$$

(iii) At no load, $E_1 = V_o = V_1 = 4.44 f\phi_m N_1$
Substituting the given values, we get

$$460 = 4.44 \times 50 \times \phi_m \times 550$$

Therefore, the maximum value of the flux in the core is $\phi_m = 3.76$ mWb

3.18 VA Rating of Transformer

The iron and copper losses depend only on the supply voltage and the current flowing through the winding respectively. Since these losses do not depend on the phase angle between the supply voltage and the current, the transformer rating expressed as a product of voltage and current is called VA rating of the transformer. Therefore, the transformer rating is expressed as kVA and not as kW. Depending upon the size of transformer, it can carry a maximum current called full load current, I_{FL} without overheating. Therefore, the transformer rating is given by

$$\text{kVA} = \frac{V_{\text{rated}} \times I_{\text{FL}}}{1000}$$

3.19 Losses and Efficiency of the Transformer

3.19.1 Losses in the Transformer

The different losses exist in the transformer are: (i) Core or iron losses (ii) Ohmic or copper losses (iii) Stray losses and (iv) Dielectric losses. The classification of losses in transformer is shown in Figure 3.56.

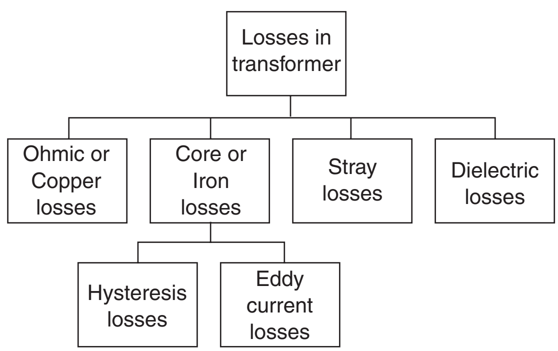


Figure 3.56 Losses in transformer

(i) Core or Iron Losses

The losses which take place in the transformer due to the alternating magnetic flux produced in the magnetic core is called core or iron losses, P_i. These losses are further classified as hysteresis and eddy current losses.

Hysteresis Losses, P_h

Since there is generation of alternating flux in the core of the transformer, magnetisation and demagnetisation of the core takes place in an alternate manner. For each cycle of alternating flux, a hysteresis loop is obtained. Due to the effect of this hysteresis, a loss of energy in the form of heat occurs in the transformer called hysteresis loss as given by

$$P_h = K_h B_m^{1.67} f v \tag{3.17}$$

where K_h is the hysteresis constant which varies for each material used in the transformer, B_m is the maximum flux density in Wb/m^2, f is the frequency in Hertz and v is the volume of the core in m^3.

Eddy Current Losses, P_e

An induced emf is generated in the core when the alternating magnetic flux links with a closed circuit. Due to this induced emf, a current called eddy current circulates within the core. Its magnitude depends on the induced emf and the winding resistance of the transformer. The I^2R losses which occur in the transformer due to this eddy current is called eddy current loss as given by

$$P_e = K_e B_m^2 f^2 t^2 v \tag{3.18}$$

where K_e is the eddy current constant and t is the thickness of the core in m.

Since the transformer is a constant flux machine for a given supply voltage V_1 with constant frequency f, the flux density, B_m is also constant. Hence, it is clear that both the hysteresis and eddy current losses are constant for a given transformer. Therefore, the core or iron losses are also called constant losses. These constant losses can be minimised by using high grade silicon steel material which possesses low hysteresis loop with thin laminations for core construction of the transformer.

(ii) Ohmic or Copper Losses

The losses which occur in the transformer due to its winding resistances is called copper losses, P_c. Therefore, the total copper loss in the transformer is given by

$$P_c = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_{1e} = I_2^2 R_{2e}$$

Since the currents flowing through the windings vary with respect to the load, these losses are called variable losses.

(iii) Stray Losses

The losses exist in the transformer due to the leakage of magnetic flux is called stray losses. Since the percentage of stray losses is very less when compared to iron and copper losses, this loss can be neglected.

(iv) Dielectric Loss

The losses which occur in the insulating material of the transformer are called dielectric losses which affect the efficiency of the transformer. Similar, to stray loss, the dielectric loss can be eliminated as its percentage is very small than that of iron and copper losses. Therefore, the total loss in the transformer, P_T is

$$P_T = P_i + P_c$$

3.19.2 Efficiency of Transformer

Due to iron and copper losses in the transformer, the power output of the transformer is less when compared to the power supplied to it. Therefore,

Power output = Power input – Total losses

or Power input = Power output + Total losses

In general, the efficiency of the transformer is defined as the ratio of the power output to power input and as given by

$$\eta = \frac{\text{Power output}}{\text{Power intput}}$$

i.e.,
$$\eta = \frac{\text{Power output}}{\text{Power output + total losses}} = \frac{\text{Power output}}{\text{Power output} + P_i + P_c}$$

The power output of the transformer is given by

$$\text{Power output} = V_2 I_2 \cos \phi_2$$

where $\cos \phi_2$ is the power factor of the load.

Similarly, the expression for efficiency of the transformer when it is subjected to different fractional loads can be obtained as follows:

Let $n = \dfrac{\text{Actual load}}{\text{Full load}}$. If the load connected to the transformer changes, the load current varies proportionately as

$$I_2 = n(I_2)_{\text{FL}}$$

where $(I_2)_{\text{FL}}$ is the load current at full load condition.

Similarly, the VA rating of the transformer and copper loss depending on the current also varies as

$$\text{VA rating} = n \times (\text{VA rating})_{\text{FL}}$$

and
$$P_c = n^2 (P_c)_{\text{FL}}$$

Therefore, the general expression for efficiency of the transformer is

$$\%\eta = \frac{n \times (\text{VA rating})_{\text{FL}} \cos \phi_2}{n \times (\text{VA rating})_{\text{FL}} \cos \phi_2 + P_i + n^2 (P_c)_{\text{FL}}} \times 100$$

As the load connected to the transformer increases from the no-load condition, the load current also increases, thereby increasing the efficiency of the transformer. The efficiency of the transformer attains a maximum value at a particular load current. If the load current, I_2, is further increased beyond the maximum value I_{2m}, the efficiency of the transformer starts decreasing. The plot between the efficiency and load current is shown in Figure 3.57.

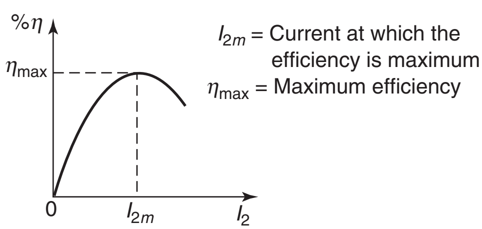


Figure 3.57 Efficiency vs load current

Here, I_{2m} is the load current at which the efficiency of the transformer is maximum, $\eta_{\max}$.

3.20 ALL-DAY EFFICIENCY **[AU Nov/Dec, 2011]**

The normal efficiency of the transformer is given by the ratio of output power to input power, which is not the true measure of the performance of distribution-transformers that serve residential and commercial loads. The true measure of distribution-transformer performance is given by all-day efficiency, which is the ratio of total energy output in kWh to total energy input in kWh.

Therefore, $$\% \text{ All day } \eta = \frac{\text{Output energy in kWh}}{\text{Input energy in kWh}} \times 100$$

Based on this all-day efficiency, the performance of various transformers is compared.

Example 3.7

A 600 kVA single-phase transformer has an efficiency of 94% both at full load and half load at unity power factor. Determine the efficiency at 75% of full load, at 0.9 power factor. **[AU April/May, 2015]**

Solution

Given rating of the transformer = 600 kVA, $\cos \phi_2 = 1$, $\eta_{\text{FL}} = \eta_{\text{HL}} = 94\%$

The full-load efficiency of the system is

$$\%\eta_{\text{FL}} = \frac{\text{kVA} \cos \phi_2}{\text{kVA} \cos \phi_2 + P_i + (P_c)_{\text{FL}}} \times 100$$

Substituting the known values, we get

$$94 = \frac{(600 \times 10^3 \times 1)}{(600 \times 10^3 \times 1) + P_i + (P_c)_{\text{FL}}} \times 100$$

Therefore, $$P_i + (P_c)_{\text{FL}} = 38297.87 \qquad (1)$$

It is known that, $n = \dfrac{\text{Given load}}{\text{Full load}}$. Hence, for half load, $n = 0.5$.

Therefore, for half load,

$$\%\eta_{\text{HL}} = \frac{n \times \text{kVA} \cos \phi_2}{n \times \text{kVA} \cos \phi_2 + P_i + n^2 \times (P_c)_{\text{FL}}} \times 100$$

Substituting the known values, we get

$$94 = \frac{0.5 \times 600 \times 10^3 \times 1}{0.5 \times 600 \times 10^3 \times 1 + P_i + (0.5)^2 (P_c)_{\text{FL}}} \times 100$$

Therefore, $$P_i + 0.25(P_c)_{\text{FL}} = 19148.93 \qquad (2)$$

Solving Eqn.(1) and Eqn.(2), we get

$$(P_c)_{\text{FL}} = 25531.92 \text{ W and } P_i = 12765.95 \text{ W}$$

Therefore, the efficiency at 75% full load, i.e., when $n = 0.75$ and $\cos \phi_2 = 0.9$ is

$$\%\eta = \frac{n \times \text{kVA} \cos\phi_2}{n \times \text{kVA} \cos\phi_2 + P_i + n^2 \times (P_c)_{\text{FL}}} \times 100$$

$$= \frac{0.75 \times 600 \times 10^3 \times 0.9}{(0.75 \times 600 \times 10^3 \times 0.9) + 12765.97 + ((0.75)^2 \times 25531.91)} \times 100 = 93.72\%$$

3.21 Three-Phase Transformer [AU Nov/Dec, 2014]

The transformer used to supply or transfer large amounts of power to three-phase connections, to meet the required demand economically, is called a three-phase transformer. In power systems, it is used in different stages for stepping up or stepping down higher voltages. There are numerous advantages of a three-phase transformer, when compared to a single-phase transformer.

3.21.1 Construction

There are two methods to construct a three-phase transformer, as explained below:

1. ***Using a bank of three single-phase transformers:*** In this method, three single-phase transformers are connected such that the primary windings of each transformer are connected to each other. Similarly, the secondary windings of each transformer are connected to each other. The phase angle between these three single phases is 120°. In this method, if a fault occurs in any one of the single-phase transformers, the continuity of the supply is maintained by the other two single-phase transformers.
2. ***Using a single three-phase transformer:*** Here, the three-phase transformer has a single core, where all the three windings get wounded. This method of constructing a three-phase transformer is preferable since it is economical and more convenient. In this method, if a fault occurs, there will be discontinuity in the supply.

3.21.2 Winding Connection

The three-phase transformer can be constructed using a common magnetic core for both primary and secondary windings. Based on the type of construction of primary and secondary windings, three-phase transformers are classified as:

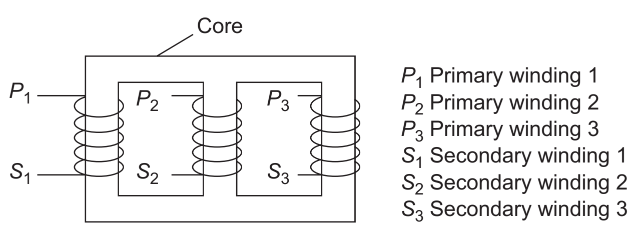


Figure 3.58 Core-type Three Phase Transformer

1. ***Core-type three-phase transformer:*** A core-type three-phase transformer is shown in Figure 3.58. Here, three limbs or legs and two yokes, which form a magnetic path, exist in the core. For each phase, both the low and high-voltage windings formed using circular cylindrical coils are concentrically wound on each limb. Here, the three-phase transformer is constructed using stack lamination. The low-voltage windings are wound near the core with suitable insulation between them, whereas, the high-voltage windings are wound over the low-voltage windings with suitable insulation between them. When the system is balanced, the magnetic flux generated in the primary windings gets added up to make the resultant flux as zero. But if the system is unbalanced, the resultant flux exists, and it circulates high current.

2. ***Shell-type three-phase transformer:*** In this type, since each phase has an individual magnetic circuit, the three phases are more independent. Here, the construction is similar to the single-phase shell-type transformer and each phase is placed on top of another, as shown in Figure 3.59. Here, the winding direction of the units 'a' and 'c' are same when compare to unit 'b'. The effect of unbalanced condition is less and each laminated core surrounds its corresponding coil.

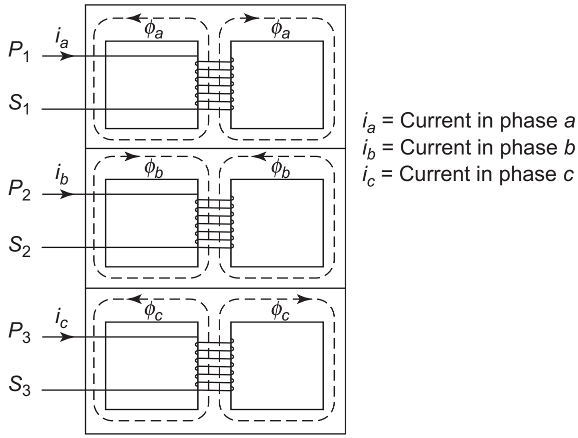


Figure 3.59 Shell-type Three-phase Transformer

3.21.3 Working of Three Phase Transformer

Consider that the primary windings of a three-phase transformer are connected in star on the cores, which are displaced by 120°, as shown in Figure 3.60. Here, for simplification purpose, only primary windings connected to the three-phase AC supply are shown. By connecting the empty leg of each core, it forms the centre leg.

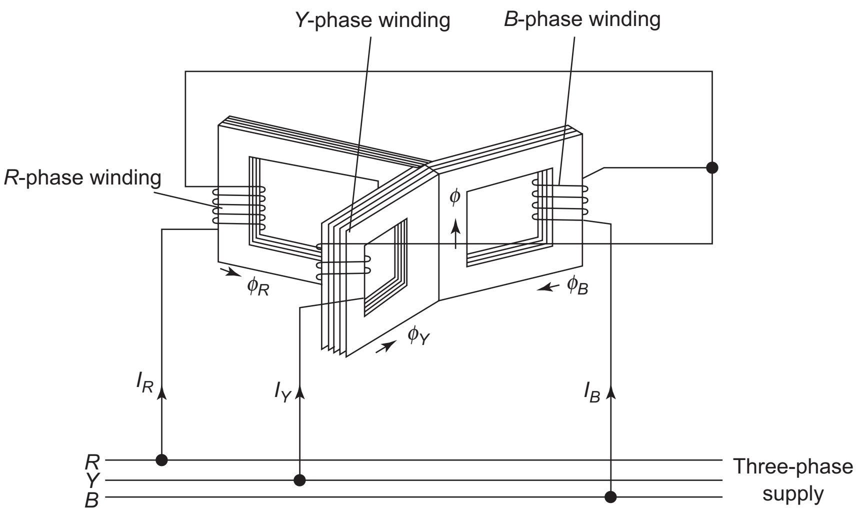


Figure 3.60 Working of a Three-phase Transformer

When the primary windings are excited using a three-phase supply, currents I_R, I_Y and I_B flow through its respective windings, which produce the magnetic fluxes, ϕ_R, ϕ_Y and ϕ_B in the respective cores. Since the centre leg is common for all the cores, the sum of all three fluxes is carried by it. These fluxes induce an emf in the primary winding and based on the principle of transformer, an emf is induced in its respective secondary winding. The phase angle between the induced emf in primary and secondary windings is based on the winding connections. The induced emf in the secondary winding drives the currents to the load connected to it. Stepping up or stepping down of primary voltage can be achieved, based on the winding connection and the number of turns in each winding.

3.21.4 Advantages and Disadvantages of Three Phase Transformer

Advantages

1. Smaller and easier to install.
2. Requires less core material and hence it is more economical.
3. Provides higher efficiency.
4. Easier to transport, as its weight is relatively less.
5. Protective device installation is easier.
6. When compared to a bank of single-phase transformers, it requires less space for the same rating.

Disadvantages

1. Repairing cost is more.
2. When it is self-cooled, its capacity reduces.

3.22 AC Machines

The induction motor is an important class of electrical machines in our day-to-day applications. More than 85% of the motors used in industries are induction motors. Three-phase and single-phase induction motors are most widely used in industrial and domestic applications respectively. In this chapter, the working principles, constructions, equivalent circuits and torques developed by three-phase and single-phase induction motors are discussed. Also, the performance characteristics of the induction motor obtained by blocked rotor, no-load and load test are discussed. At the time of starting, the induction motor draws a large amount of current that causes damage to the equipment. Hence, a starter is needed to limit the starting current. Therefore, the necessity of a starter, along with its type, is discussed elaborately. In order to obtain constant speed, irrespective of load and a variable power-factor operation, a synchronous motor is employed. The detailed working principle, construction, emf equation, characteristics, staring methods and applications of a synchronous motor are also discussed.

3.23 Three-Phase Induction Motor

A three-phase induction motor is an AC motor consisting of a three-phase winding and it works on the principle of a rotating magnetic field. The magnetic field rotates at a speed known as synchronous speed. Since the induction motor rotates at a speed less than the synchronous speed, it is also called an asynchronous motor. The working principle, construction, working and characteristics of a three-phase induction motor are discussed in this section.

3.23.1 Construction **[AU Nov/Dec, 2014; April/May, 2014]**

The two important parts of a three-phase induction motor are: (i) stationary three-phase windings, called stator and (ii) rotating component, called rotor. The rotor is connected to the mechanical load through a shaft. The schematic representation of a three-phase induction motor is shown in Figure 3.61.

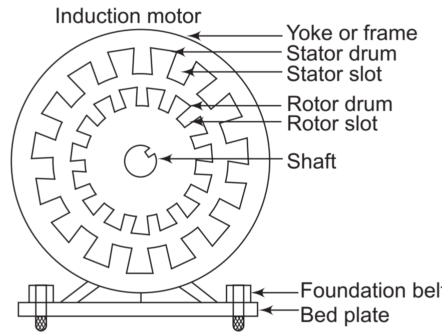


Figure 3.61 Schematic Representation of a Three-phase Induction Motor

Stator

It is the stationary part of the three-phase induction motor. The outer, solid, circular, steel metal part of the stator is called a yoke or frame. Also, it has a laminated cylindrical drum with insulated stampings, called the stator drum. These silicon-steel stampings are insulated from each other, with about 0.5 mm thickness, to reduce the iron losses and hysteresis losses. These stampings are embossed together to build the stator drum and fitted in a yoke or frame. Slots are provided in the stampings to carry the required number of stator conductors. These conductors are connected in series to form balanced three-phase windings called stator windings, which are star or delta-connected windings. These windings are wound on a definite number of poles. The stator windings, when excited using a three-phase AC supply, produce the required rotating magnetic field.

Rotor

It is the rotating part of the three-phase induction motor and is placed inside the stator. This cast iron rotor is cylindrical, laminated and provided with slots to carry rotor conductors or windings. The air gap between the stator and the rotor is kept as low as possible. The two different types of rotors are: (i) squirrel-cage rotor and (ii) slip-ring or wound rotor.

Squirrel-cage Rotor

The construction of a squirrel-cage rotor and its symbolic representation is shown in Figures 3.62 (a) and (b) respectively. In this type, the rotor core is cylindrical in shape with slots to carry the bar-shaped un-insulated copper or aluminium rotor conductors. Using a conducting copper end-ring, these conductors are permanently shorted at its ends to provide good mechanical strength. Since the closed electrical circuit resembles a cage, this rotor is called a squirrel-cage rotor or a short-circuited rotor.

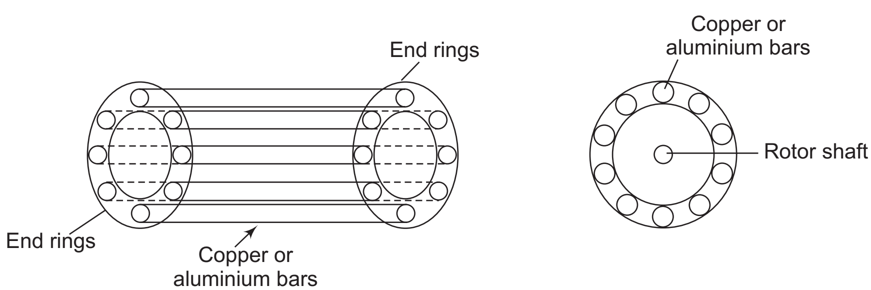


Figure 3.62 Squirrel-cage Rotor (a) Construction (b) Symbolic Representation

As the conductors are permanently shorted, the total rotor resistance of the rotor is minimum and no external resistance can be added in the rotor resistance. In general, the slots are skewed, as shown in Figure 3.63, rather than in parallel to the shaft.

The reasons for skewing the slots in the squirrel-cage rotor are:

- Smooth rotor operation is possible.
- Magnetic locking between stator and rotor gets reduced.
- Effective transformation ratio between stator and rotor increases.

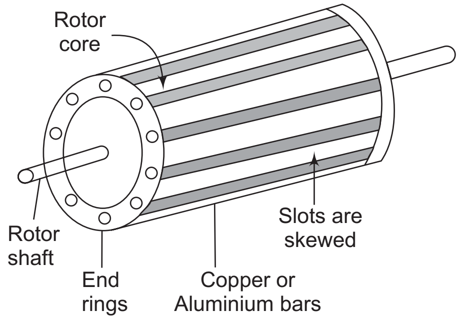


Figure 3.63 Skewed Squirrel-cage Rotor

Slip-ring or Wound-ring Rotor

The schematic diagram of a slip-ring rotor is shown in Figure 3.64. Similar to the stator of a three-phase induction motor, the rotor conductors placed in the rotor slots are electrically connected to form a balanced three-phase winding. Same number of stator poles is used in the rotor to carry the rotor windings. These star or delta-connected rotor windings are permanently connected to a slip ring and brush assembly, which are mounted on the shaft. During running condition of the motor, these slip rings are used to create a short-circuit condition by connecting a metal collar. In this rotor, the external resistances can be added in series with rotor resistance per phase through slip and brush assembly and hence, the total rotor resistance per phase can be controlled.

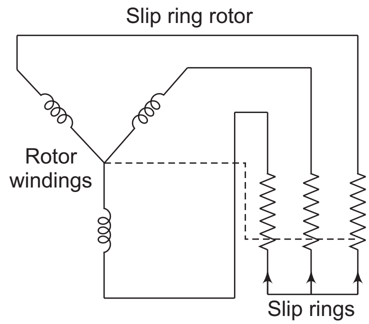


Figure 3.64 Slip-ring Rotor

Slip Ring and Brush Assembly

The slip ring and brush assembly are used to connect the rotor, which is rotating continuously, to the stationary external circuit. The three slip-rings are mounted on the shaft where the three-phase rotor winding is rotating and each slip ring is connected to an individual winding. The stationary brushes make contact with these slip rings, as shown in Figure 3.64. Hence, the rotating windings are available at the brushes, to which the external circuit can be connected. Also, the voltage can be supplied to the rotor windings by connecting the supply to the brushes.

3.23.2 Comparison between Slip-ring and Squirrel-cage Rotor

[AU Nov/Dec, 2009]

The comparison between slip-ring and squirrel-cage rotors is listed in Table 3.5.

Table 3.5 *Comparison Between Slip-ring and Squirrel-cage Rotors*

Slip-ring Rotor	Squirrel-cage Rotor
The rotor of the motor is constructed as a slip-ring type.	The rotor of the motor is a squirrel-cage type.
Has a three-phase winding, similar to a stator.	Has bar-shaped rotor conductor and is shorted using end rings.
Also called phase-wound rotor.	Also called cage motor.
Complicated construction and costly.	Simple construction and cheap.

(*Contd.*)

Possible to add external resistance.	It is not possible to add external resistance.
Requires slip ring and brush assembly to connect the rotor to the external circuit.	Slip ring and brush assembly is not required.
The rotor resistance starter can be used.	Rotor resistance starter cannot be used.
High starting-torque and low starting-current.	Low starting-torque and high starting-torque.
Requires frequent maintenance due to the existence of brushes.	Requires less maintenance.
Rotor copper loss is high.	Rotor copper loss is less.
Efficiency of the motor is low.	Efficiency of the motor is high.
Used in applications like lifts, hoists etc., where high starting-torque is required.	Used in lathe machines, fans, blowers, profiting machines, etc.

3.23.3 Working

When a three-phase balanced AC voltage is applied to a balanced three-phase winding, a rotating magnetic field with constant magnitude and speed is produced. The three-phase induction motor works on the this principle. The speed in rpm at which the rotating magnetic field rotates is called synchronous speed, denoted as N_{syn}. If the supply frequency, f and number of poles, P are known, then the synchronous speed is given by

$$N_{syn} = \frac{120f}{P}\text{rpm} \tag{3.19}$$

The synchronous speed in rps, n_{syn} is given by

$$n_{syn} = \frac{N_{syn}}{60} = \frac{2f}{P}$$

The rotating magnetic field (RMF) produced by the three phase windings can be rotated either in clockwise or anticlockwise direction. When any two phase windings of induction motor are interchanged, the direction of rotating magnetic field gets reversed. This concept of direction of rotating magnetic field is shown in Figure 3.65 (a) and (b).

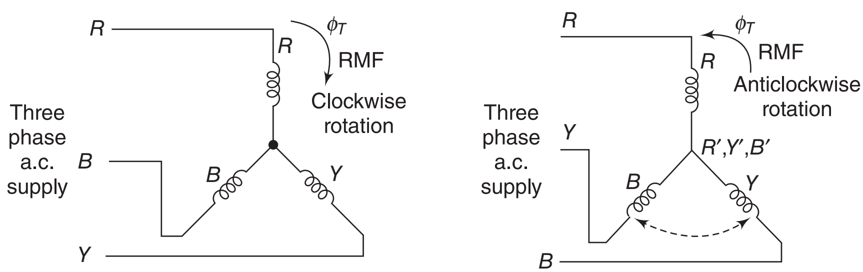


Figure 3.65 Direction of rotating magnetic field (a) clockwise direction (b) anticlockwise direction

When a three-phase balanced AC voltage is applied across the balanced three-phase stator winding, a rotating magnetic field which rotates at synchronous speed, N_{syn} is generated and it passes through the stator, air gap and rotor as shown in Figure 3.66(a). When the time varying rotating magnetic field links with the stationary rotor conductors, an emf is induced in the rotor. When the rotor is connected to an external circuit, a current called rotor current flows through the rotor conductors as shown in Figure 3.66(b). It is assumed that the rotating magnetic field is rotating in clockwise direction and the current flows inside the rotor conductor that is indicated as '×' in Figure 3.66(b). It is obvious that the current carrying conductor generates its own magnetic flux. Hence, a flux called rotor flux is produced whose direction is determined using Fleming's rule as shown in Figure 3.66(c). Since there are two fluxes, an interaction between these fluxes is possible as shown in Figure 3.66(d) where at the right of rotor conductor, the two fluxes cancel each other and at the left of rotor conductor, the two fluxes are added up. Therefore, low and high flux densities are seen at the right and left of rotor conductor. The high flux density area exerts a push on rotor conductor, and thus it starts revolving from left to right as shown in Figure 3.66(d). The direction of rotation of rotor conductor is same as the direction of rotating magnetic field. In other words, according to Lenz's law, the nature of the rotor-induced current is to oppose the cause producing it, i.e., the rotating magnetic field. Therefore, the rotor rotates in the same direction as that of the rotating magnetic field with speed N that is less than N_{syn} i.e., $N < N_{syn}$ and the rotor conductor tries to match up the speed of the rotating magnetic field but never reaches N_{syn}.

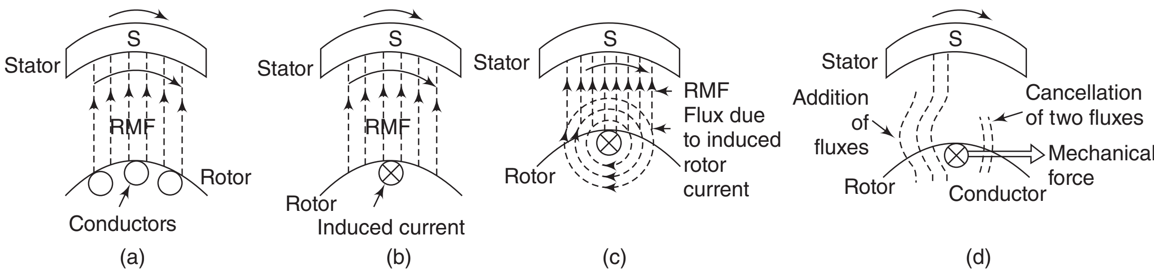


Figure 3.66 Working of three phase induction motor

3.23.4 Reason for $N < N_{syn}$

The following sequence of events will happen if the rotor conductor rotates with speed N_{syn} i.e., when $N = N_{syn}$:

- The relative motion between the rotor and the rotating magnetic field becomes zero i.e., $N_{syn} - N = 0$
- Since the relative speed is zero, no emf will be induced in the rotor and there will be no rotor current or rotor flux.
- Therefore, no torque will be produced on the rotor and eventually the induction motor stops.

Therefore, when the rotor is rotating at N_{syn}, the motor eventually stops. Hence, the rotor rotates at speed less than N_{syn} called sub-synchronous speed.

3.23.5 Slip of an Induction Motor

The difference between the synchronous or rotating magnetic-field speed, N_{syn} and sub-synchronous or rotor speed, N is called slip speed. Therefore,

$$\text{slip speed} = (N_{syn} - N) \text{ rpm}$$

The magnitude of induced emf, rotor current and torque developed in the rotor are decided based on this slip speed.

The slip is defined as the difference between the synchronous speed, N_{syn} and the rotor or motor speed, N expressed as a fraction of N_{syn}. It is also called absolute or fractional slip, denoted by s. Therefore,

$$s = \frac{N_{syn} - N}{N_{syn}} \tag{3.20}$$

Also, the percentage slip is given by

$$\%s = \frac{N_{syn} - N}{N_{syn}} \times 100 \tag{3.21}$$

Using Eqn. (3.20), the motor speed is given by

$$N = N_{syn}(1 - s) \tag{3.22}$$

3.23.6 Three-Phase Induction Motor vs Transformer [AU April/May, 2003]

The comparison of a three-phase induction motor with a transformer is listed in Table 3.6.

Table 3.6 *Comparison Between a Three-phase Induction Motor and a Transformer*

Three-phase Induction Motor	Transformer
Stator and rotor are the two parts of an induction motor.	Primary and secondary windings are the two parts of a transformer.
AC supply is given to the stator	AC supply is given to the primary winding
Also called rotating transformer	Also called stationary transformer
A distinct air gap exists between the stator and the rotor.	No air gap exists between the stator and the rotor
Frequencies of induced emf and current in the stator and the rotor vary with respect to slip	Frequencies of induced emf and current in the primary and secondary windings are same
Part of the energy in the rotor circuit is electrical and some part is converted into mechanical form.	The total energy in the secondary winding is in electrical form.
If E_1 and E_2 are the stator and rotor emfs per phase, then the transformation ratio is $K = \frac{E_2}{E_1}$	If E_1 and E_2 are the primary and secondary induced emfs per phase, then the transformation ratio is $K = \frac{E_2}{E_1}$
$K = \frac{\text{Rotor turns per phase}}{\text{Stator turns per phase}}$	$K = \frac{\text{Number of turns in secondary winding}}{\text{Number of turns in primary winding}}$

Example 3.8

A three-phase six-pole 50 Hz squirrel-cage induction motor is running with a slip of 4%. Determine the speed of the rotating field relative to the stator winding, motor speed and frequency of emf induced in the rotor. **[AU Nov/Dec, 2006]**

Solution

Given $P = 6$, $f = 50$ Hz and $s = 4\% = 0.04$

(i) The speed of rotating field with respect to stator winding is given by

$$N_{syn} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

(ii) The speed of the motor is given by

$$N = N_{syn}(1 - s) = 1000(1 - 0.04) = 960 \text{ rpm}$$

(iii) The frequency of emf induced in the rotor is given by

$$f_r = sf = 0.04 \times 50 = 2 \text{ Hz}$$

Example 3.9

A 373 kW three-phase 440 V, 50 Hz induction motor has a speed of 950 rpm on full load. The motor has 6 poles. Determine slip of induction motor and the number of complete alternations made by the rotor voltage per minute. **[AU April/May, 2007]**

Solution

Given $P = 6$, $f = 50$ Hz and $N = 950$ rpm

The synchronous speed of the induction motor or the speed of stator is given by

$$N_{syn} = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm.}$$

The slip of the induction motor is given by

$$s = \frac{N_{syn} - N}{N_{syn}} = \frac{1000 - 950}{1000} = 0.05 = 5\%$$

The stator frequency is given by

$$f_s = sf = 0.05 \times 50 = 2.5 \text{ Hz} = 2.5 \text{ cycles/sec.}$$

Here, the induced voltage in the rotor has the same frequency as the supply or stator frequency.
Therefore, the number of complete alternations made by the rotor voltage per minute is = 2.5 × 60 = 150

3.24 Torque Equation **[AU Nov/Dec, 2012]**

Similar to DC motor, the torque developed in the rotor of the induction motor depends on: (i) Flux per stator pole, ϕ, (ii) Rotor current, I_{2r} and (iii) Power factor of the rotor, $\cos \phi_{2r}$.

Therefore, $T \,\alpha\, \phi I_{2r} \cos \phi_{2r}$ (3.23)

It is known that the flux produced in the stator, ϕ is directly proportional to the stator voltage, E_1 i.e., $\phi \,\alpha\, E_1$.

Hence, $T \,\alpha\, E_1 I_{2r} \cos\phi_{2r}$

We know that the transformation ratio is $K = \frac{E_1}{E_2}$ i.e,. $E_1 = KE_2$.

Substituting $E_1 = KE_2$ in the above equation, we get

$$T \,\alpha\, KE_2 I_{2r} \cos \phi_{2r} \tag{3.24}$$

Substituting Eqn. (4.15) and Eqn. (4.17) in the above equation, we get

$$T\ \alpha\ KE_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

Therefore, $$T = kE_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

where k is the proportionality constant given by $k = \dfrac{3}{2\pi n_{\text{syn}}}$ where n_{syn} is the synchronous speed in rps.

Hence, the torque developed in the rotor during running condition becomes

$$T = \frac{ksE_2^2 R_2}{R_2^2 + (sX_2)^2} = \frac{3}{2\pi n_{\text{syn}}} \times \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \tag{3.25}$$

Therefore, if the slip at any particular load and standstill rotor parameters are known, the torque developed in the rotor can be obtained.

3.25 Characteristics of Three-Phase Induction Motor

3.25.1 Torque-Slip Characteristics **[AU Nov/Dec, 2011; April/May, 2007]**

The characteristics of the induction motor obtained by plotting the torque against slip when the motor is operated from a standstill position to a synchronous running position are called torque-slip characteristics i.e., the slip of the induction motor varies from $s = 1$ to $s = 0$. In general, since E_2 is constant, the torque of the induction motor is given by

$$T\ \alpha\ \frac{sR_2}{R_2^2 + (sX_2)^2} \tag{3.26}$$

Based on the slip value, the torque-slip characteristics are divided into two regions as: (i) low-slip region and (ii) high-slip region.

Low-slip Region

In low-slip region, the slip s is very small and hence $R_2 >> sX_2$. Therefore, neglecting sX_2 term in Eqn. (3.32), we get

$$T\ \alpha\ \frac{s}{R_2}$$

i.e., $$T\ \alpha\ s \text{ (since } R_2 \text{ is constant)} \tag{3.27}$$

Therefore, in low-slip region, the torque is directly proportional to slip s and it varies linearly. Hence, the torque-slip characteristic is a straight line in this region. In this region, as the load increases, the speed of the motor decreases. This decrease in speed increases the slip and as a result the torque increases. This region is also called stable region of operation.

High-slip Region

In high-slip region, the slip, s, is very high and hence $R_2 << sX_2$. Therefore, neglecting R_2 term in the denominator of Eqn. (3.32), we get

$$T \, \alpha \, \frac{sR_2}{(sX_2)^2}$$

i.e., $$T \, \alpha \, \frac{1}{s} \text{ (since } R_2 \text{ and } X_2 \text{ are constants)} \tag{3.28}$$

Therefore, in high-slip region, the torque is inversely proportional to slip, s, and it varies linearly. Hence, the torque-slip characteristic is rectangular hyperbolic in this region.

Here, as the load increases, speed of the motor decreases. This decrease in speed increases the slip and hence, the torque decreases. Due to extra loading effect, there is further decrease in speed and increase in slip. This increase in slip causes the torque to decrease further and eventually motor comes to a halt position i.e., standstill condition. Therefore, the induction motor cannot be operated at any point in this region and hence it is called unstable region of operation.

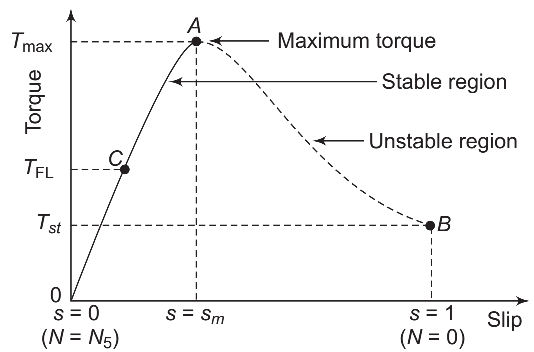


Figure 3.67 Torque–Slip Characteristic Curve

The complete torque–slip characteristic curve of an induction motor is shown in Figure 3.67.

In low-slip or stable region, as the load increases, the slip increases and the torque increases linearly till maximum torque, T_{max} is achieved. The slip at which T_{max} is achieved is s_m. If the load is increased beyond T_{max}, the motor shifts to the high-slip or unstable region of operation and hence, the induction motor comes to a standstill condition at such a load. Therefore, the maximum torque developed in the motor is also called breakdown torque or pull out torque.

Full-load condition is the load at which the current drawn by the motor is within its safe limits. The torque developed at this condition is called full-load torque as denoted by T_{FL}. It is clear from Figure 3.67 that the load can be increased beyond full-load condition till maximum torque is achieved. But if the motor is operated continuously in the region C to A, there is a possibility of breakdown of winding insulation due to large current.

Therefore, the region OC corresponds to full-load condition in which the induction motor is operated safely for longer duration. The region CA is used to achieve maximum torque in which the induction motor is operated only for a short duration of time.

3.25.2 Speed-Torque Characteristics **[AU Nov/Dec, 2008]**

The speed-torque characteristic curve of an induction motor is shown in Figure 3.68. When the motor is running at synchronous speed i.e., $N = N_{syn}$, the relative speed and the slip becomes zero and hence the torque developed in the motor is zero. Also, when the motor is in a standstill position i.e., $N = 0$, the relative speed is maximum i.e., $s = 1$ and therefore, a torque called starting torque, T_{st} is developed in the motor.

The speed at which the motor is running at no-load condition is N_{NL} and the torque developed at this condition is zero. When the induction motor is loaded, the motor speed gets reduced and hence a torque is developed in the motor. When the motor is fully loaded, the torque developed in the motor is T_{FL} and the speed at which the motor runs is N_{FL}. It is seen that the drop in motor speed from T_{NL} to N_{FL} is very minimum, in the range of 4% to 6%, and hence, the three-phase induction motor is called a constant-speed motor.

The induction motor can be loaded till the maximum torque, T_{max} is developed in the motor. The motor speed at which T_{max} is developed in the motor is N_m. If the motor is further loaded, the motor enters the unstable region and comes to a standstill position, where the speed is zero and torque developed is T_{st}. This unstable region is shown as a dotted line in Figure 3.68.

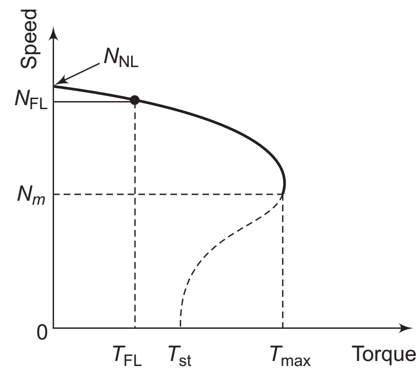


Figure 3.68 Speed-Torque Characteristic Curve

3.25.3 Effect of Change in Rotor Resistance on Torque

In a slip-ring induction motor, the extra resistance can be added externally to control the value of rotor resistance and thereby the torque developed in the rotor is controlled. The different torque equations of induction motor are:

$$T_{st}\,\alpha\,\frac{E_2^2 R_2}{\sqrt{R_2^2+X_2^2}},\ T\,\alpha\,\frac{sE_2^2 R_2}{\sqrt{R_2^2+(sX_2)^2}}\ \text{and}\ T_{max}\,\alpha\,\frac{E_2^2}{2X_2} \tag{3.29}$$

Also, the slips corresponding to the torques given in the above equations are:

$$s=1,\ s=\frac{N_{syn}-N}{N_{syn}}\ \text{and}\ s_m=\frac{R_2}{X_2} \tag{3.30}$$

Hence, the change in rotor resistance does not affect the slip and T_{max}. But the other parameters, including s_m change. The pictorial representation of the effect of change in rotor resistance on torque–slip characteristics is shown in Figure 3.69.

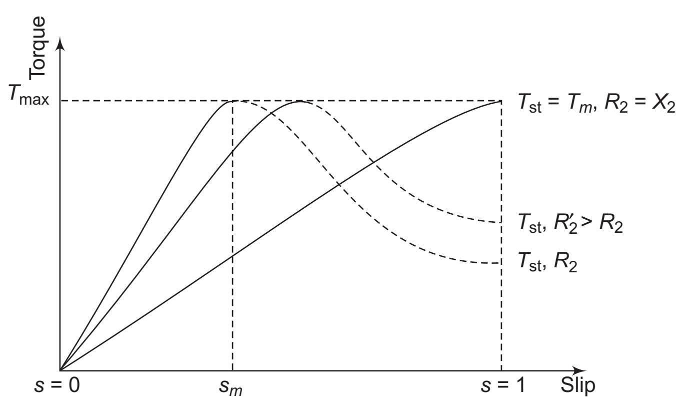


Figure 3.69 Effect of Rotor Resistance on Torque–Slip Characteristics

Here, the slip at a standstill condition is 1. Therefore, if the starting and maximum torques are same, then the slip s_m is equal to 1. Hence, the required condition for the starting and maximum torques to be same is $R_2 = X_2$.

3.25.4 Effect of Change in Rotor Reactance on Torque

Similar to the rotor resistance R_2, the rotor reactance X_2 can be changed, which will affect the torque and slip of the induction motor. From Eqns. (3.35) and (3.36), it is clear that if the rotor reactance X_2 changes, there will be a change in the torque and slip. The torque-slip characteristics of a three-phase induction motor for different values of X_2 are shown in Figure 3.70.

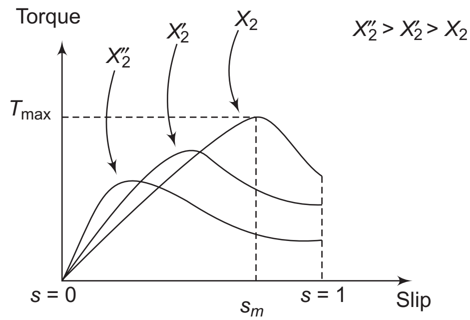


Figure 3.70 Effect of Rotor Reactance on Torque–Slip Characteristics

3.26 Speed Control of Induction Motor

It is known that a three-phase induction motor is a constant-speed motor, whose speed cannot be controlled using rheostats. When the speed varies, it affects the motor performance in terms of its power factor, efficiency etc.

The motor speed and torque equation of an induction motor are given by

$$N = N_{syn}(1 - s) \quad \text{and} \quad T \;\alpha\; \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \tag{3.31}$$

From the above equation, it is seen that by varying either the synchronous speed or slip of induction motor, the motor speed can be controlled. Also, if the slip of induction motor varies, the torque developed in the motor varies. Therefore, to maintain constant torque at constant load condition, $R_2$and X_2can be varied.

3.26.1 Speed Control Methods

The speed of induction motor can be controlled using different methods. These methods are classified into two categories, as follows:

1. ***From stator side:***
   - Supply frequency control or *V/f* control
   - Supply voltage control
   - Controlling number of stator poles
   - Adding rheostats in stator circuit
2. ***From rotor side:***
   - Adding external resistance in rotor circuit
   - Cascade control
   - Injecting slip frequency voltage into the rotor circuit

3.26.2 Supply Frequency Control or *V/f* Control

The synchronous speed of the induction motor is given by

$$N_{\text{syn}} = \frac{120f}{P}$$

From the above equation, it is clear that by controlling the supply frequency *f*, the speed of the induction motor can be controlled. Similar to a transformer, the air-gap flux in the induction motor is given by

$$\phi_{ap} = \frac{1}{4.44K_1T_{\text{ph1}}}\left(\frac{V}{f}\right)$$

where K_1 is the stator winding constant and T_{ph1} is the stator turns per phase.

Hence, if the supply frequency is changed to control the speed of the induction motor, the air-gap flux changes and it results in saturation of stator and rotor cores. Due to this saturation, the no-load current increases, which damages the stator windings. Therefore, to keep the air-gap flux constant, the supply voltage is varied. Hence, by varying the supply voltage, the ratio *V*/*f* is maintained constant, thereby ensuring a constant air-gap flux in controlling the speed of motor. The electronic method for *V*/*f* control is shown in Figure 3.71.

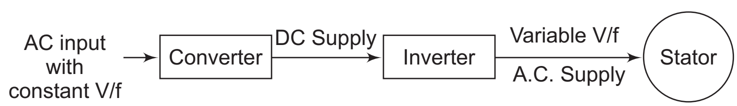


Figure 3.71 Electronic Method for *V/f* Control

It is noted that the supply voltage and its frequency can be varied in this method. Converter and inverter are the devices used in this scheme to maintain constant air-gap flux, by maintaining *V*/*f* as constant. The torque–slip characteristics of this method, by varying *f* and maintaining *V*/*f* as constant, is shown in Figure 3.72.

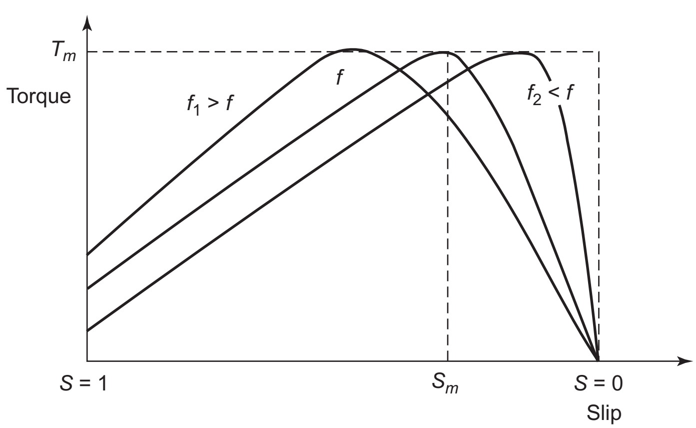


Figure 3.72 Torque–slip Characteristics Using *V/f* Method

3.26.3 Supply Voltage Control

It is known that by varying the supply voltage, the emf induced in the rotor E_2 can be varied, since $E_2 \alpha E_1 \alpha V$. Therefore, the torque equation of Eqn. (3.37) becomes,

$$T \alpha \frac{sV^2 R_2}{R_2^2 + (sX_2)^2}$$

Since the induction motor is normally operated in low-slip region, the term sX_2 is neglected, as $sX_2 << R_2$. Hence,

$$T \alpha \frac{sV^2}{R_2}$$

For a constant R_2, if the supply voltage is reduced, the torque developed in the motor decreases. But, to supply the same load, increasing the slip in induction motor increases the torque developed in the motor. Increase in slip indicates the decrease in speed of the motor. Therefore, the motor is able to develop the required torque at a lower speed and the speed–torque characteristics of the motor using this speed control are shown in Figure 3.73.

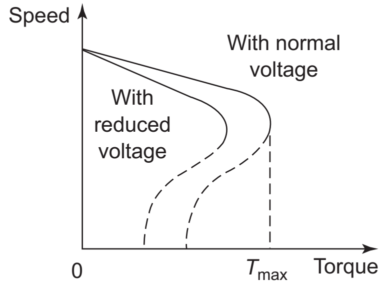


Figure 3.73 Speed–Torque Characteristics for Different V

3.26.4 Controlling Number of Poles

In this method, varying the number of stator poles controls the speed of the induction motor. Hence, this method is also called as pole changing method. Using this method, different speeds can be obtained in the motor. The number of stator poles can be changed using: (i) Consequent poles method (ii) Multiple stator-winding method and (iii) Pole amplitude modulation method. The speed–torque characteristics of a three-phase induction motor by varying the number of stator poles are shown in Figure 3.74.

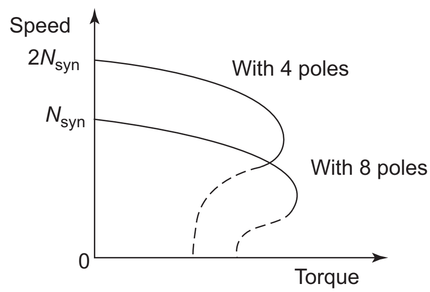


Figure 3.74 Speed–Torque Characteristics for Different Stator Poles

3.26.5 Adding Rheostats in Stator Circuit

The schematic diagram of a three-phase induction motor with rheostat in the stator circuit to control the speed is shown in Figure 3.75. Using this method, a reduced voltage is applied to the stator, as there will be a drop in the external rheostats connected to the circuit. Since there is a direct relation between the applied voltage and the speed, the speed of the motor reduces.

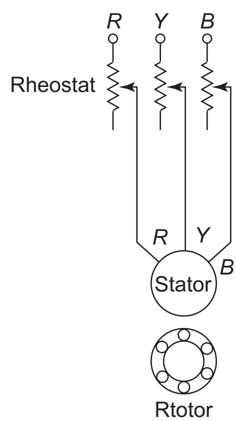


Figure 3.75 Addition of External Rheostats in the Stator Circuit

3.26.6 Adding External Resistance in Rotor Circuit

It is applicable only to slip-ring induction motors. In low-slip region, the torque developed in the motor is given by

$$T \alpha \frac{s}{R_2} \quad \text{(since } sX_2 << R_2\text{, the term } sX_2 \text{ is neglected)}$$

For a constant supply voltage, if adding external rheostats increases the rotor resistance, the torque developed in the motor decreases. But to supply the same load, increasing the slip of induction motor increases the torque developed in the motor, which decreases the speed of the motor. Therefore, the motor is able to develop the required torque at a lower speed and the speed–torque characteristics of the motor using this speed control are shown in Figure 3.76.

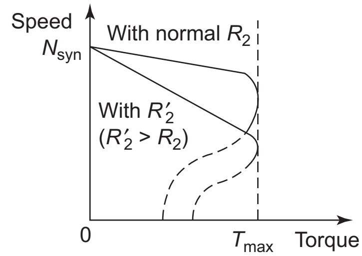


Figure 3.76 Speed–Torque Characteristics for Different R_2

This type of speed control is rarely used, due to the following reasons:

- Speed greater than normal speed is not possible.
- Rotor copper loss increases if the external resistance is added to the circuit and hence efficiency of the motor is less.
- It is applicable only to slip-ring induction motors.
- Extra component is to be added to the motor for cooling purpose and hence this method becomes expensive.

3.26.7 Cascade Control

The schematic diagram of cascade control of a three-phase induction motor is shown in Figure 3.77.

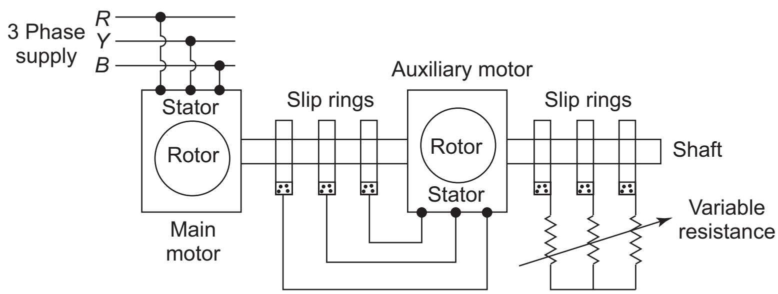


Figure 3.77 Cascade Speed Control of an Induction Motor

In this method, two induction motors—main and auxiliary motors—are mounted on the same shaft. A slip-ring rotor is used in the main motor while the auxiliary motor can be either squirrel-cage or slip-ring rotor. Main supply is given to the stator winding of the main motor. While stator winding of auxiliary motor receives a voltage obtained at a slip frequency from the slip rings of the main motor. This process of connecting two motors is called cascading of motors. The cascading of motors can be either cumulative cascading or differential cascading. If torque developed in the motors act in the same direction it is called cumulative cascading. If torque developed in the motors act in the opposite direction it is called differential cascading. Using this method, the different speeds of auxiliary induction motor are:

$$N = \frac{120f}{P_A + P_B} \text{ for cumulative cascading}$$

and

$$N = \frac{120f}{P_A - P_B} \text{ for differential cascading}$$

3.26.8 Injecting Slip Frequency EMF to the Rotor Circuit

This method is used to control the motor speed by injecting a voltage in the rotor circuit. Since the rotor frequency is slip times the supply frequency, the injected voltage to the circuit must be at slip frequency. This injected voltage can either oppose or aid the induced emf in the rotor circuit. If the injected voltage opposes the induced emf in the rotor, the effective rotor resistance increases and if the injected voltage aids the induced emf in the rotor, the effective rotor resistance decreases. Hence, the rotor resistance can either be increased or decreased, thereby controlling the speed of the motor.

3.27 Single-Phase Induction Motor

A single-phase AC supply is commonly used in residential and commercial loads. The motor that works using a single-phase AC supply is called a single-phase induction motor and is more commonly used when compared to DC motor. Since this motor uses a single-phase AC supply, it has very small power rating. Most commonly used applications are: small toys, small fans, hair dryers etc.

3.27.1 Construction

Similar to a three-phase induction motor, the single-phase induction motor has two main components: a stator and a rotor. The stator and rotor construction of the single-phase induction motor is similar to the three-phase induction motor, as discussed in Section 3.23.1. Also, only squirrel-cage rotor is used in a single-phase induction motor and its construction is explained in Section 3.23.1. The schematic representation of a single-phase induction motor is shown in Figure 3.78.

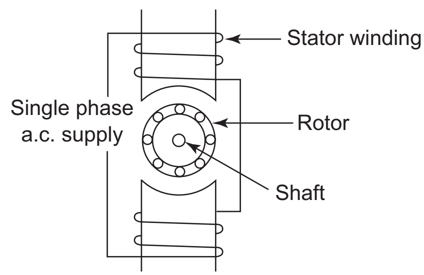


Figure 3.78 Single-phase Induction Motor

3.27.2 Working

A single-phase induction motor must have two fluxes for its operation. A rotating magnetic flux called main flux is produced in the stator winding when a single-phase AC supply is supplied to the stator windings. When the main flux links with the stationary rotor conductors, using the transformer action, an emf is induced in the rotor. Since the squirrel-cage rotor is closed at the end, the induced emf drives the current through the rotor. As this rotor current flows through its conductor, a flux called rotor flux is produced. When these two fluxes i.e., main and rotor fluxes interact, a torque is developed in the rotor and hence the rotor rotates.

The major differences between a DC motor and a single-phase induction motor are: (i) two supplies are required in a DC motor when compared to a single supply in AC motor; (ii) DC motors are self-starting while single-phase induction motors are not self starting. Double-revolving field theory is used to explain the reason why a single-phase induction motor is not self-starting.

3.27.3 Double Revolving Field Theory **[AU April/May, 2014; Nov/Dec, 2011]**

It states that any alternating quantity is resolved into two rotating components such that the magnitude of these components is exactly half the magnitude of the original alternating quantity and these two components rotate in opposite direction. Consider that the maximum magnitude of alternating flux ϕ_1, produced by exciting the stator windings in a single-phase induction motor is ϕ_{1max}.

According to double revolving field theory, this flux ϕ_1 is resolved into two components each with magnitude $\phi_{1max}/2$. They are called forward and backward components, represented as ϕ_f and ϕ_b respectively. The speed of these two components is N_{syn} and it depends on the frequency and the number of stator poles. The components ϕ_f and ϕ_b are rotated in anti-clockwise and clockwise directions respectively, so that the resultant of these components gives the instantaneous value of the stator flux at any instant.

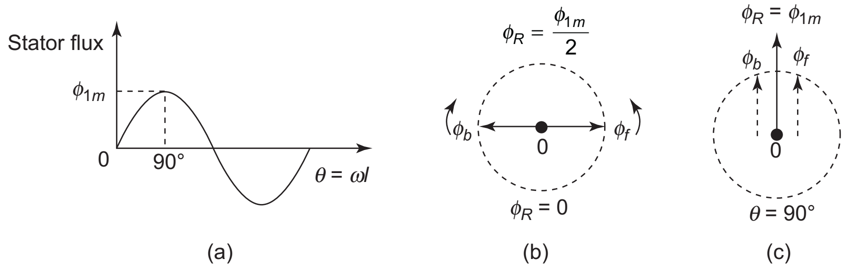


Figure 3.79 Stator Flux (a) Original Quantity (b) At Start (c) At $\theta = 90°$

The original stator flux ϕ_1, produced due to the excitation of stator windings, is shown in Figure 3.79(a). Consider two instants to prove that the single-phase induction motor is not a self-starting motor. First, at start, the two components ϕ_f and ϕ_b, are shown opposite to each other, as shown in Figure 3.79(b), so that the resultant flux is zero and second at $\theta = 90°$, where the two components ϕ_f and ϕ_b, are shown in the same direction, as shown in Figure 3.79(c), so that the resultant flux is maximum.

Since these two components rotate at speed N_{syn}, the rotor conductors disturb it and hence an emf is induced in the rotor that further circulates rotor current in the circuit. This rotor current, when passing through the rotor conductor, produces rotor flux. The rotor flux that interacts with ϕ_f produces a torque in the clockwise direction, and that interacts with ϕ_b produces a torque in the anti-clockwise direction. Since at start, these torque act opposite to each other, the resultant torque is zero. As the resultant torque is zero, the rotor does not rotate and it proves that the single-phase induction motor is not self-starting. These two opposite torques and their resultant torque at different motor speed is shown in Figure 3.80.

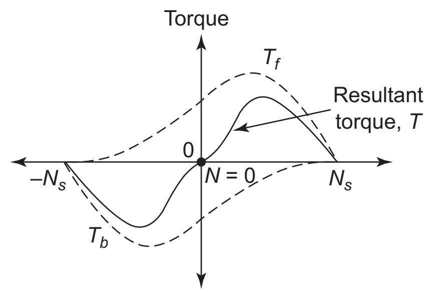


Figure 3.80 Forward, Backward and Resultant Torque at Different Speeds

When a rotor is given an initial rotation, the torque developed in the motor increases and hence the motor starts rotating in the direction in which the rotor is initially rotated. However, rotating the rotor initially is not practically possible. Therefore, some modifications are required in the construction of a single-phase induction motor to make them self-starting.

3.28 Types of Single-Phase Induction Motor

An arrangement is to be provided in the single-phase induction motor to make the alternating stator flux as rotating so that the stator flux rotates in a particular direction. Hence, the torque produced in the single-phase induction motor will be unidirectional. Therefore, the single-phase induction motor becomes self-starting under the influence of a rotating stator magnetic field. Based on the method of producing a rotating magnetic field, the single-phase induction motors are classified as:

- Split-phase induction motor

- Capacitor induction motor
  - Capacitor-start induction motor
  - Capacitor-start capacitor-run induction motor
- Shaded pole induction motor

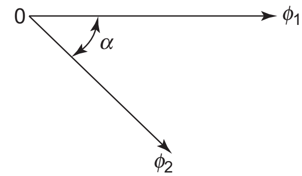


Figure 3.81 Alternating Fluxes with α as Phase Difference

The necessary condition for a rotating magnetic field to be generated in a single-phase induction motor is to have a minimum of two alternating fluxes with phase difference α between them, as shown in Figure 3.81.

Due to interaction of these two fluxes, a resultant flux is produced. Since the fluxes are alternating, the resultant flux will change its direction at every instant. This results in a resultant flux rotating in a particular direction. Therefore, in all the types of single-phase induction motors, suitable arrangement is made to generate an additional flux so that the required rotating magnetic field is produced. Here, if the phase angle α is more, the starting torque of the single-phase induction motor is high.

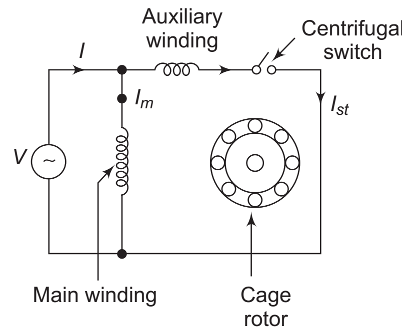


Figure 3.82 Circuit Diagram of a Split-phase Induction Motor

3.28.1 Split-phase Induction Motor

[AU Nov/Dec, 2012]

In this type of single-phase induction motor, the stator carries two windings: main and auxiliary or starting windings. Here, the main winding is purely inductive and the auxiliary winding is highly resistive, since it carries a series resistance. The circuit diagram of a split-phase induction motor is shown in Figure 3.82.

Consider that the current carried by the main and starting windings are I_m and I_{st}. The phasor diagram of these currents, with respect to the supply voltage, is shown in Figure 3.83. Since the main winding is purely inductive, the current I_m lags the voltage by ϕ_m and since the auxiliary windings are highly resistive, the current I_{st} lags the voltage by a very small angle. Hence, the phase difference between I_m and I_{st} is more i.e., α is high.

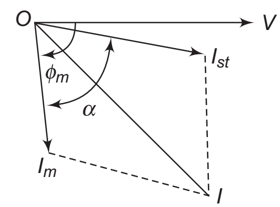


Figure 3.83 Phasor Diagram of a Split-phase Induction Motor

These currents, while flowing through their respective windings, produce two fluxes, which are displaced by phase angle α. Therefore, the resultant of these fluxes produces a rotating magnetic field and the induction motor gets started due to the production of a starting torque. The split-phase induction motor has a centrifugal switch that is used to disconnect the auxiliary windings from the induction motor when the motor attains 75 to 80% of synchronous speed. The torque–speed characteristic curve of a split-phase induction motor is shown in Figure 3.84.

The starting torque of split-phase induction motor is poor, which is 125% to 150% of T_{FL}. Here, changing the terminals of either starting or main windings can vary the direction of motor speed.

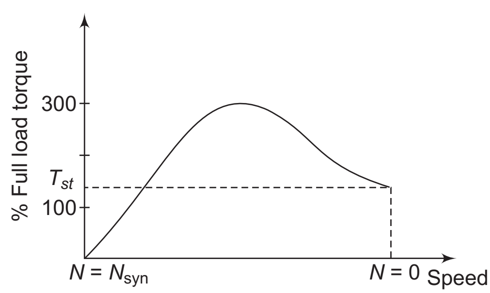


Figure 3.84 Torque–Speed Characteristics of a Split-phase Induction Motor

Applications

Low starting-current and moderate starting-torque are the characteristics of a split-phase induction motor. Therefore, it is used in easily started loads like fans, blowers, grinders, centrifugal pumps, washing machines, oil burners, office equipment etc.

3.28.2 Capacitor Induction Motor **[AU Nov/Dec, 2010; April/May, 2008]**

The only difference between the split-phase and the capacitor induction motor is the centrifugal switch connected in series with the capacitor. Due to the presence of the capacitor in series with the starting winding, a leading current I_{st} is drawn by the starting winding so that the phase difference between I_{st} and I_m is more when compared to a split-phase induction motor. The phasor diagram of the capacitor induction motor is shown in Figure 3.85.

This type of induction motor is classified based on the capacitor that remains in the circuit permanently or disconnected, as:

- Capacitor-start induction motor
- Capacitor-start capacitor-run induction motor

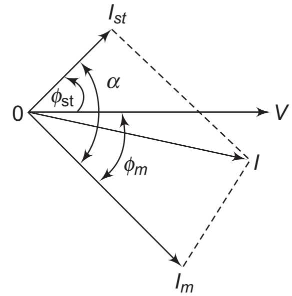


Figure 3.85 Phasor Diagram of a Capacitor Induction Motor

Capacitor-start Induction Motor

The circuit diagram of the capacitor-start induction motor is shown in Figure 3.86.

Since the phase angle, α, between the currents is very large, the starting torque proportional to α is also very high. In addition, when the motor reaches 75 to 80% of the synchronous speed, using a centrifugal switch, the starting winding and the capacitor are disconnected from the main winding. Since the capacitor remains in the motor only at the beginning, it is called a capacitor-start induction motor.

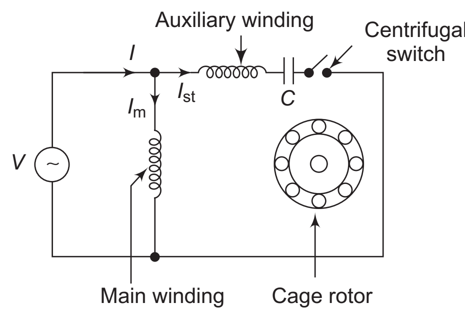


Figure 3.86 Capacitor-start Induction Motor

Capacitor-start Capacitor-run Induction Motor

The construction of this motor is similar to a capacitor-run induction motor, except for the absence of a centrifugal switch in the motor, as it exists throughout the motor operation. The schematic diagram of a capacitor-start capacitor-run induction motor is shown in Figure 3.87. Since the capacitor exists throughout the motor operation, the power factor of the motor is improved.

The capacitor value is chosen in such a way that it should compromise between the best starting and running conditions. Hence, the capacitor value is chosen such that the starting torque is only 50 to 100% of full-load torque T_{FL}, which is less when compared to a capacitor-start induction motor.

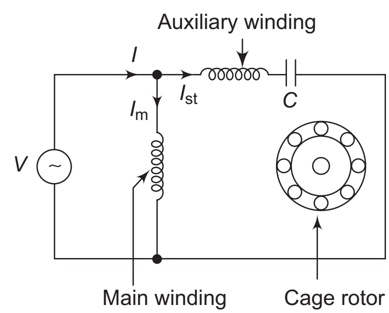


Figure 3.87 Capacitor-start Capacitor-run Induction Motor

Since capacitor is added to the circuit, this motor is costly when compared to a split-phase induction motor. The torque–speed characteristics of a capacitor induction motor are shown in Figure 3.88.

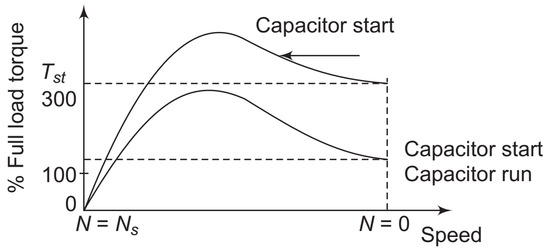


Figure 3.88 Torque–Speed Characteristics of a Capacitor Induction Motor

Applications

Since the capacitor-start induction motor has a high starting torque, it is used in hard starting loads like compressors, conveyors, grinders, fans, blowers, refrigerators, air conditioners etc. Also, the capacitor-start capacitor-run motor is used in ceiling fans, blowers and air-circulation equipment, where the requirement of starting torque is less.

3.28.3 Shaded Pole Induction Motor **[AU Nov/Dec, 2010; April/May, 2009]**

In a shaded pole induction motor, the stator has projected salient poles where the stator windings are wound. Here, each projected pole is provided with a copper band. The schematic diagram of a shaded pole induction motor and the enlarged view of a projected pole are shown in Figures 3.89(a) and (b) respectively.

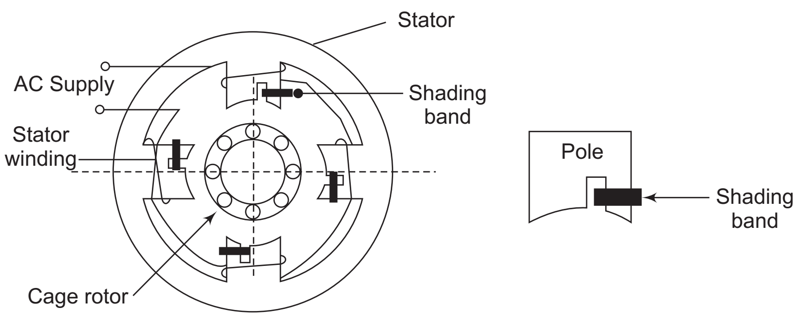


Figure 3.89 Shaded Pole Induction Motor (a) Schematic Diagram (b) Projected Pole

When a single-phase AC supply is provided to the stator winding, a rotating magnetic field is produced due to a copper band provided to the poles. The explanation of production of rotating magnetic field is given below:

When the current flows through the stator winding, a flux called main flux is produced, as shown in Figure 3.90.

Also, when the current flows through the copper band, a flux called shaded-ring flux is produced. Here, the magnetic axis lies at the position where there is more flux. When the magnetic axis shifts, a rotating magnetic field is produced.

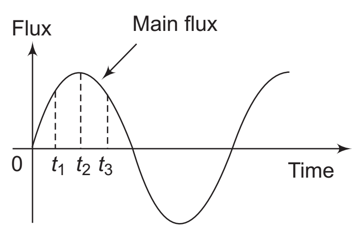


Figure 3.90 Main Flux of Induction Motor

Applications

The single-phase induction motor has the following characteristics: very low starting-torque, low power-factor and low efficiency. Hence, it used in small fans, advertising displays, film projectors, record players, gramophones, hair dryers, photo-copying machines etc.

3.29 Alternator or Three-Phase AC Generator

The AC machine, which generates an alternating emf, is called a three-phase AC generator or an alternator. Since the alternator rotates at a synchronous speed, N_s, it is also called a synchronous generator. In general, an alternator is of three-phase type, as a three-phase power system has more advantages when compared to a single-phase power system. This generator is mainly used to generate large power at the power stations.

3.29.1 Construction **[AU Nov/Dec, 2010; April/May, 2009]**

Similar to an induction motor, the two important parts of an alternator are the stator and the rotor, where the stationary part of the machine that carries the armature winding is called a stator and the rotating part of the machine that produces the field is called a rotor. Here, the output or the induced emf is generated in the stator, whereas the main field required to generate the induced emf is produced at the rotor. The construction of an AC generator or an alternator is shown in Figure 3.91.

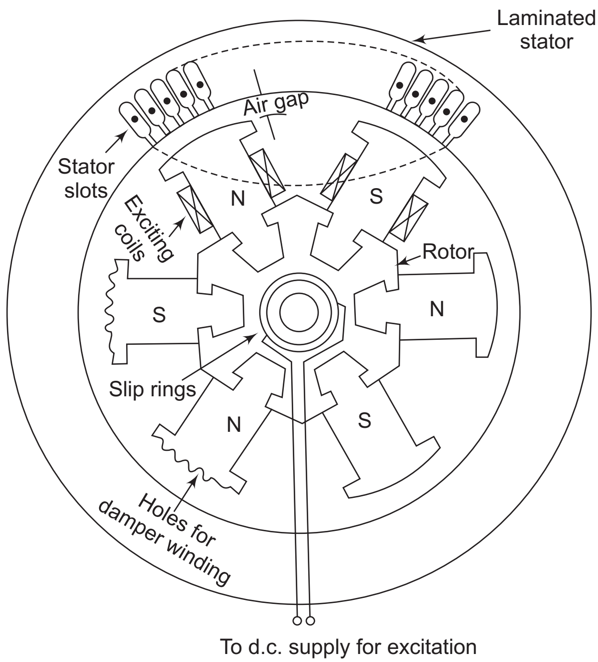


Figure 3.91 Schematic Diagram of an Alternator

Stator

The stator is the stationary part of the alternator and it consists of different parts like stator frame, stator core and stator windings. Stator frame is the outer cover of the alternator, made up of a cast iron or mild steel-frame and helps in protecting the inner parts of the alternator and supports the stator core. In addition, it provides a closed path for the magnetic flux to pass through stator windings. The stator or armature core is made of laminated steel or magnetic iron sheets. The armature core is slotted in the inner periphery to accommodate the stator or armature winding. It also provides a path for the magnetic flux. The armature core is laminated to reduce the constant losses in the alternator. The three-phase balanced star-connected stator or armature winding may be single layered or double layered. Since the windings are balanced, the number of turns and the size of wire used in the windings should be the same. The induced emf in the alternator is brought out of the alternator using the stator windings. Ventilation is provided using the holes in the stator frame. The stator construction of an alternator is shown in Figure 3.92.

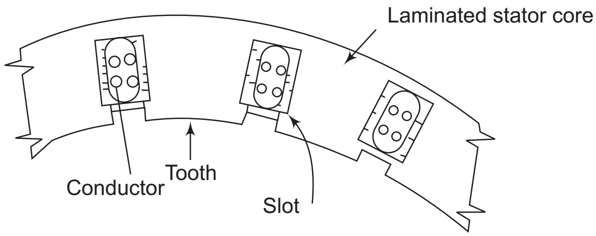


Figure 3.92 Construction of a Stator

Rotor

The rotating part of the alternator is called a rotor and it is like a flywheel with alternate *N* and S-pole electromagnets on it. These electromagnets are magnetised using DC excitation. Since the rotor is rotating, the excitation voltage is given through slip rings and brushes, which are fixed on the frame. The two different types of rotors used in an alternator are:

Salient or Projecting Pole

This type of rotor consists of even number of heavy-iron poles projecting from the rotor core surface. The typical construction of a salient pole rotor is shown in Figure 3.93.

It is primarily used in alternators that rotate at low and medium speeds. This type of rotor construction is used as a prime mover in hydraulic and internal combustion turbines. The axial length of the rotor is less whereas the diameter of the rotor is high. The field or rotor windings are provided on the pole face and excited by the DC supply through slip rings. Steel spider attached to the shaft helps in providing a path for the magnetic flux.

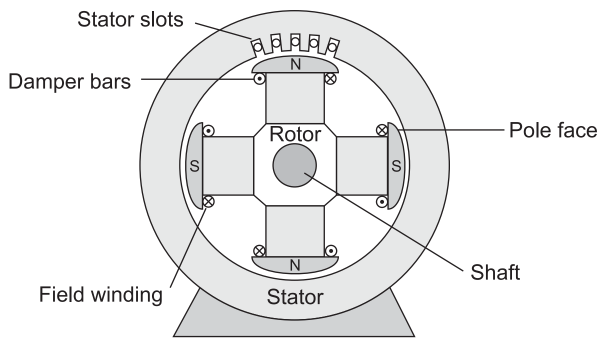


Figure 3.93 Salient or Projecting Pole Rotor

3.29.2 Working

An alternator generates the emf using the principle of electromagnetic induction, which states that, if a stationary conductor is placed in a moving magnetic field, an emf is induced in it, as shown in Figure 3.94.

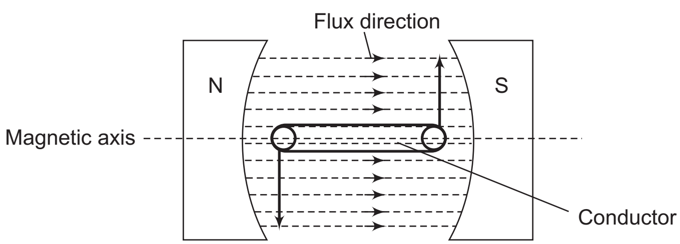


Figure 3.94 Principle of an Alternator

Working of Alternator **[AU April/May, 2013]**

When the field windings are energised using a DC supply through slip rings, alternate N and S poles are generated in the case of a smooth rotor and hence magnetic flux is produced. The prime movers are used to rotate the rotor and field windings. As the rotor rotates, the stationary armature conductors are cut by the magnetic flux. Due to the principle of electromagnetic induction, an emf is induced in the stator conductors. Since the rotor poles are alternative in nature, the induced emfs in the stator conductors are also alternating in nature and their directions are given by Fleming's rule. The frequency of the induced emf depends on the number of N and S poles that the armature conductors pass in one second. Therefore, the frequency of induced emf in the stationary armature conductors is given by

$$f = \frac{PN_s}{120} \tag{3.32}$$

3.29.3 EMF Equation of an Alternator **[AU April/May, 2011; Nov/Dec, 2007]**

In an alternator, when the stationary armature conductor cuts the magnetic flux generated by the rotor which is rotated using the prime mover, an emf is induced in the armature conductor. This induced emf is called generated emf, denoted as E_{ph}. The derivation of E_{ph} is obtained as follows:

Consider that P is the total number of poles of the alternator, ϕ is the flux produced per pole in Wb, Z is the total number of armature conductors, N_{syn} is the rotor speed or synchronous speed in rpm and Z_{ph} is the number of armature conductors per phase connected in series.

In one revolution of armature core, the total flux cut by one conductor of the armature is given by

$$\phi_T = P\phi \tag{3.33}$$

The time taken by the rotor to complete one revolution is given by

$$t = \frac{60}{N_{\text{syn}}} \tag{3.34}$$

According to Faraday's law, the average emf induced in one armature conductor is given by

$$e_{\text{ph}} = \frac{\phi_T}{t} \tag{3.35}$$

Substituting Eqns. (3.39) and (3.40) in the above equation, we get

$$e_{\text{ph}} = \frac{P\phi N_{\text{syn}}}{60} \tag{3.36}$$

Using Eqn. (3.38), we get

$$e_{\text{ph}} = 2f\phi$$

Therefore, the emf induced per turn is given by

$$e_{T\text{ph}} = 2 \times e_{\text{ph}}$$

i.e.,

$$e_{T\text{ph}} = 4f\phi$$

If Z_{ph} is the total number conductors in a phase and T_{ph} is the total number of turns connected in series such that $T_{ph} = \frac{Z_{ph}}{2}$. Then, the net average emf induced per phase is given by

$$(E_{ph})_{avg} = 4f\phi T_{ph}$$

In an AC circuit, the RMS value of the induced emf is given by

$$E_{ph} = 1.11 \times (E_{ph})_{avg}$$

i.e., $$E_{ph} = 4.44\, f\phi T_{ph} \tag{3.37}$$

which is the basic induced emf per phase of an alternator.

Therefore, the generalised emf equation of an alternator is given by

$$E_{ph} = 4.44 K_C K_d f\phi T_{ph}$$

where K_C is the pitch coil factor and K_d is the distribution factor.

Pitch coil factor, K_C: It is given by the ratio of emf induced when the coil is short pitched to the emf induced when the coil is full pitched. It is denoted as K_C which is always less than one. It is represented by

$$K_C = \cos\left(\frac{\alpha}{2}\right)$$

where α is the short pitch angle, i.e., the angle by which the coil is short pitched.

Distribution factor, K_d: The factor by which the emf induced in the armature gets reduced due to coil distribution is called distribution factor. It is denoted as K_d which is always less than one. It is represented by

$$K_d = \frac{\sin\left(\frac{m\beta}{2}\right)}{m\sin\left(\frac{\beta}{2}\right)}$$

where m is the number of slots per pole per phase and β is the slot angle as given by $\beta = \frac{180^\circ}{n}$, where n is the number of slots per pole.

It is noted that for a full pitch coil and concentrated winding, $K_C = 1$ and $K_d = 1$.

Note: If the armature winding is star connected, the line voltage of the alternator is given by

$$E_L = \sqrt{3}E_{ph}$$

Similarly, if the armature winding is delta connected, the line voltage of the alternator is given by

$$E_L = E_{ph}$$

3.30 Synchronous Motor

When a three-phase supply is given to the stator of a three-phase alternator, it works as a motor. Now, if an electromagnet is present in the rotating magnetic field, the electromagnet is magnetically locked with this field and rotates at the same speed. These machines are called synchronous motors. This machine produces a steady-state torque at constant speed and frequency. The rotor speed of this machine is equal to the synchronous speed of the stator magnetic field and it is used in situations where constant speed drive is required. The speed of this motor is constant irrespective of the load. Its speed changes for an instant at the time of loading and regains its original speed shortly.

3.30.1 Types of Synchronous Motors

The synchronous motors are classified into two types, namely:

1. Single-phase synchronous motors
   - Reluctance motor
   - Hysteresis motor
2. Three-phase synchronous motors

In synchronous motors, the speed of the rotor is same as the rotating magnetic field. Basically, it is a fixed-speed motor with only one speed, which is synchronous speed and there is no intermediate speed. In other words, it is in synchronism with the supply frequency. Synchronous speed is given by

$$N_{syn} = \frac{120 \times f}{P}$$

where N_{syn} is the synchronous speed of the motor, P is the number of poles and f is the frequency.

3.30.2 Construction **[AU April/May, 2007]**

Similar to an alternator, the synchronous motor has a three-phase winding on the stator and a DC field winding on the rotor. The basic construction of a synchronous motor consists of two parts. The schematic representation of a three-phase synchronous motor is shown in Figure 3.95.

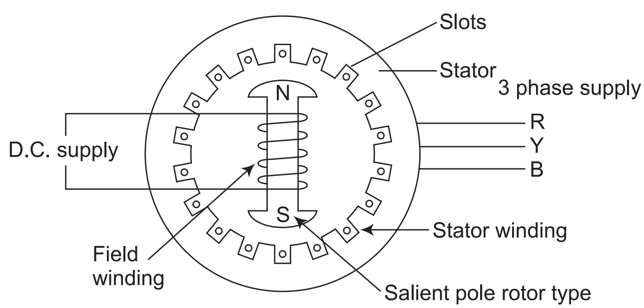


Figure 3.95 Schematic Representation of a Three-phase Synchronous Motor

Stator

It is the stationary part of the synchronous motor and it is built with a stack of laminated steel sheets. The inner periphery of the stator consists of slots to accommodate armature winding. The armature winding is star-connected and the neutral conductor is connected to ground.

Rotor

It carries a field winding and it is excited by a separate DC supply through the slip-ring arrangement. Rotor construction is classified into two types, namely:

- Salient or projected pole type
- Non-salient or cylindrical pole type

Practically, most of the motors use salient or projected pole type of construction.

The detailed description of these rotors is given in Section 3.29.1.

3.30.3 Working

[AU April/May, 2007]

It is a doubly excited machine i.e., two electrical inputs are provided to it. The three-phase stator winding is fed by a three-phase supply and the rotor is provided with a DC supply. The three-phase currents in the stator windings produce a rotating magnetic flux in the air gap at synchronous speed and the rotor carrying the DC supply produces a constant flux. The rotor has a tendency to align with the rotating field produced by the stator at all times, in order to present the path of least reluctance. Thus, if the field is rotating, the rotor will tend to rotate with the field and experiences interlocking between these two magnetic fields.

Consider a two-pole stator machine excited by an AC supply with the poles, N_s and S_s, and their positions marked at p and q respectively. The stator produces a rotating magnetic field at synchronous speed in clockwise direction, as shown in Figure 3.96(a). The rotor produces a constant magnetic field excited by a DC supply, with poles N_r and S_r, as shown in Figure 3.96(b). When two like-poles N_s and N_r, as well as S_s and S_r are brought nearer to each other, they will experience a repulsive force within each other and the rotor tends to rotate in anti-clockwise direction.

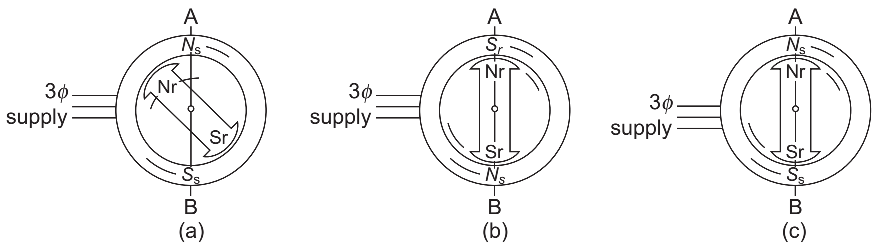


Figure 3.96 Synchronous Motor for Various Rotor Positions (a) Repulsion of Rotor in Anti-clockwise Direction (b) and (c) Attraction of Unlike Poles in Clockwise Direction

During the next half-period, the position of the stator poles gets interchanged i.e., N_s and S_s shift their positions to p and q respectively. Under these conditions, N_s attracts S_r and S_s attracts N_r. Hence, the rotor tends to rotate in clockwise direction, which is reverse of the first direction. Here, it is concluded that due to rapid and continuous interchange of stator poles, the rotor is subjected to a rapidly reversing torque. Owing to the larger rotor inertia, it cannot instantaneously respond to such quick-reversing torque. Therefore, the rotor remains in a standstill condition and the motor is not self-starting in nature.

The stator and rotor poles are attracting each other, as shown in Figure 3.96(c). The rotor is not in standstill but rotating in clockwise direction. The rotor poles shift their positions along with the stator poles and they will experince a continuous unidirectional torque in the clockwise direction, as shown in Figure 3.96(b).

3.30.4 Synchronous Motor: Not Self-Starting

Without energising the rotor, the synchronous motor is started and it is speeded upto synchronous speed. Upon reaching the synchronous speed, the rotor is excited by the DC source and it is magnetically locked in position with the stator. During this condition, the rotor poles get engaged with stator poles and both run at synchronous speed in the same direction. Due to this interlocking, the stator and rotor poles either run at synchronuous speed or not at all.

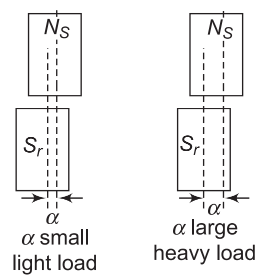


Figure 3.97 Load Angle for Light and Heavy-load Conditions

However, the arrangement of the stator and the rotor poles is not an absolutely rigid one. When the load on motor is increased, the rotor tends to fall back in phase by load or coupling angle α, but still continues to run at synchronous speed. The torque developed by the motor depends on this load angle, as shown in Figure 3.97.

3.30.5 Procedure for Starting a Synchronous Motor **[AU April/May, 2013]**

The procedure to start a synchronous motor is as follows:

1. Field winding of the motor is short-ciruited.
2. Stator winding is applied a reduced voltage using an auto-transformer, when the motor is starting up.
3. Once the motor speed reaches a steady state, a weak DC excitation is applied by removing the short-ciruited field winding.
4. Upon reaching a sufficient excitation level, the machine will be interlocked and pulled into synchronism.
5. Rated voltage is applied across the stator terminals.
6. Based on the desired power factor, the DC excitation of the motor is adjusted.

3.30.6 Methods of Starting Synchronous Motors **[AU Nov/Dec, 2014]**

Synchronous motors are not self-starting in nature and it is necessary to rotate the rotor at a speed nearer to synchronous speed. Various methods to start synchronous motors are given below.

Using Pony Motors

A pony motor is a small induction motor used as an external device to rotate the rotor to attain synchronous speed and then the DC excitation to the rotor is switched ON subsequently. After establishment of synchronism, the pony motor is decoupled from the system and the main motor continues to rotate at its synchronous speed.

Using Damper Winding

Along with the main field winding, an additional compensating winding consisting of copper bars is placed in the slots and these bars are short-circuited at its end rings. These windings are called damper windings. The short-circuited winding acts as a squirrel-cage rotor winding of an induction motor. The schematic representation of a damper winding is shown in Figure 3.98.

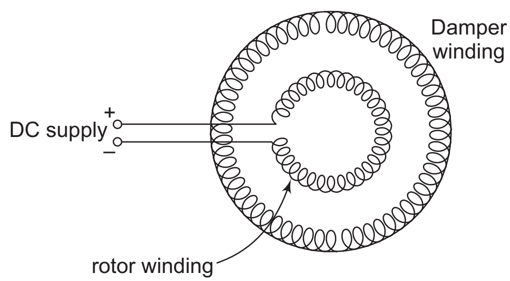


Figure 3.98 Schematic Representation of a Damper Winding

Upon excitation of the stator by a three-phase supply, the motor starts rotating as an induction motor at sub-synchronous speed and the DC supply is excited to the field winding. Once the field winding is excited, the motor is pulled into synchronism and starts rotating at its synchronous speed. During this condition, the relative motion between the damper winding and the rotating magnetic field becomes zero. Now, the motor behaves as a synchronous motor and it does not induce any emf in the damper winding. Therefore, at the time of starting, the damper winding is used to run the motor as an induction motor and afterwards, it goes out of the circuit.

As a Slip-ring Induction Motor

Due to the nature of squirrel-cage induction motor, the above method does not provide high starting torque at the time of starting a synchronous motor. In order to overcome the above limitation, it is designed to form a three-phase star or delta-connected winding and their terminals are brought out through slip rings.

An external rheostat is connected in series with the rotor circuit and gradually cut off once the motor speeds up. This arrangement limits the high inrush starting-current and attains a high starting-torque at the starting time. Once the motor reaches a near synchronous speed, the rotor is excited by the DC supply. The motor gets pulled into synchronism and starts rotating at synchronous speed. Then the slip rings will short-circuit the damper winding.

The schematic diagram of a slip-ring arrangement for synchronous motor starting is shown in Figure 3.99.

Using Small DC Machine Coupled to it

In this method, large synchronous motors are coupled with a DC motor and that is used to rotate the synchronous motors at their synchronous speed. After attaining synchronous speed, the excitation is provided to the rotor and it behaves as a synchronous motor. During this condition, the coupled DC motor acts as a DC generator (exciter). The field of the synchronous motor is then excited by this DC generator itself.

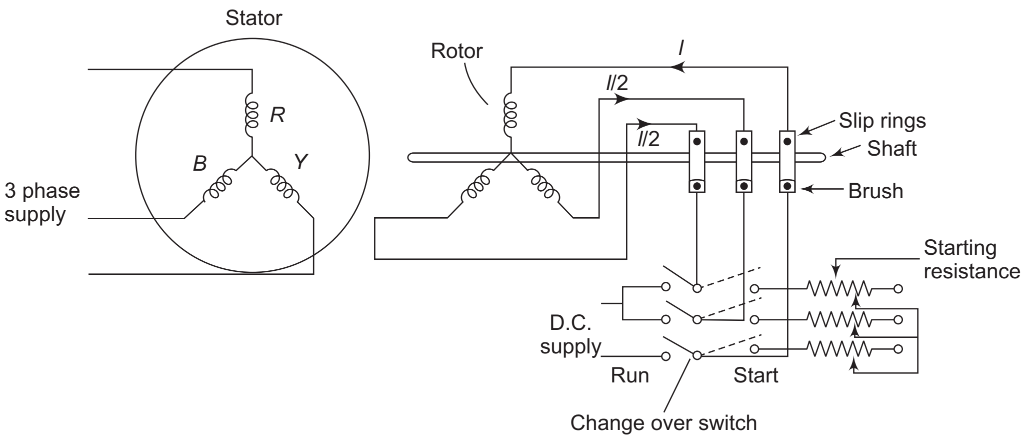


Figure 3.99 Schematic Diagram of a Slip-ring Arrangement for Synchronous Motor Starting

3.30.7 V and Inverted-V Curves

The excitation of a synchronous motor varies with the magnitude of the armature current and it has a large magnitude for low and high-excitation values. The over-excited and under-excited motor runs with leading and lagging power-factors respectively. The excitation at which the magnitude of the induced emf is less than the applied voltage ($E_{bph} < V_{ph}$) is called under-excitation. Similarly, when the excitation changes in such a way that if the magnitude of the induced emf is less than the applied voltage ($E_{bph} < V_{ph}$), then it is called over-excitation.

In between the under and over-excitation values, the magnitude of current reaches minimum value $I_{a\min}$, at unity power-factor. The excitation at this point is called critical excitation, where I_{aph} is in phase with V_{ph}.

The variation of armature current I_a, with different excitation values, is shown in Figure 3.100(a). The excitation can be varied by changing the field current of the motor. A graph is drawn between armature current, I_a, of the motor against the field current, I_f, for various loading conditions. Since the shape of the curve looks like the alphabet ‘V’, these curves are called V curves of the synchronous motor.

Similarly, a collection of inverted V-curves is obtained by plotting power factor, cos ϕ against the field current, I_f for various loading conditions. Since the shape of these curves looks like an inverted ‘V’, these curves are called inverted-V curves, as shown in Figure 3.100(b).

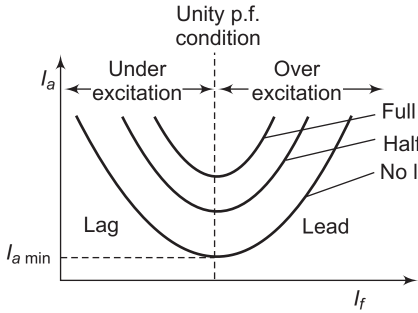


Figure 3.100(a) V-Curves for Various Loads

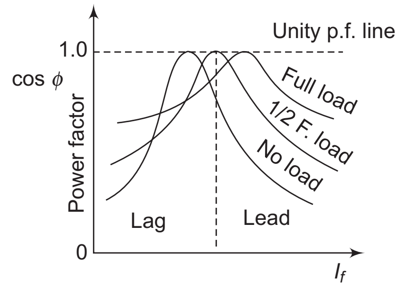


Figure 3.100(b) Inverted V-Curves for Various Loads

3.30.8 Comparison Between Synchronous and Induction Motors

[AU April/May, 2013]

The comparison between synchronous and induction motors is given in Table 3.7.

Table 3.7 *Comparison Between Synchronous and Induction Motors*

S. No.	Synchronous Motor	Induction Motor
1.	Requires damper winding for self-starting.	Inherently self-starting.
2.	Separate DC source is required for motor excitation.	Rotor windings are not fed by separate source and it is excited by the induced emf.
3.	Runs at a constant speed, irrespective of the load.	Speed of the motor decreases with increase in load.
4.	By changing excitation, the motor can be operated over a wide range of power factors.	Motor always runs with lagging power-factor.
5.	Can be used as a synchronous condenser for power-factor correction.	Not possible with this motor.
7.	Very sensitive to sudden load change results in hunting.	No hunting phenomenon exists in this motor.
8.	Complex in construction and expensive.	Simple in construction , rugged, low maintenance, cheap particularly in case of squirrel-cage motor.

Appliations of Synchronous Motors

The applications of synchronous motors are:

1. *Power-factor correction*
   Over-excited synchronous motor delivers a leading power-factor. It is used as a power-factor correcting device in systems equipped with induction motors, like welding machines, flouresent lamps etc.
2. *Constant-speed application*
   Due to constant speed characteristics, high efficiency and high speed, it is used in centrifugal pumps, reciporating compressors, paper mills and metal-rolling mills etc.
3. *Voltage regulation*
   Especially for long-distance transmission lines, receiving end voltage drastically varies based on the load condition and it developes over and under-voltages. Inorder to contain these voltages within the limit, excitation of the synchronous motor is adjusted accordingly.

TWO MARK QUESTIONS AND ANSWERS

1. State the Faraday's law of electromagnetic induction. **[AU Nov/Dec, 2011]**

Faraday's Law of electromagnetic induction states that whenever a current carrying conductor cuts the magnetic flux, a dynamically induced emf gets generated in the conductor.

2. How are DC machines classified? **[AU Nov/Dec, 2016]**

The DC machines are classified as DC motor and DC generator.

3. What is the function of a DC generator? **[AU Nov/Dec, 2009]**

The DC generator is a dynamic DC machine which generates electrical energy from mechanical energy using Faraday's law of electromagnetic induction.

4. List the main parts of DC machine. **[AU Nov/Dec, 2010]**

The different part of DC machine is listed in section 3.3.1.

5. What are the functions of yoke? What is the choice of material for the yoke? **[AU Nov/Dec, 2009]**

Refer to section 3.3.1 for the functions of yoke and its choice of material.

6. State the functions of commutator or specify the role of commutator in DC generator. **[AU Nov/Dec, 2009; April/May, 2011]**

The functions of commutator are:

(i) Through the brushes, it provides a connection between the rotating armature conductor and stationary external circuit.

(ii) The alternating current induced in the armature conductor is converted into unidirectional current in a DC generator.

7. State the emf equation of a DC machine stating the meaning of each term. **[AU Nov/Dec, 2009]**

The emf equation of a DC machine is $\dfrac{\phi ZNP}{60\text{ A}}$

where ϕ is the flux per pole, Z is the number of armature conductors, N is the speed of the machine in rpm, P is the number of poles in the machine and A is the number of parallel paths.

8. An 8-pole, wave -connected armature has 600 conductors and is driven at 625 rpm. If the flux per pole is 20 mWb, determine the generated emf. **[AU Nov/Dec, 2013]**

Solution: Given N = 625 rpm, P = 8, Z = 600 and ϕ = 20 mWb.

Since the wave windings are wave connected, A = 2. Therefore, the generated emf is

$$E_g = \frac{\phi ZPN}{60\text{ A}} = \frac{20\times 10^{-3}\times 8\times 600\times 625}{60\times 2} = 500\text{ V}$$

9. Mention the types of excitation in DC machine. Or What are the different methods of excitation of generator? **[AU Nov/Dec, 2012]**

The different methods of excitation of DC generator are: (i) Self excitation and (ii) Separate excitation.

10. What is self excited generator? How does it get excited? **[AU April/May, 2011]**

If a separate DC source is not required for excitation, it is called self excited generator. Here, the required power for exciting the field winding is obtained from the power developed in the armature of the DC generator.

11. What is DC compound generator?

[AU Nov/Dec, 2012; April/May, 2011]

Refer to section 3.4 for DC compound generator

12. Draw the open circuit characteristic curve of self excited DC generator. [AU Nov/Dec, 2011]

The open circuit characteristic of self excited DC generator is shown in Figure UQ3.12.

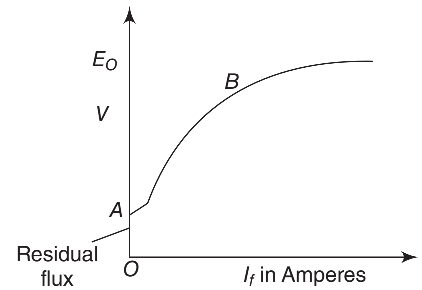


Figure UQ3.12

13. Draw the open circuit characteristics of DC generator. [AU Nov/Dec, 2014]

The open circuit characteristics of separately and self excited DC generators are shown in Figure UQ3.13.

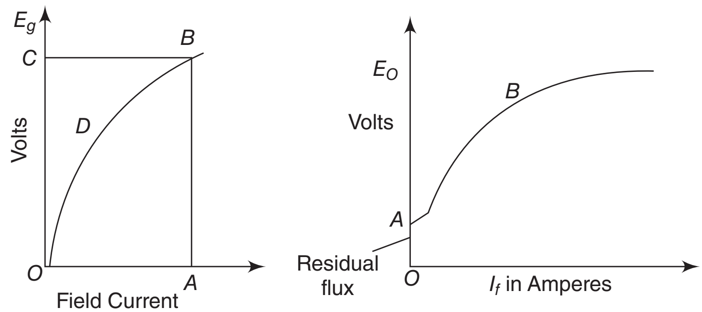


Figure UQ3.13

14. Write the necessary conditions to be satisfied for the self excited DC generator to build up emf. Or What are the conditions to be fulfilled for the self excitation of a DC shunt generator?

[AU April/May, 2014; April/May, 2010; April/May, 2011]

Refer to Section 3.5.2 for the conditions to be fulfilled for the self excitation of a DC shunt generator.

15. Sketch the load characteristics of a DC series generator. [AU April/May, 2011]

The load characteristics of a DC series generator are shown in Figure UQ3.15.

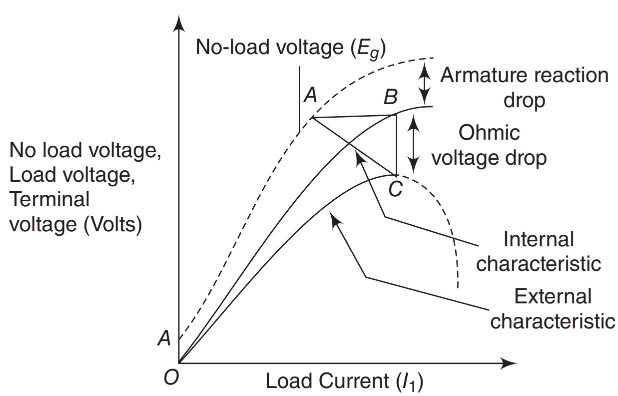


Figure UQ3.15

16. What is an electric motor? State its principle of working. **[AU Nov/Dec, 2011]**

A DC motor is a dynamic DC machine which generates mechanical energy from electrical energy. Refer to section 3.7.2 for its working principle.

17. Write the working principle of DC motor. **[AU Nov/Dec, 2016]**

Refer to section 3.7.2 for the working principle of a DC motor.

18. Define back emf of DC motor. Or What is back emf? **[AU Nov/Dec, 2012; April/May, 2012]**

Refer to section 3.7.3 for the back emf of DC motor.

19. What is the significance of back emf. **[AU April/May, 2013]**

Refer to section 3.7.3 for the significance of back emf.

20. A DC motor operates from 240 V supply. The armature resistance is 0.2 Ω. Determine the back emf when the armature current is 50 A. **[AU Nov/Dec, 2013]**

Given $R_a = 0.2\ \Omega$, $I_a = 50$ A and $V = 240$ V.

Therefore, the back emf of DC motor is

$$E_b = V - I_a R_a = 240 - (50 \times 0.2) = 230 \text{ V}$$

21. A 200 V DC motor has an armature resistance of 0.06 Ω and series field resistance of 0.04 Ω. If the motor input is 20 kW, find the back emf of the motor and power developed in armature. **[AU May/June, 2016]**

The line or load current of the armature is $I_L = \dfrac{P_i}{V} = \dfrac{20 \times 10^3}{200} = 100$ A. Since $I_a = I_L$, the back emf of the motor is $E_b = V - I_a R_a = 200 - (100 \times 0.06) = 194$ V.

The power developed in armature is $E_b I_a = 194 \times 100 = 19.4$ kW.

22. For a DC motor, write the expression for speed. **[AU Nov/Dec, 2016]**

The expression for speed of DC motor is

$$N = \frac{V - I_a R_a}{\phi} \times \frac{60\text{ A}}{ZP} = \frac{E_b}{\phi} \times \frac{60\text{ A}}{ZP}$$

where V is the supply voltage, E_b is the back emf of DC motor, I_a is the armature current, R_a is the armature resistance, ϕ is the flux per pole, Z is the number of armature conductors and A is the number of parallel paths.

23. List the types of DC motors. Give any one difference between them. Or Mention the types of DC motor. **[AU Nov/Dec, 2014; Nov/Dec, 2016]**

Refer to section 3.8 for the types of DC motor its description.

24. Draw the speed torque characteristics of DC shunt and series motors. **[AU April/May, 2012]**

Refer to section 3.9.1 and 3.9.2 for the speed torque characteristics of a DC series and shunt motors.

25. List the different methods of speed control of DC shunt motor. **[AU Nov/Dec, 2011]**

Refer to section 3.10.2 for the speed control of a DC shunt motor.

26. State the various applications of DC motors. **[AU April/May, 2013]**

Refer to section 3.11 for the various applications of DC motors.

27. What is stepper motor? **[AU April/May, 2009]**

A stepper motor is a special type of synchronous motor designed to rotate through a specific angle for each applied electrical pulse.

28. What are the different types of stepper motor? **[AU Nov/Dec, 2010]**

The different types of stepper are variable reluctance, permanent magnet and hybrid stepper motor.

29. List the applications of stepper motor. **[AU April/May, 2007]**

The stepper motors are used in

- Computer peripherals like printer, disk drive etc., X-Y plotter, scientific instrument and machine tool
- Quartz-crystal watch
- Many supporting roles in the manufacture of packaged foodstuff, commercial end product and even in the production of science fiction movies

30. What is a brushless DC motor? **[AU Nov/Dec, 2009]**

The Brushless Direct Current (BLDC) motor is a derivative of DC motor and it shares the same torque and speed performance curve characteristics. The major difference between BLDC and DC motor is the use of brushes for commutation.

31. What are advantages and disadvantages of BLDC motor? **[AU Nov/Dec, 2008]**

Refer to section 3.13.4 for the advantages and disadvantages of BLDC motor

32. Write down the emf equations of a single phase transformer.

[AU Nov/Dec, 2009; Nov/Dec, 2012]

The rms value of the induced emf in the primary winding is given by

$$E_1 = 4.44 f \phi_{\mathrm{m}} N_1$$

Similarly, the rms value of the induced emf in the secondary winding is given by

$$E_2 = 4.44 f \phi_m N_2$$

33. Draw the no-load phasor diagrams of a transformer. **[AU Nov/ Dec, 2014; April/May, 2006]**

The no-load phasor diagram of ideal and practical transformer without winding resistances and leakage reactances is shown in Figures. UQ3.33(a) and (b) respectively.

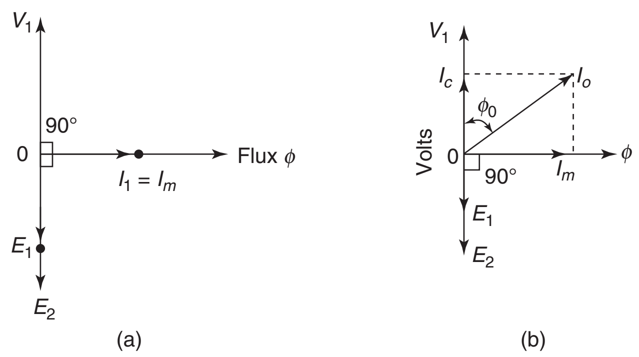


Figure UQ3.33

34. Define voltage transformation ratio of transformer. **[AU Nov/ Dec, 2012]**

The voltage transformation ratio of transformer, K is defined as the ratio of the secondary induced voltage E_2 to the primary induced voltage E_1 as

$$K = \frac{E_2}{E_1}$$

35. Distinguish between core and shell-type transformers. **[AU April/May, 2015; Nov/Dec, 2008]**

Refer to section 3.15.1 for the differences between core and shell-type transformers.

36. What is ideal transformer and how does it differ from a practical transformer? **[AU April/May, 2015]**

Refer to section 3.17.1 and 3.17.2 for the ideal and practical transformers.

37. What is a transformer? **[AU Nov/Dec, 2006]**

A transformer is a static device used for coupling two or more electric circuits. It is working on the principle of mutual induction and it transfers the electric energy from one circuit to another when there is no electrical connection between the two circuits.

38. Give the principle of a transformer. **[AU April/May, 2010]**

Refer to section 3.15.2 for the principle of transformer.

39. Why are transformers rated in kVA instead of kW? **[AU April/May, 2012]**

The iron and copper losses depend only on the supply voltage and the current flowing through the winding respectively. Since these losses do not depend on the phase angle between the supply voltage and the current, the transformer rating expressed as a product of voltage and current is called VA rating of the transformer. Therefore, the transformer rating is expressed as kVA and not as kW.

40. What is an ideal transformer? **[AU April/May, 2011]**

Refer to section 3.16 for ideal transformer.

41. What will happen if transformer primary is excited by DC voltage? **[Nov/Dec, 2010]**

Refer to section 3.15.4 for the sequence of events when the transformer primary is excited by DC voltage.

42. What are turns ratio and transformation ratio of transformer? **[AU Nov/Dec, 2011]**

Turns ratio: It is the ratio of the number of turns in primary N_1 to number of turns in secondary N_2. Hence,

$$\text{Turns ratio } = \frac{N_1}{N_2}$$

If $N_2 > N_1$, the transformer is called step-up transformer and if $N_2 < N_1$, the transformer is called step-down transformer.

Transformation Ratio (K): It is defined as the ratio of the secondary induced voltage (E_2) to the primary induced voltage (E_1). Hence,

$$K = \frac{E_2}{E_1}$$

43. What are step-up and step-down transformers? **[AU April/May, 2013]**

Step-up transformer: It transfers a high current, low AC voltage into a low current, high AC voltage. Here, $N_2 > N_1$ and $V_2 > V_1$. Therefore, the transformation ratio is greater than 1 i.e., $K > 1$.

Step-down transformer: It is vice versa of step-up transformer, i.e., it transfers a low current, high AC voltage into a high current, low AC voltage. Here, $N_2 < N_1$ and $V_2 < V_1$. Therefore, the transformation ratio is less than 1 i.e., $K < 1$

44. State the different core constructions of a transformer. **[AU Nov/Dec, 2011]**

Refer to section 3.15.1 for different core constructions of a transformer.

45. What are the different winding connections in three phase transformer? [AU Nov/Dec, 2009]

Refer to section 3.21.2 for the different winding connections in three phase transformer.

46. What are the advantages and disadvantages of three phase transformer? [AU April/May, 2008]

Refer to section 3.21.4 for the advantages and disadvantages of three phase transformer.

47. State the principle of three phase induction motor. [AU Nov/Dec, 2014; April/May, 2011]

Refer to section 3.23.3 for the principle of three phase induction motor.

48. A three phase induction motor does not run at synchronous speed. Why? [AU April/May, 2011; Nov/Dec, 2011]

Refer to section 3.23.4 for the reason of three phase induction motor not running at synchronous speed.

49. Write down the relation between speed and frequency. [AU Nov/Dec, 2012]

The relation between the speed and frequency is given by

$$N_{\text{syn}} = \frac{120f}{P}$$

50. How to reverse the direction of rotation of three phase induction motor? [AU April/May, 2012]

Refer to section 3.23.3 to reverse the direction of rotation of three phase induction motor.

51. Why are the slots on the induction motors are usually skewed? [AU April/May, 2006]

Refer to section 3.23.1 for the reasons of skewing the slots in induction motor.

52. Explain why at synchronous speed, the torque developed by the induction motor is zero. [AU April/May, 2009]

Refer to section 3.23.4 for the reason why the torque developed in the motor is zero at synchronous speed.

53. What is slip of an induction motor? State its expression. [AU April/May, 2013; Nov/Dec, 2012]

Refer to section 3.23.5 for the slip of an induction motor and its expression.

54. Draw the torque -slip characteristic of three phase induction motor and show the various regions of operation. [AU April/May, 2013]

Refer to section 3.25.1 for the torque-slip characteristics of three phase induction motor.

55. What is the speed of the rotor field in space? [AU Nov/Dec, 2008]

The speed at which the rotor field rotates in space is the rotor speed, which is denoted as N.

56. In which type of motor can external resistance be introduced in the rotor circuit? What is the effect of it? [AU April/May, 2005]

Refer to section 3.23.1 for the motor type to which the external resistance can be added.

57. Draw the torque-speed curve of the induction motor. [AU April/May, 2012]

The torque-speed curve of the induction motor is shown in Figure UQ3.57.

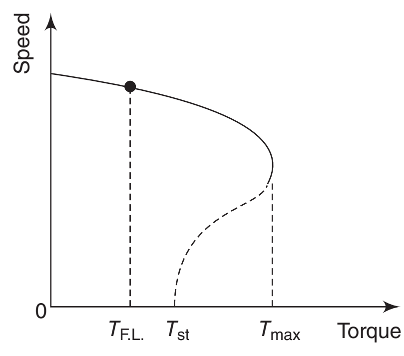


Figure UQ3.57

58. Distinguish between squirrel cage and slip ring induction motors. [AU April/May, 2014]

Refer to 3.23.2 for the differences between squirrel cage and slip ring induction motors.

59. Why is the single phase induction motor not self starting? [AU Nov/Dec, 2011; Nov/Dec, 2010]

Refer to section 3.27 for the reason why three phase induction motor is not self starting.

60. Where are split phase motors used? [AU Nov/Dec, 2006]

Refer to section 3.28.1 for the usage of split phase motors

61. List out four applications of single phase induction motor.
[AU Nov/Dec, 2011; April/May, 2008]

Refer to section 3.27 for the application of single phase induction motor

62. Name the starting methods for single phase induction motor.
[AU April/May, 2014; April/May, 2011; Nov/Dec, 2010]

Refer to section 3.28 for the different starting methods for single phase induction motor.

63. Why are centrifugal switches provided on single phase induction motors?
[AU April/May, 2011]

Refer to section 3.28.1 for the reasons why centrifugal switches are required in single phase induction motor

64. Name the types of alternator. [AU Nov/Dec, 2014]

The different types of alternator are salient pole alternator and non-salient pole alternator

65. Write down the emf equation of an alternator. [AU Nov/Dec, 2010]

The generalised emf equation of an alternator is

$$E_{\text{ph}} = 4.44 K_C K_d f \phi T_{\text{ph}}$$

66. State the difference between salient and cylindrical type of rotors. [AU April/May, 2011]

Refer to section 3.29.1 for the difference between salient and cylindrical type of rotors.

67. Determine the speed at which the 6 pole alternator is driven to obtain the frequency of emf induced to be 50 Hz. [AU April/May, 2014]

Given $P = 6$ and $f = 50$ Hz

The speed at which the alternator is to be driven is called synchronous speed and is given by

$$N_s = \frac{120f}{P} = \frac{120 \times 50}{6} = 1000 \text{ rpm}$$

68. How a synchronous motor can be made self starting? [AU Nov/Dec, 2011]

Refer to section 3.30.5 for the procedure to make synchronous motor a self starting motor.

69. State the characteristic features of synchronous motor. [AU Nov/Dec, 2011]

Refer to section 3.30 for the characteristic features of synchronous motor

70. What are the different excitations of a synchronous motor? [AU Nov/Dec, 2011]

The different excitations of a synchronous motor are: (i) Normal excitation, (ii) Under excitation and (iii) Over excitation.

71. State the applications of synchronous motor. [AU Nov/Dec, 2013]

Refer to section 3.31.8 for the applications of synchronous motor.

72. Why synchronous motor is called so? [AU April/May, 2013]

Since the motor rotates at a speed called synchronous speed, the motor is called synchronous motor.

REVIEW QUESTIONS

1. Discuss the basic concepts of emf generation in a DC machine and derive it.
2. Draw and explain the characteristics of a DC generator.
3. What is back emf? How does back emf in a DC motor make the motor self regulating? State the significance of back emf.
4. Derive from the first principle an expression for the torque developed in a DC motor.
5. Draw the speed-current and torque-current characteristics of a DC motor.
6. Explain the methods of speed control of a DC motor with the help of neat diagrams.
7. What do you mean by stepper motor? Discuss the different types of stepper motor.
8. Explain the operation of stepper motor.
9. Discuss a brushless DC motor.
10. Define a static transformer and explain its principle of operation. How is energy transferred from one circuit to another? Which of the two windings is called primary and secondary?
11. Explain the construction of single phase transformer.
12. Derive the emf equation of a transformer. On what factors does the induced emf in a winding depend?
13. How do you define an ideal transformer? How does an actual transformer differ from it?
    Explain the operation of ideal transformer at no-load condition.
14. Explain the operation of practical transformer at no-load condition.
15. Explain the construction and working of three phase transformer.
16. State the working principle of three phase induction motor.
17. What is the speed of revolving magnetic field? Can it be reversed? How?
18. Explain the construction and working of three phase induction motor.
19. Is it possible for the rotor to run at synchronous speed? Why?
20. Explain the torque-slip and speed-torque characteristics of an induction motor.
21. How does the torque in induction motor change with respect to rotor resistance and rotor reactance?
22. How can the induction motor speed be controlled and explain it.
23. Explain the construction and working of a single phase induction motor.
24. Discuss double revolving field theory.
25. What are the different types of single phase induction motor and explain them.
26. Explain the construction and working of an alternator.
27. Distinguish between salient pole and smooth cylindrical rotor. Discuss their point of relative merits and demerits.
28. Derive the emf equation of an alternator.
29. Explain the construction and working of synchronous motor.
30. Why synchronous motor is not a self starting motor?
31. Explain the procedure and method to start synchronous motor.
32. What are V and inverted V curves?
33. How does a synchronous motor differ from an induction motor?
34. Discuss the applications of synchronous motor.
35. The emf per turn of a single phase 6.6 kV, 440 V, 50 Hz, transformer is approximately 12 V. Calculate the number of turns in the HV and LV windings and the net cross sectional area of the core for a maximum flux density of 1.5 T. [*Ans.* 550, 37 and 360 sq. cm]

36. A single phase 50 Hz transformer has 80 turns on the primary and 400 turns on the secondary winding. The net cross sectional area of the core is 200 sq.cm. If the primary winding is connected to a 240 V, 50 Hz supply, determine: (i) emf induced in the secondary winding and (ii) Maximum flux density in the core. [*Ans.* 1200 V and 0.6756 Wb / sq.m]
37. The no-load current of a transformer is 10 A at a power factor of 0.25 lagging, when connected to 400 V, 50 Hz supply. Determine: (i) Magnetising component of no-load current (ii) Iron loss and (iii) Maximum amount of flux in the core. Assume number of turns in the primary winding is 500. [*Ans.* 9.6824 A, 1000 W and 3.6036 mWb]
38. A 600 kVA, single phase transformer while working at unity power factor has an efficiency of 92 per cent at full load and also at half load. Determine its efficiency when it operates at unity power factor and 60 per cent of full load. [*Ans.* 92.3282%]
39. A 250 kVA, single phase transformer has 98.135 per cent efficiency at full load and power factor of 0.8 lagging. The efficiency at half load and power factor of 0.8 lagging is 97.751 per cent. Determine the iron loss and full load copper loss. [*Ans.* P_i = 2000.18 W and P_c = 1800.69 W]

CHAPTER 4

Utilisation of Electrical Power

4.1 Introduction

The electrical power is generated, transmitted, distributed and utilized as sinusoidal voltages and currents in most of the commercial, industrial and domestic applications. The two broad classification of electrical energy sources are: (i) conventional and (ii) renewable energy sources. Due to some disadvantages of conventional energy sources like coal, fossil fuel, etc., it is necessary to go for alternate energy sources, i.e., renewable energy source. Renewable energy sources use the abundantly available natural sources like solar, wind, etc. to generate electrical power. In this chapter, a detailed description of solar and wind energy is discussed. Electrical energy generated using conventional and renewable energy source can be used for lighting, refrigeration, air conditioning, etc. Lighting loads include different lamps like sodium vapour lamp, mercury vapour lamp and fluorescent tube. The detailed description of these different lamps is presented in this chapter.

Battery plays a significant role in storing the generated electrical energy for future use. The different types of batteries explained in this chapter are: (i) Nickel – Cadmium, (ii) Lead – acid, and (iii) Lithium – ion. The charging and discharge characteristics of these batteries are explained. Further, the transmission and distribution of electric power, the necessity of protecting the power system and operation of various protective devices like earthing, circuit breaker, fuse, and relay are explained. Tariff refers to the cost of electrical energy with which the consumer is charged. Tariff plays a major role in maintaining a healthy relation between the supplier and consumer. Hence, due consideration has to be given in fixing the tariff and the different consumers must be charged with different tariff. The different objectives and characteristics of tariff, factors affecting the tariff, and different types of tariff are discussed.

Power Generation

Power generation has always been a challenge to the modern world. The status of a country's development is proportional to its energy consumption. On the whole, at most of the times, the power requirement exceeds the power generation. This power requirement is met using either the conventional sources of energy or the renewable energy resources. A conventional source of energy is a resource that does not renew itself at a sufficient rate for sustainable economic extraction. The various reasons for finding an alternative source for conventional resources are:

(i) Conventional energy sources are limited, i.e., once they are used, it is very difficult to replace them.

(ii) When the conventional energy sources are used, it generates greenhouse gases like carbon dioxide that leads to severe climatic changes in the environment.

(iii) Usage of the conventional energy sources increases the pollution and causes severe lung diseases.
(iv) Cost of electrical power generated using conventional energy sources is high.
(v) Initial as well as maintenance costs of conventional power generation are high.
(vi) When the generated power is transmitted to the consumers, it suffers high transmission losses.
(vii) Creates a threat to the environment as it causes global warming.

4.2 Renewable Energy

A renewable resource, an alternative for conventional resource, is a natural resource which can replenish with the passage of time, either through biological reproduction or other naturally occurring processes. The renewable energy sources that are used for generating electrical power are: solar, wind, hydro, biomass, hydrogen fuel cells and geothermal power.

4.2.1 Solar Energy

One of the most readily available sources of renewable energy is solar energy. It is one of the most promising renewable generations and its usage is increasing rapidly. In some countries, before 1980, some academic research has proposed to use solar energy efficiently. After the sudden increase in the oil prices in 1980s, many countries started the practical research to use solar energy. One of the cleanest and highly reliable sources of renewable energy that can be used in different forms to generate electrical energy to meet the demand is solar energy. There is a rapid increase in the usage of solar energy as the amount of solar energy reaching the earth is many million times greater than the energy consumed by a single man. If all the solar energy falling on the earth for one day is converted into electricity, it is possible to meet the demand of the entire planet for one year.

Solar Photovoltaic (PV) Power Generation System

The most efficient way of generating electrical power is the solar power generation since it requires less number of steps when compared to other electrical power generation. In this generation, incident solar energy is directly converted into electricity using PV or solar cells. The principle on which the PV cells generate electrical energy is photoelectric effect. Thus, the converted electrical energy can be used to meet the residential or industrial demands. In this system, using photoelectric effect, the incident solar energy gets converted into electrical energy and it can be used either for the load or can be connected to the grid. The overall block diagram of solar PV power generation system is very flexible. The main building blocks of solar PV power generation system are PV modules that can be arranged in arrays to generate large electrical power. However, some additional components are required to store produced electrical energy for future use. The major components existing in the solar PV power generation system are:

(i) **Solar PV Panels:** Solar PV cell or solar PV panels are used to convert the incident solar energy into electrical energy using photoelectric effect. Maximum energy is obtained from the sun by mounting or tilting the solar PV panels. In general, the rated voltage of each solar panel is 12 V. Solar array is used to produce 24 V or 48 V for standalone system and several hundreds of volts in grid connected system by connecting solar PV panels in series. Similarly, more power can be generated by increasing the current and maintaining the same voltage, if the solar PV panels are connected in parallel. The concept of solar PV panels in series and parallel is explained in Figure 4.1(a) and (b) respectively. Consider the rating of a solar panel as 12 V, 12 W and 1 A. If 4 such panels are connected in series as shown in Figure 4.1(a), the rating of the solar array will be 48 V, 48 W and 1 A. But if they are connected in parallel as shown in Figure 4.1(b), the resultant rating of the solar array will be 12 V, 48 W with 4 A.

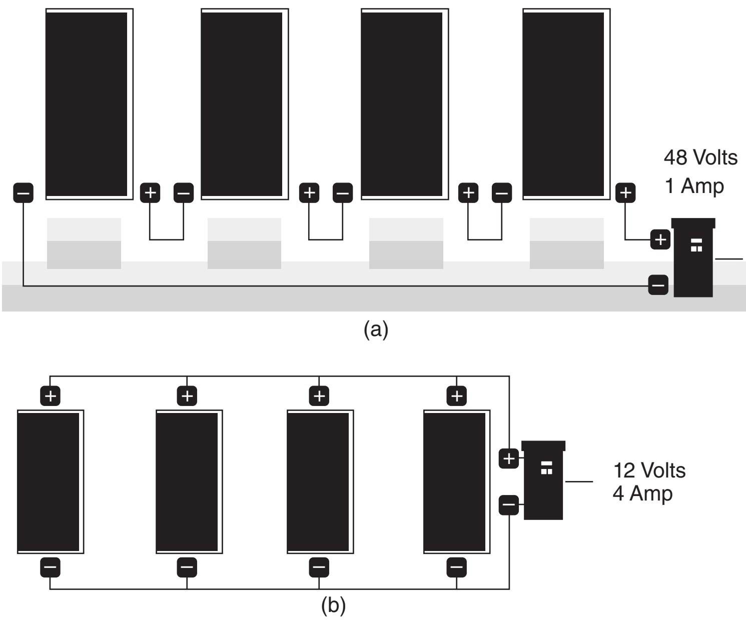


Figure 4.1 Solar panels connected in (a) series (b) parallel

(ii) **Batteries:** In standalone solar PV systems, batteries are required to store the energy generated from solar PV panels or solar PV array. Once fully charged, batteries provide a constant power supply even when the generated solar power is variable. In general, lead acid batteries are used in solar PV systems. Similar to solar PV panels, batteries can be connected either in series or parallel to form a battery bank.

(iii) **Controller:** Controller is required in the solar PV system to regulate the current flowing in and out of the battery. It plays a major role in safe guarding the battery from overcharging. Also, it prevents the usage of battery once it is completely discharged. This module helps in balancing the electric power generated to use in domestic appliances. In addition, a signal in the form of alarm is given by the controller when the system is not functioning properly.

(iv) **Inverter:** The electric power generated by the solar PV system is DC. Since most of the electrical loads operate on AC, an inverter is required to convert DC to AC. Also, in grid connected system, to integrate the generated solar power and the main grid, an inverter is mandatory. In recent days, solar PV panels are attached with micro inverters to provide a high AC voltage.

Types of Solar PV Power Generation Systems

This solar PV power generation system is classified into two major groups:

(i) **Off-Grid or Stand-Alone System:** This PV power generation system is simple and comparatively small. Main grid is isolated from this system. Hence, the generated electrical energy is used to meet the demands. It is the most popular type of solar PV power generation system to replace the main grid. It is mainly used in remote areas where the usage of main grid power is difficult. The main purpose of this system is to charge the battery that helps in supplying the load. The power from the battery can be used either for DC loads or AC loads when it is attached with the inverter. The block diagram of the stand-alone system is shown in Figure 4.2.

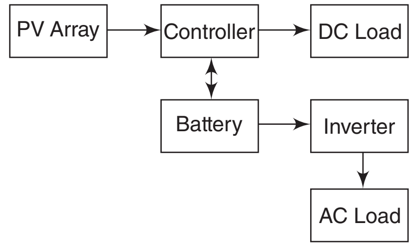


Figure 4.2 Stand-alone system

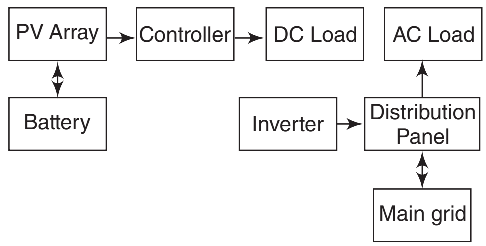


Figure 4.3 Grid connected system

(ii) **Grid Connected System:** In this solar PV power generation system, the generated electrical power is transmitted to the main grid. This type of system is available in urban areas where electric power from main grid is easily available. During day time, if the generated electric power is more, then the excess power is exported to the main grid. Also, if the generated electric power is less, the required power is imported from the grid. The size of the grid connected system varies since this system does not have enough power to meet the electrical demand. The block diagram of grid connected system is shown in Figure 4.3.

This system receives the payment for each kilowatt of power which is supplied to the electricity providers. This type of installation reduces the dependence on electric utilities and hence reduces the electricity bills. The major components in this solar installation include a PV array or solar cells, inverter and the metering system. The main disadvantage of the grid-connected solar system is that it will switch off in the event of power cut, because this system is a part of utility grid and hence if there is a power cut, the power from the solar array is also switch off. If this system does not stop, current flowing back into the grid could lead to serious faults.

Advantages, Disadvantages and Applications of Solar Energy

Advantages

The advantages of solar energy are:

(i) Time taken to install solar system is less when compared to conventional power system.
(ii) Does not pollute the air or water while generating electrical power.
(iii) Since there is rotating components, there is no noise pollution.
(iv) Running cost of the solar power is less.
(v) Solar energy is effectively free in nature and it is inexhaustible.
(vi) Helps in preserving the wildlife environment as it helps in maintaining our eco-system.
(vii) Helps in preserving water since it does not require water to cool its components.
(viii) Has wide range of applications.

Disadvantages

The disadvantages of solar energy are:

(i) Initial cost of solar energy is high.

(ii) Purely depends on the solar radiation which is highly random in nature.

(iii) Some back-up power is required since the solar energy is not reliable.

(iv) Large space is required to install solar PV panels.

(v) Efficiency of the solar power is less.

(vi) Size of the solar PV systems varies depending on the geographical location.

Applications

The major applications of solar energy are:

(i) **Solar Water Heating:** The solar PV panels absorb incident solar radiations and transfers the heat generated to the circulating water in the tubes. This solar water heating is used in commercial areas, domestic and industrial units.

(ii) **Solar Heating of Buildings:** When the building requires heat it gets transferred to the load like fan, ducts, air outlets, radiators, etc., to warm up the living spaces of a building.

(iii) **Solar Distillation:** Here, solar power is used to convert the impure water into a distilled water and is used to supply it to colleges, schools, defense labs, petrol pumps, etc., where scarcity of water exist.

(iv) **Solar Pumping:** In this, solar power is used to pump the water for irrigation purposes.

(v) **Solar Drying of Agricultural and Animal Products:** Uses the solar power to dry potato-chips, ginger, milk, fish, etc.

(vi) **Solar Furnaces:** Used in metallurgical and chemical operations to measure the ceramics properties at extremely high temperatures.

(vii) **Solar Cooking:** Due to energy crisis of conventional source, the solar power is used for cooking purposes as solar cookers.

(viii) **Solar Green Houses:** Utilises the solar power to grow plants.

(ix) **Solar Thermal Power Generation System:** Here, solar energy is used as a source of heat to generate the steam required to drive the steam turbine that generate electrical energy.

4.2.2 Wind Energy

The other form of solar energy which is generated from wind is called wind energy. It is known that winds are produced due to uneven heating of the earth's atmosphere, abnormalities existing in the surface of the earth and earth's rotation. Earth's terrain, water bodies and vegetation changes the wind flow pattern. This movement of wind or flow of wind is converted into the mechanical power using wind turbines. The mechanical power can either be converted into an electrical power using a generator or it can be directly used for pumping loads.

Power Curve of the Wind Turbine

The amount of wind power generated using a wind turbine varies based on the wind speed. Power curve of the wind turbine is a relationship curve between wind power and wind speed. This curve varies for each type of wind turbine. Generally, the wind turbine gets operated continuously by delivering different power levels since wind speed varies instant by instant. There exist some operating characteristics for each wind turbine which decides its power curve. They are start-up speed, cut-in speed, rated speed and cut-out speeds.

Start-Up Speed: The wind speed at which the rotor and blade assembly of the wind turbine starts to rotate without generating any power is called start-up speed.

Cut-In Speed: The minimum wind speed at which the rotor and blade assembly of the turbine starts rotating and generates usable wind power is called cut-in speed. In general, the cut-in speed is typically between 3 m/s and 5 m/s for the most of wind turbines.

Rated Speed: The minimum wind speed at which the wind turbine rotates and generates the rated wind power is called rated speed. For an example, a 10 kW wind turbine can generate 10 kW only if the wind speed reaches or crosses the rated speed. In general, rated speed is typically between 11 m/s to 16 m/s for most wind turbines. Between the cut-in and rated speed, the wind power generated by the wind turbine increases with respect to increase in wind speed. It is to be noted that, beyond the rated speed, the wind power obtained from the turbine remains stable.

Cut-Out Speed: The wind speed at which the wind turbine stops generating the wind power is called cut-out speed. In general, the cut-out speed is between 20 m/s to 35 m/s for most of the wind turbines. Here, the wind turbine stops generating the power to prevent it from get damaged. At this speed, in some cases the wind turbine gets shutdown. There are different ways to shut down the wind turbine. Once the wind speed falls back to cut-in speed or start-up speed, the wind turbine starts its normal operation.

Based on the above definitions, the power curve of the wind turbine is given in Figure 4.4.

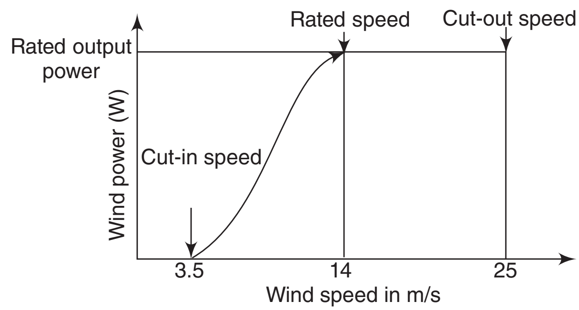


Figure 4.4 Power curve of a wind turbine

Calculation of Wind Power From Wind Turbine

The wind power obtained from the wind turbine is given by

$$P = \frac{1}{2} C_p \rho A V^3$$

where C_p is the maximum power coefficient of the wind turbine ranging from 0.25 to 0.45, ρ is the density of the air, A is the swept area of the rotor and V is the velocity of the wind or wind speed in m/s.

Since a non-linear factor exist in the above equation, it can be ignored and a simplified linear piecewise function as given in the following equation is used for exhibiting the relation between the wind speed and output of wind generation unit. The simplified relation for the wind power is given by

$$\begin{cases} P = 0 & V < V_{\text{in}} \text{ or } V > V_{\text{out}} \\ P = P_R \left(\dfrac{V - V_{\text{in}}}{V_r - V_{\text{in}}} \right), & V_{\text{in}} \le V \le V_r \\ P = V_R & V_r \le V \le V_{\text{out}} \end{cases}$$

where V_{in}, V_{out} and V_r are the cut-in, cut-out and rated wind speed respectively in m /s; P_R is the rated output of wind generation unit in watts and P is the wind generation output in watts.

Components of Wind Turbine

The general block diagram of the wind turbine that generated power from wind is shown in Figure 4.5.

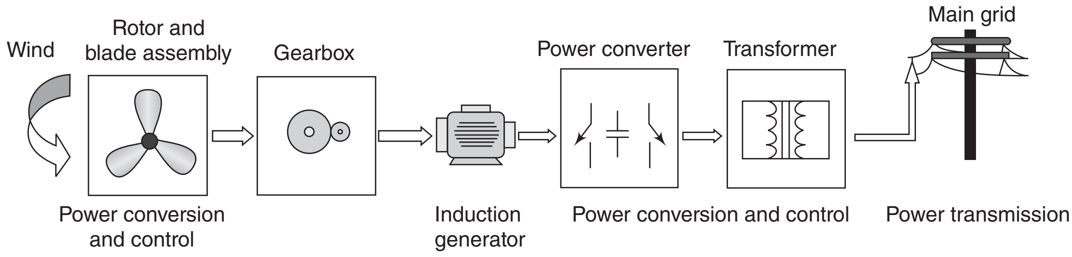


Figure 4.5 Block diagram of a wind turbine

The block diagram shows the step by step procedure in converting the wind into electrical power and the way of injecting it into the main grid. Power converter and transformer are the external devices required to inject the wind power generated using induction generator to the main grid. The schematic diagram of the wind turbine is shown in Figure 4.6.

The other components of wind turbine are:

(i) **Nacelle:** The key components of the wind turbine are placed inside the nacelle. It includes gearbox and the electrical generator. Service men will enter the nacelle using the tower of the turbine. Rotor blades and the hub to place the rotor blades exist near to the nacelle.

(ii) **Blades:** Light weight, robust and corrosion free materials like fibreglass or reinforced plastic are used in making the blades. It is designed in such a way that, it captures the wind and spins. It is available in different sizes. A typical blade size varies from 10 meters to 70 meters based on the power generation. Blades of the wind turbine are turned or pitched to control the rotor speed according to the wind speed.

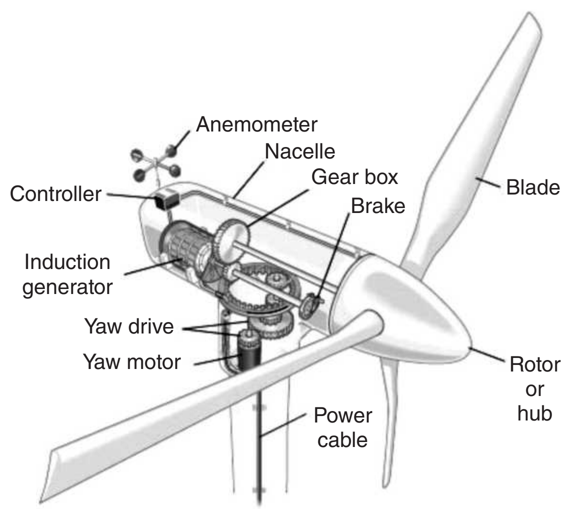


Figure 4.6 Schematic diagram of Wind turbine

(iii) **Rotor:** The hub around which the blades are placed is called rotor. In wind turbine, rotor refers to hub and the blades. It is the most important component of the wind turbine as at this place the conversion of kinetic energy takes place.

(iv) **Brake:** It is placed in the wind turbine so that the rotor can be stopped either electrically or mechanically or hydraulically during emergency situation.

(v) **Gear Box:** It helps in connecting the low and high speed shafts. Generally, the speeds of low and high speed shafts are 30 to 60 rpm and 400 to 1800 rpm respectively. It is necessary as the high speed shaft helps in rotating the induction generator and hence wind energy gets generated.

(vi) **Induction Generator:** The AC generator that rotates at asynchronous speed is called the induction generator or asynchronous generator. It helps in generating the electric power from the wind. In modern wind turbine, the maximum electric power generated is usually between 600 and 3000 kilowatts.

(vii) **Controller:** It helps in controlling the wind turbine, i.e., blades of the wind turbine either rotates or stops based on the signal given by the electronic controller. The wind speed is measured using *Anemometer* and necessary control action will be performed by the controller.

(viii) **Wind Vane:** It helps in changing the orientation of the wind turbine by measuring the direction of wind and communicating it to yaw drives so that maximum wind energy is produced. Yaw drive is activated using yaw motor.

Classification of Wind Turbine

Based on the size of the wind, turbine it is classified as:

(i) **Utility-Scale:** Turbine with the capacity of more than 900 kW used in this category to generate wind power in large scale.

(ii) **Industrial-Scale:** Turbine with the capacity of 50 kW to 750 kW used in this category to generate wind power along with diesel generator so that it reduces power consumption from grid and peak loads existing in the grid.

(iii) **Residential-Scale:** Turbine with the capacity of 400 watts to 50 kW used in this category to generate constant power along with solar photo-voltaic, batteries, and inverters at remote locations where utility power is not easier to install.

Based on the axis of rotation of wind, turbine it is classified as:

(i) **Horizontal Axis Wind Turbine (HAWT):** In HAWTs, both wind axis and rotational axis of the rotor are parallel to each other. Here, the rotor shaft and electrical induction generator are placed at the tower top such that it is pointed towards the wind. Most commercial used wind turbines belong to this category with propeller rotor and it can possess either two or three blades. HAWT is further classified as: (i) Upwind turbines and (ii) Downwind turbines.

(ii) **Vertical Axis Wind Turbine (VAWT):** In VAWTs, both wind axis and rotational axis of the rotor are perpendicular to each other. It is very rarely used wind turbine in generating wind power. In VAWT, the blades can be either curved blades as in Darrieus wind turbine or straight blades as in Savonius wind turbine. In this type of wind turbine, the gear box and induction generator are placed at the bottom level. The schematic diagram of HAWT and VAWT is shown in Figure 4.7 (a) and (b) respectively.

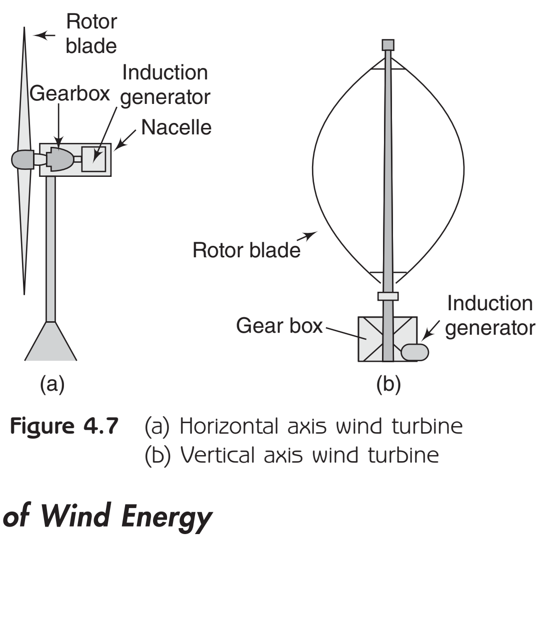


Figure 4.7 (a) Horizontal axis wind turbine (b) Vertical axis wind turbine

Advantages, Disadvantages and Applications of Wind Energy

Advantages

The advantages of wind energy are:

(i) It is available abundantly and it is an inexhaustible source of energy.

(ii) Generating large amount of electrical energy is possible.

(iii) As it is pollution free, it doesn't lead to acid rain or greenhouse effect.

(iv) It can be used as mechanical energy without any conversion.
(v) It is easier to provide electricity in remote areas.
(vi) A steady power can be generated when it is combined with solar energy.

Disadvantages

The disadvantages of wind energy are:

(i) To meet out the peak demand, expensive storage device is required.
(ii) Source of wind energy is uncertain and unpredictable.
(iii) Installation cost is high.
(iv) Creates large noise in generating the power.
(v) It cannot be installed in every place since source of wind energy is random in nature.
(vi) Transmission cost of wind energy is high as it is installed away from consumer.
(vii) Efficiency of the wind turbine is less when compared to other sources.
(viii) Creates a threat to wildlife as birds gets hurt due to rotation of blades.
(ix) Since more mechanical parts exist in the wind turbine, maintenance cost is high.

Applications

The major applications of wind energy are:

(i) Used in propelling the sailboats in water bodies.
(ii) Used in running pumps to draw ground water.
(iii) It is converted to mechanical energy that helps in running flourmills to grind wheat.
(iv) Used in lighting load and to lift different loads.

4.3 Illumination

Light influences all areas of our day-to-day activities and it gives us well- being to our body. It controls the biological processes and is responsible for our sleep-wake rhythm. Good illuminated light gives us a relaxing and regenerative effect which will increase the stamina and concentration in work. Moreover good light guarantees safety in the public areas. Before the invention of the electric bulb, illuminating the world after the sunset was difficult, unpleasant and hazardous task. It was not possible to do day-time work after dusk. During those days crude system of lighting was used to illuminate after the sunset. During the middle of the nineteenth century, a gas mantle and an oil lamp were used as a source of light and it torches to light up a good-sized room. After the invention of electric filament lamps, it proves to be the best competitor to gas as a source of light. The electric lamps are preferable over the other sources of illumination for the following reasons such as cleanliness, convenience, steady light output and reliability. Before entering into the field of illumination engineering, it is essential to know the glimpse of light energy.

4.3.1 Nature of Light

Sun is the main source of light in earth. It is the radiant energy which gives heat and light at a very high rate. However, only a fraction of this light reaches the earth and only 40 per cent of the energy is converted into light. The sunlight reaches earth without heating or lighting the space during its travel and releases its energy only when it strikes a solid object. The energy radiated in this manner is called radiant energy. When the temperature of the object is increased, the energy is radiated in the form of light. The light is a part of radiant energy, which propagate as a wave motion in an either medium, similar to that of an electromagnetic wave. The velocity of propagation of the radiant energy is approximately 3×10^8 m/sec for all practical purposes. When the radiant energy hits the obstacle on its way, either the energy is reflected or absorbed by it. If this energy is absorbed by the obstacle, then this energy is converted into heat. The properties and behaviour of the radiant energy depends upon the wavelength. The wavelengths in the range of 4×10^{-5} cm -7.5×10^{-5} cm can produce the sensation of sight. The relationship between wavelength λ, velocity of propagation v and frequency f is given by

$$\lambda \times f = c$$

i.e., $$\lambda \times f = 3 \times 10^8 \text{ m/sec}$$

The wavelength is also represented in the following two units, namely

(1) Micron

(2) Angstrom (Å) so that

$$1 \text{ micron} = 10^{-6} \text{ m}$$

$$1 \text{ Å} = 10^{-8} \text{ cm}$$

Therefore, visible radiation lies between the limits of 4000 Å to 7500 Å. The energy radiated by the hot body is monochromatic, i.e., the radiation has only one wavelength. Table 4.1 shows the wavelength and its corresponding colour.

Table 4.1 *Wavelength and corresponding colour*

S. No.	Wavelength Å	Colour
1	4000	Violet
2	4750	Blue
3	5500	Green
4	6000	Yellow
5	7000	Red

The important definitions used in illumination of light are as follows:

Definitions

Plane Angle

The source emits light in all directions and it is not limited to a particular plane. Therefore, knowledge of plane and solid angle has become essential.

Angle between two straight lines lying in the same plane meet at a point and converge. This angle is called plane angle and as shown in Figure 4.8. The plane angle is expressed in radians.

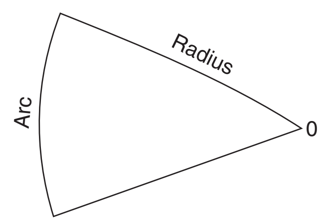


Figure 4.8 Illustration of plane angle

$$\text{Plane angle} = \frac{\text{arc}}{\text{radius}}$$

Solid Angle

The angle subtended at a point in space by an area is called solid angle. The solid angle is expressed in steradians as shown in Figure 4.9.

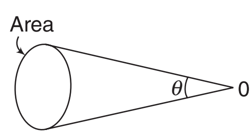


Figure 4.9 Illustration of solid angle

$$\text{Solid angle} = \frac{\text{area}}{(\text{radius})^2} = \frac{A}{r^2}$$

The solid angle subtended by a point in all directions in space is given by

$$\text{Solid angle} = \frac{\text{area of sphere}}{(\text{radius})^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

Light

The radiant form of energy which produces sensation of vision upon the human eye is called light. It is made up of many small particles-like packets that have energy and momentum but no mass.

Luminous Flux

The energy in the form of light waves radiated per second from a luminous body is called luminous flux or luminous power. It is a measurement of energy released in the form of visible light from a light source. For example, an incandescent lamp acts as the luminous body.

When an electrical power is supplied to the lamp, some portion of the power is lost by heat conduction, convection and absorption. The remaining portion of power radiates in the form of light waves whose wavelength lies in the visual range between 4000 Å and 7000 Å. Luminous flux interprets the sensitivity of a human eye by taking the power of each wavelength present in the light. It is used to describe the eye's response to each wavelength. Figure 4.10 shows the representation of flux diagram for incandescent lamp.

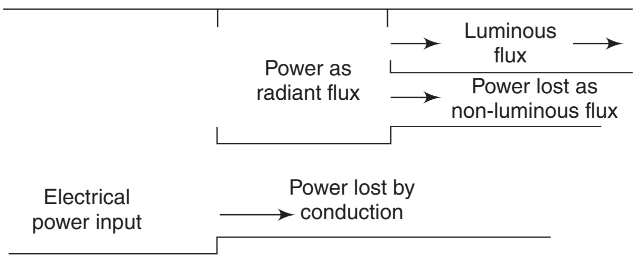


Figure 4.10 Flux diagram for incandescent lamp

Luminous Intensity

The amount of light power emitting from a point source within a solid angle of one steradian is called luminous intensity.

Figure 4.11 shows point source of light emitting luminous flux.

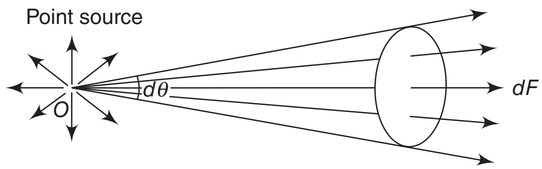


Figure 4.11 Source of light emitting luminous flux

Consider a point source of light as O. Let dF be luminous flux crossing any section of a narrow cone of solid angle $d\theta$ steradians and the apex of the cone so formed is at the source. The luminous intensity I in the direction of the cone is given by the ratio of luminous flux dF to the solid angle $d\theta$ is expressed as $I = \dfrac{dF}{d\theta}$. It is also defined as the flux emitted by the source per unit solid angle.

Lumen

The luminous flux per unit angle from a source of one candle power is called lumen and it is a unit of flux.

$$\text{lumens} = CP \times \theta$$

where CP is the candle power and θ is the solid angle.

Candle Power

The number of lumens per unit solid angle θ is candle power and one CP is equal to 4π lumens.

i.e., $$CP = \frac{\text{lumens}}{\theta}$$

Illumination

The luminous flux received by a surface per unit area is called illumination. It is also called illuminance or degree of illumination and its unit is lux.

$$\text{Ilumination} = \frac{\text{luminous flux}}{\text{area}} = \frac{CP \times \theta}{\text{area}}$$

Substituting $\theta = \frac{\text{area}}{r^2}$ in the above equation, we get

$$= \frac{CP}{\text{area}} \times \frac{\text{area}}{r^2} = \frac{CP}{r^2}$$

where r is the radius i.e., distance between the area and the point where solid angle is formed.

Brightness

It is defined as the luminous intensity per unit projected area of the surface in the given direction. The unit of brightness is candles per sq. metre or candles per sq.cm or candles /ft^2.

The brightness B_r is given by

$$B_r = \frac{\text{Luminous intensity}}{\pi \times A}$$

$$B_r = \frac{CP \times \omega}{\pi \times \text{area}} \text{ candle per sq. ft.}$$

Mean horizontal Candle Power (MHCP)

Mean horizontal candle power (MHCP) is the mean of the candle power in all directions in the horizontal plane containing source of light.

Mean Spherical Candle Power (MSCP)

Mean spherical candle power (MSCP) is the mean or average of candle power of a source of light in all directions in all the planes.

Mean Hemispherical Candle Power (MHSCP)

Mean hemispherical candle power (MHSCP) is the mean of the candle power in all directions within the hemisphere either above the horizontal plane or below the horizontal plane.

Reduction Factor

It is defined as the ratio of mean spherical candle power to mean horizontal candle power.

$$\text{Reduction Power} = \frac{\text{MSCP}}{\text{MHCP}}$$

4.3.2 Laws of Illumination

The following are the two laws of illumination, namely: (i) Inverse square-law and (ii) Lambert's cosine law

Inverse Square Law

Consider a point source of light *S* as shown in Figure 4.12. The three parallel surfaces of area a_1, a_2 and a_3 are placed at a distance of d, $2d$ and $3d$ respectively from the point source as shown in Figure 4.12.

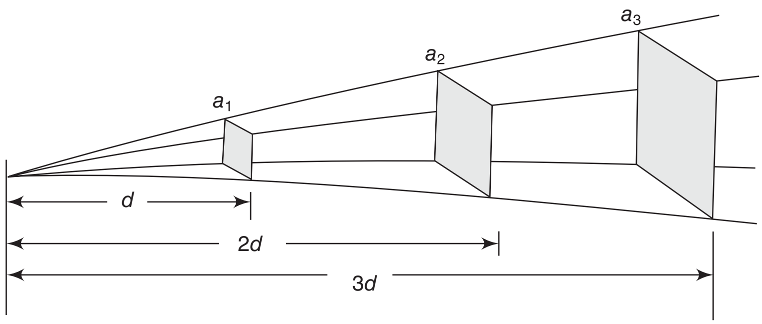


Figure 4.12 Illustration of inverse square law

The intensity of illumination I of the source in that direction is given by

$$\frac{1}{d^2} : \frac{1}{(2d)^2} : \frac{1}{(3d)^3}$$

Therefore, illumination of a surface is inversely proportional to the square of the distance of the surface from the source of light and this law is applicable only for a point source.

Lambert's Cosine Law

Practically, surface is not normal to the light flux and inverse square law finds its limitation in this condition. Illumination in such cases is given by Lambert's cosine law. It is also called $\cos^3$ law. The illumination of surface by Lambert's square law is given by

$$\text{Illumination of surface} = \frac{CP}{h^2}\cos^3\theta$$

where CP is the candle power at a point P and h is the distance of point source from the plane.

From the above Figure 4.13, illumination of a surface at any point is dependent upon the cube of cosine of the angle between the line of flux and the normal at that point.

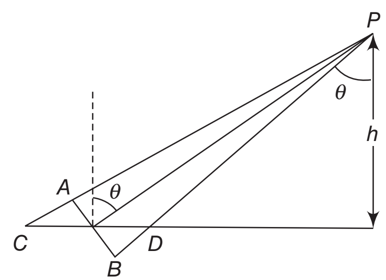


Figure 4.13 Illustration for Lambert's cosines law

Example 4.1

A lamp having mean spherical candle power (MSCP) of 1000 is suspended at a height of 10 metres. Calculate the (i) total flux of light, and (ii) the illumination just below the lamp.

Solution

Given MSCP = 1000 and height h = 10 m

(i) Total flux of light = MSCP × 4π = 1000 × 4π = 12566.37 lumens

(ii) The illumination just below the lamp $= \dfrac{\text{MSCP}}{h^2} = \dfrac{1000}{(10)^2} = 10$ lux

Example 4.2

When a 240 V lamp takes a current of 0.8 ampere, it produces a total lux of 2880 lumens. Calculate (i) MSCP of the lamp and (ii) the efficiency of the lamp.

Solution

Given Voltage = 240 V, Total lumens = 2880 and I = 0.8 A

(i) $\text{MSCP} = \dfrac{\text{Total lumens}}{4\pi} = \dfrac{2880}{4\pi} = 229.18$

(ii) The wattage of lamp = $V \times I$ = 240 × 0.8 = 192 W

Therefore, efficiency of the lamp $= \dfrac{\text{Lumens}}{\text{Wattage}} = \dfrac{2880}{192} = 15$

Example 4.3

A lamp of 600 candle power is placed at the centre of a room, 20 m × 10 m × 5 m. Calculate the illumination in each corner of the floor.

Solution

Given Candle power (CP) = 600, Dimension of room = 20 × 10 × 5 m

To calculate illumination in each corner of the floor

Figures E4.3(a) to (c) show the position of the lamp in a room. Lamp is placed at O, and O' is the position just below the lamp.

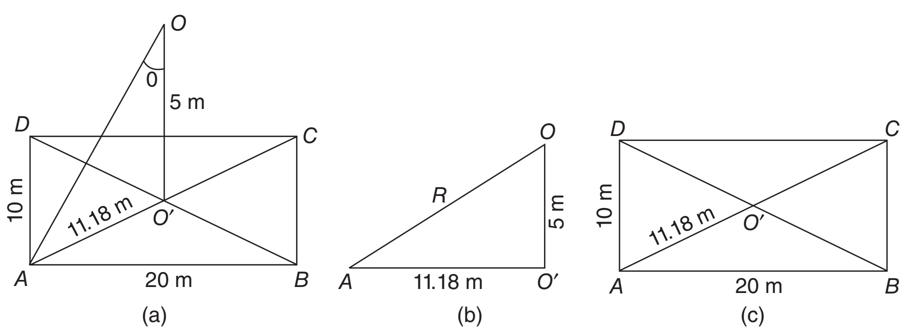


Figure E4.3 (a) to (c)

The diagonal length of the room = $\sqrt{(20)^2 + (10)^2} = 22.36$

Length $\quad AO' = \dfrac{22.36}{2} = 11.18$ m

$$AO = R = \sqrt{(11.18)^2 + (5)^2} = 12.247 \text{ m.}$$

Letting θ be the length between the normal on the floor and the line of flux, we have

$$\cos\theta = \frac{5}{12.247} = 0.4083$$

Illumination in each corner is given by

$$= \frac{CP}{R^2}\cos\theta = \frac{600}{(12.25)^2} \times 0.4083 = 1.6325 \text{ lux}$$

Example 4.4

Two lamps of 200 and 300 candle powers are arranged as shown in the Figure E4.4. Calculate the illumination in the middle of the lamps.

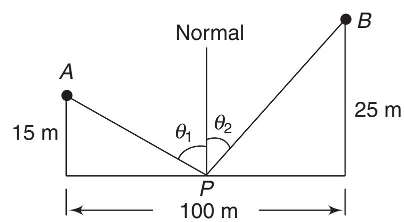


Figure E4.4

Solution

Illumination at point P due to lamp $A = \dfrac{CP}{AP^2}\cos\theta_1$

$$AP = \sqrt{15^2 + 50^2} = \sqrt{2.725} = 52.2 \text{ m}$$

$$\cos\theta_1 = \frac{15}{52.2} = 0.288$$

Illumination at point P due to lamp $A = \dfrac{200}{(52.2)^2} \times 0.288 = 0.0211$ lux

Illumination at point P due to lamp $B = \dfrac{CP}{BP^2}\cos\theta_2$

$$BP = \sqrt{25^2 + 50^2} = \sqrt{625 + 2500} = \sqrt{3125} = 55.9 \text{ m}$$

$$\cos\theta_2 = \frac{25}{55.9} = 0.4472$$

Hence, illumination at point $P = \dfrac{300}{55.9^2} \times 0.4472 = 0.0429$ lux

So, the total illumination at point P = 0.0211 + 0.0429 = 0.064 lux

Example 4.5

A room 20 m × 15 m is to be illuminated by eight lamps and the average illumination is to be 50 lumens/m^2. If the utilisation factor is 0.45 and depreciation factor is 1.2, calculate the mean spherical power per lamp.

Solution

Given room dimension = 20 m × 15 m, average illumination = 50 lumens/m^2, utilisation factor = 0.45 and depreciation factor = 1.2

The area to be illuminated = 20 × 15 = 300 sq.m.
Total number of lux required = 300 × 50 = 15000 lumens

Total lumen to be given out by lamps = $\dfrac{15000 \times 1.2}{0.45} = 40000$ lumens

MSCP of each lamp = $\dfrac{15000 \times 1.2}{0.45 \times 4\pi \times 8} = 697.88$ lumens

Example 4.6

It is required to provide an illumination of 100 candle power in a factory hall 40 m × 10 m. Assume that the depreciation factor is 0.8, co-efficient of utilisation is 0.4 and efficiency of lamp is 13 lumens per watt. Calculate the number of lamps and their disposition.

Solution

Given area of the room 40 × 10 = 400 sq.m., depreciation factor = 0.8, co-efficient of utilisation = 0.4 and lamp efficiency = 13 lumens/watt
Total lumens required = 400 × 100 = 40000 lumens

Gross lumens output by the lamps = $\dfrac{40000}{0.4 \times 0.8} = 125000$ lumens

Therefore, total wattage required = $\dfrac{125000}{13} = 9615.38$

If 300 watt lamps are used, number of lamps = $\dfrac{9615.387}{300} \approx 32$

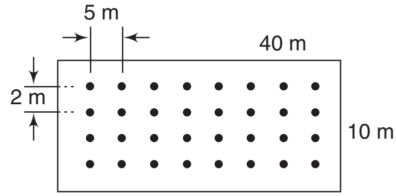


Figure E4.6

The lamps are arranged, 4 along the width and 8 along the length to give quite uniform illumination.
Total number of lamps used = 4 × 8 = 32
The disposition of the lamps is as shown in Figure E.2.9.

4.4 Lighting Schemes

The distribution of the light emitted by electric lamps is controlled by means of reflectors, translucent diffusion screens and even lenses. Depending upon the requirement, different lighting schemes are classified as shown in Figure 4.14.

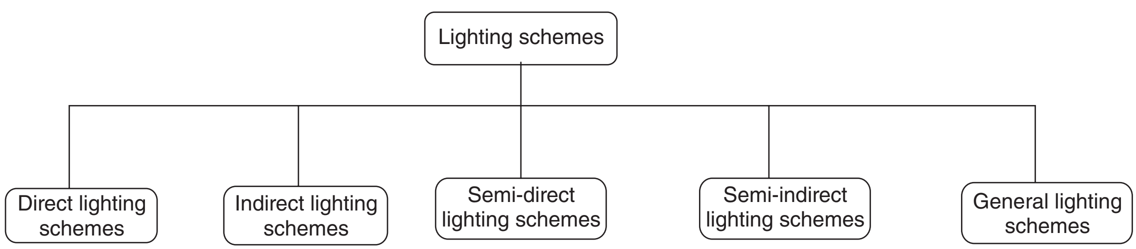


Figure 4.14 Classification of lighting schemes

Direct lighting scheme is the most commonly used lighting scheme.

Direct Lighting Scheme

When more than 90 per cent of total light flux is made to fall directly on the working object with the help of deep reflectors it is called direct lighting scheme as shown in Figure 4.15. It is the most commonly used lighting schemes and it will cause strain on the eyes. This lighting scheme is designed in such a way that glare on eyes are eliminated. It is usually employed in industrial lighting, residential lighting and commercial lighting. Working constantly under a direct light will cause strain on the eyes. Therefore, suitable size of lamp along with suitable filtering is selected. The accumulation of dirt on the lamp, shade or diffuser will be avoided to decrease the luminous intensity and prevent equal distribution of illumination. Therefore, it is advisable to clean the fittings regularly.

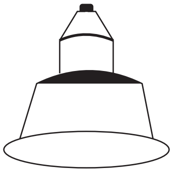


Figure 4.15 Shades for direct lighting schemes

Indirect Lighting Scheme

When total light flux does not fall on the objects directly is called indirect lighting as shown in Figure 4.16. In this scheme, at least 80 per cent of the total light is directed upwards and no light is directly thrown downwards. In this lighting, the lamps are placed in opaque type shade and the maximum light is thrown towards the ceiling and it reaches the object by diffusion or reflection from the ceiling. It is usually employed in illuminating drawing offices, workshops and other places where shadows are to be eliminated. However, in this type of lighting, the requirement of the light is usually more than that of direct lighting scheme. Due to this reason, it is suggested to allow some percentage of direct lighting in addition to indirect scheme. The illumination in this scheme will reduce tremendously if the fittings are not properly cleaned.

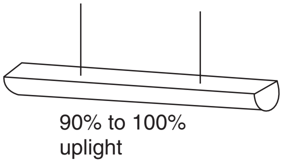


Figure 4.16 Indirect Scheme

Semi-Direct Lighting Scheme

In this scheme, the shades are used in such a way that about 60 per cent of the light is illuminated downwards and 40 per cent is projected upwards as shown in Figure 4.17. This scheme is efficient and it will reduce the chances of glare to the eye to a reasonable extent. The important characteristic of this system is that it provides a uniform distribution of the light which in turn increases the efficiency of the system.

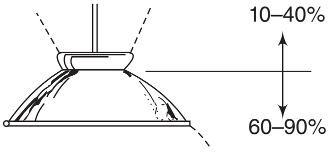


Figure 4.17 Semi-direct lighting scheme

Semi-Indirect Lighting Scheme

In this scheme shown in Figure 4.18, about 10 per cent–40 per cent light from the source is directly thrown downwards and about 60 per cent–90 per cent is projected upward. Therefore, the lights received by the object is due to diffused reflection and directly thrown. It does not possess the defect of indirect lighting scheme.

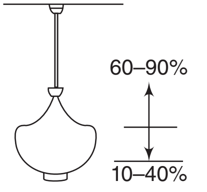


Figure 4.18 Semi-indirect lighting scheme

General Diffusing System

In this scheme, the shades are provided for illumination and it will produce equal distribution of lights in both upwards and downwards direction and in fact, it is an ideal scheme for illumination.

4.4.1 Design of Lighting Scheme

The lighting schemes arrangement should be designed in such a way that it must provide sufficient illumination, uniform distribution of light, avoid glare and shadow. For designing a lighting scheme, the following factors should be taken into consideration.

Space–Height Ratio

It is defined as the ratio of horizontal distance between lamps and the mounting height of the lamps as given by

$$\text{Space-height ratio} = \frac{\text{Horizontal distance between lamps}}{\text{Mounting height of the lamps}}$$

Practically, this ratio ranges between 1 and 2.

Utilisation Factor

It is the ratio of total lumens utilised on working planes to total lumens radiated by lamp as given by

$$\text{Utilisation factor} = \frac{\text{Total lumens utilised on working planes}}{\text{Total lumnes radiated by lamp}}$$

The value of this factor depends upon the following conditions:

(i) Illuminated area
(ii) Height at which the lamps are mounted
(iii) Colour of surrounding walls, ceiling and fittings
(iv) Type of lighting schemes (Direct or Indirect)

Utilisation factor values

Direct light = 0.25 to 0.5

Indirect light = 0.1 to 0.3

Depreciation Factor

It is the ratio of illumination under normal working condition to illumination when everything is clean. Its average value is 0.8.

$$\text{Thus, depreciation factor} = \frac{\text{Illumination under normal working condition}}{\text{Illumination when everything is clean}}$$

In case, if the value of depreciation factor is more than 1 and if it is 1.3 to 1.4, then

$$\text{Depreciation factor} = \frac{\text{Illumination when everything is clean}}{\text{Illumination under normal working condition}}$$

Waste Light Factor

When a surface is illuminated by a number of lamps, there will be a certain amount of wastage due to overlapping of light waves. It is called waste light factor. Its values for rectangular and irregular areas are 1.2 and 1.5 respectively.

Total Lumens Required

$$\text{The total gross lumens} = \frac{\text{Area (sq.metre)} \times \text{Illumination (metre} - \text{candle)} \times \text{waste light factor}}{\text{Coefficient of utilisation} \times \text{Depreciation (for values less than 1)}}$$

$$= \frac{\begin{array}{c}\text{Area (sq.metre)} \times \text{Illumination (metre} - \text{candle)} \times \\ \text{Depreciation factor (for values more than 1)} \times \text{waste light factor}\end{array}}{\text{Coefficient or utilisation} \times \text{Depreciation (for values less than 1)}}$$

4.5 Electric Lamps

Electric lamp is a device that produces visible light to provide lighting for buildings. For illumination, following types of electric lamps are employed.

(i) Incandescent lamp that produces light by a heated filament.

(ii) Arc lamp that produces light by means of an electric arc.

(iii) Electric discharge lamp that produces light by flow of electrons in a semiconductor.

Each of these lamps has been discussed in the following sections.

4.5.1 Incandescent Lamp

Incandescent lamp shown in Figure 4.19 is one of the oldest electric lighting technology. With efficacies ranging from 4 to 24 lumens per watt, incandescent lamps are the least energy-efficient electric light source and have a relatively short life (750–2500 hours). Light is produced by passing a current through a tungsten filament, causing it to become hot and glow. With use, the tungsten slowly evaporates, eventually causing the filament to break. These lamps are available in many shapes and finishes. The two most common types of shapes are the A-type lamp and the reflector-shaped lamp.

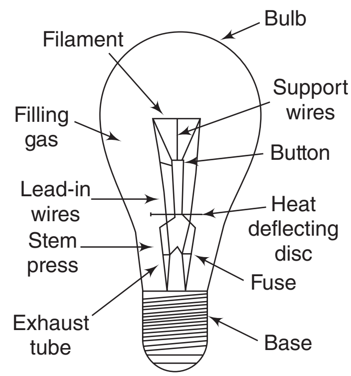


Figure 4.19 Incandescent lamp

The components of the incandescent lamp are an evacuated glass bulb and a fine metallic wire as shown in Figure 4.19. When this fine metallic wire is energised, electric current passes through which increases the temperature of the wire. At low temperature, heat energy is dissipated. When the temperature is high, heat as well as light energy will be emitted. The amount of emitted light energy depends on the temperature. The metal used as a filament in the incandescent lamp must have the following properties.

(i) It must have high melting point with low temperature co-efficient.

(ii) It must be ductile.

(iii) It must have low vapour pressure.

(iv) It must be withstand vibrations.

Tungsten is the most common material used as filament. Tantalum carbides are being added to tungsten to make it extremely hard and brittle with a melting point of about 3400° C.

4.5.2 Arc Lamps

These lamps are used for obtaining extreme brightness and have been employed for illuminating streets and roads. Following types of arc lamps are usually employed.

(i) Carbon arc lamp
(ii) Flame arc lamp
(iii) Magnetic arc lamp

4.5.3 Electric Discharge Lamps

These electric discharge lamps have a transparent enclosure that contains a gas at low pressure. The low pressure gas gets excited from the supply mains through the two electrodes. The excited atom of the gas emits light. The intensity of the light in the discharge lamps depends on the composition of the gas and its vapour pressure.

The gas discharge lamps have the following advantages compared to filament lamps.

1. The efficiency of the filament lamp depends on the temperature of the filament. But this temperature cannot be increased beyond the melting point of the filament. Also, only a portion of the input energy is converted into light energy and the remaining is wasted as heat energy. In gas discharge lamp, light energy is obtained from gas column which is excited electrically without heat.
2. Light energy with one particular wavelength is emitted in gas discharge lamp because the atoms are closely packed.

Each of the three types of electric lamps has its own merits and demerits. The factors to be considered while choosing an electric lamp are:

- **Luminous Efficacy:** It is ratio of lumen output from the lamp to the electrical power (in watt) input to the lamp. The required illuminance must be provided by the lamp in conjunction with the lighting economically.
- **Life Span**
- **Lumen Maintenance:** A parameter that measures the depreciation of brightness of the lamp.
- **Colour of an Electric Lamp:** Pleasant appearance and faithful colour reproduction of an object and its surroundings depend on the proper selection of colour of an electric lamp.
- **Auxiliary Equipment:** It is required along with the lamps. For example, the types of ballast used can affect lamp output, life, starting reliability, system efficiency and occupant comfort.

4.5.4 Fluorescent Lamp

A **fluorescent lamp** or a **fluorescent tube** or a **tube light** is a low weight mercury vapour lamp that uses fluorescence to deliver visible light. The normal luminous viability of fluorescent lighting frameworks is 50–100 lumens.

Tube light

It works on low pressure mercury vapour discharge phenomenon and converts ultraviolet ray into visible ray with the help of phosphor coated inside glass tube. As shown in Figure 4.20, the elements that are present inside the tube light are as follows:

(i) Filament coils as electrodes
(ii) Phosphor coated glass bulb

(iii) Mercury drop
(iv) Inert gases (argon)
(iv) Electrode shield
(vi) End cap
(vii) Glass stem

The tube light cannot be directly connected on power supply. It needs some auxiliary components to work. They are:

Ballast: All discharge lamps (fluorescent and high intensity discharge) require an auxiliary piece of equipment called ballast. It can be electromagnetic ballast or electronic ballast. Ballasts have three main functions.

(i) It provides correct starting voltage because lamps require a higher voltage to start than to operate.
(ii) It matches the line voltage to the operating voltage of the lamp.
(iii) It limits the lamp current to prevent immediate destruction because once the arc is struck the lamp impedance decreases.

Starter

Starters are required with some inductor type ballasts to start a fluorescent lamp. The starter connects both ends of the lamp together and preheats the lamp ends before lighting. The current flowing through the lamp causes the starter's contacts to heat and open, thus interrupting the flow of current.

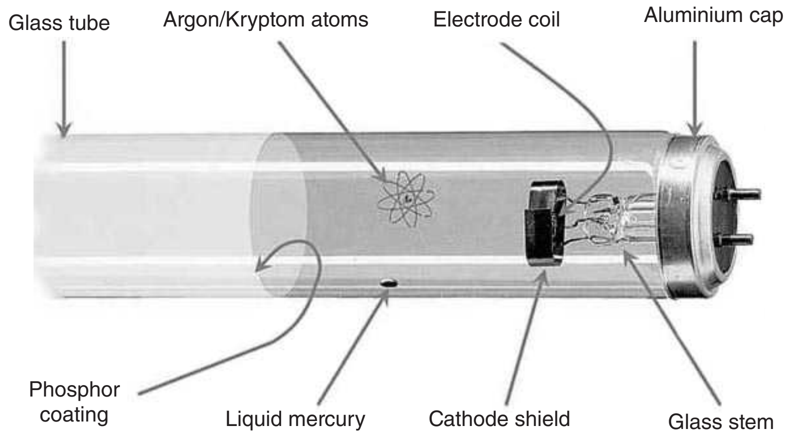


Figure 4.20 Components of a tube light

Principle and Working

When a sufficiently high voltage is applied across the electrodes, a strong electric field is set up. A small amount of current through the electrode filaments heats up the filament coil. As the filament is oxide coated, sufficient amount of electrons is produced and the electrons rush from negative electrode (cathode) to positive electrode (anode) due to this strong electric field.

During the movement of free electrons, a discharge process is established. The basic discharge process follows three steps:

(i) Free electrons are derived from the electrodes and they get accelerated by the electric field applied.
(ii) Kinetic energy of the free electrons is converted into the excitation energy of the gas atoms.
(iii) The excitation energy of the gas atoms gets converted into the radiation.

The circuit connection of fluorescent lamp is shown in Figure 4.21. When the switch is ON, current cannot pass through the tube light as the gas inside the tube is not ionised. For the gas to ionise, a high current is required across the filament of the fluorescent lamp. To provide this, starters are used. So, when the switch is ON, the circuit is closed through the ballast and the starter. The current flowing in the circuit heats up the filaments at the end of fluorescent tube. The starter acts as a time delay switch. Once the capacitor in the starter is charged fully, the starter acts as an open circuit. Therefore, the circuit is closed through the ballast, the fluorescent lamp and the switch. At that instant, preheating the ends of electrodes allows the stream of electrons to flow from cathode to anode and ionise the mercury vapour present inside the tube. The electrons flowing inside the tube collide with the gaseous mercury atoms. This collision results in the release of light photons with ultraviolet wavelength. This UV light is not visible by naked eyes. When these light photons hit the phosphor coating, the photon losses energy in the form of heat and emits the remaining energy as visible light. Without the starters, stream of electrons is never created between the two filaments, and without ballast, arc is never created between the filaments.

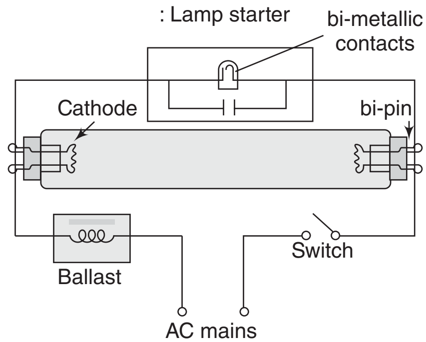


Figure 4.21 Circuit connection of fluorescent lamp

Types of Fluorescent Circuits

There are three main types of fluorescent circuits: (i) Rapid start, (ii) Instant start, and (iii) Preheat.

The ***rapid start*** circuit is the most commonly used circuit. Rapid start ballasts provide continuous lamp filament heating during lamp operation. Users notice a very short delay (less than 1/2 second) after flipping the switch before the lamp is started.

The ***instant start*** system ignites the arc within the lamp instantly. This ballast provides a higher starting voltage, which eliminates the need for a separate starting circuit. This higher starting voltage causes more wear on the filaments, resulting in reduced lamp life compared with rapid starting.

The ***preheat circuit*** technology is used except for low-wattage magnetic ballast applications such as compact fluorescents. A separate starting switch, called a starter, is used to aid in forming the arc. The filament needs some time to reach proper temperature and hence the lamp does not strike for a few seconds.

4.5.5 High Pressure Mercury Vapour Lamp (HPMV)

The general construction of a HPMV lamp is shown in Figure 4.22. The arc is contained in an inner bulb called the arc tube. The arc tube is filled with high purity mercury and argon gas. The arc tube is enclosed within the outer bulb, which is filled with nitrogen. The mercury lamps use a phosphor coating on the inner wall of the bulb to improve the colour rendering index. When voltage is applied between two main electrodes, it gets impressed across the auxiliary electrode and its adjacent main electrode resulting in a glow discharge between the two. Consequently this glow discharge sets up arc discharge between the main electrodes. Normally ballast is used to limit the current, and a capacitor is used to improve power factor of 0.5 to 0.9. The time taken for reaching the full discharge (to reach luminous flux) is 2 to 4 minutes. The light output obtained is about 50 lumens per watt. Clear mercury vapour lamps, which produce a blue-green light, consist of a mercury-vapour arc tube with tungsten electrodes at both ends. These lamps have the lowest efficacies of the high intensity discharge (HID) family, rapid lumen depreciation, and a low colour rendering index. Because of these characteristics, other HID sources have replaced mercury vapour lamps in many applications.

The *main advantage* of the HPMV lamps is that, it can withstand rating up to 2 KW with light output power of 1,00,000 lumens. In a colour corrected HPMV lamp, the outer glass envelope is coated with fluorescent powder to give additional red radiation to some amount. The composition of their light is 49 per cent of yellow 42 per cent of green and 7.5 per cent of red. As a result, colour rendition is considerably better.

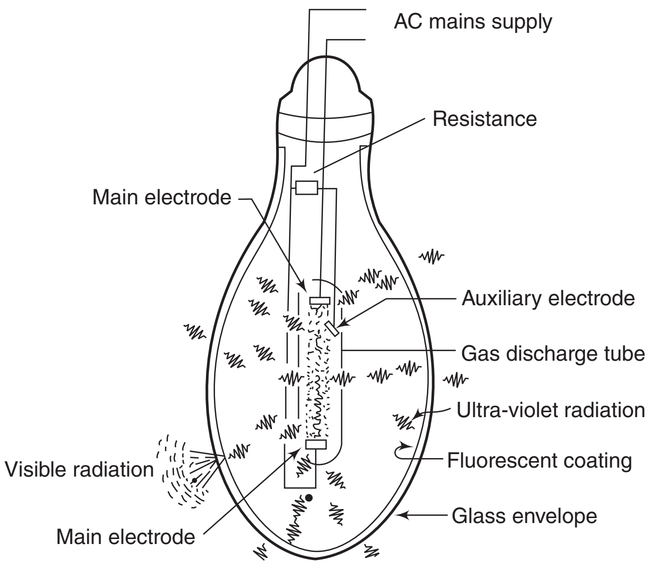


Figure 4.22 High Pressure Mercury Vapour Lamp

4.5.6 High Pressure Sodium (HPS) Vapour Lamp

When compared to mercury lamps, HPS lamps do not contain starting electrodes, and the ballast circuit includes a high-voltage electronic starter. The arc tube is made of a ceramic material which can withstand temperatures up to 2372°F. It is filled with xenon that helps start the arc, as well as a sodium-mercury gas mixture. Sodium, the major element used, produces the golden colour that is characteristic of HPS lamps. Although HPS lamps are not generally recommended for applications where colour rendering is critical, HPS colour rendering properties are being improved. Some HPS lamps are available in deluxe and white colours that provide higher colour temperature and improved colour rendition. The efficacy of low wattage white HPS lamps is lower than that of metal halide lamps (lumens per watt of low wattage metal halide is 40-50, while white HPS is less than 30 LPW). Figures 4.23 (a) and 4.24(b) show the construction and circuit connection of a typical single-ended, screwbase high-pressure sodium lamp.

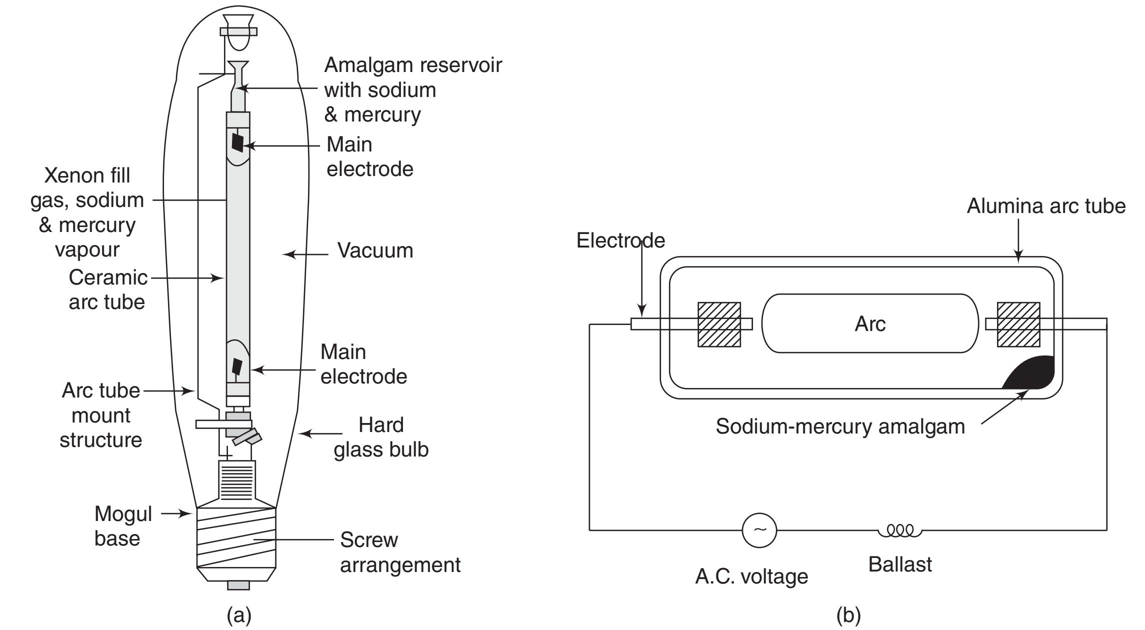


Figure 4.23 High pressure sodium vapour lamp (a) Structure (b) circuit connection

The advantages of HPS vapour lamps are:

(i) These lamps have high luminous efficacy and their life span is about 24000 hours. Its higher efficacy makes it a better choice than metal halide for these applications, especially when good colour rendering is not a priority.

(ii) They have excellent lumen maintenance capability.

(iii) The high pressure sodium (HPS) lamp is widely used for outdoor and industrial applications.

4.5.7 Low Pressure Sodium Vapour Lamp

Low pressure sodium (LPS) lamps are similar to fluorescent systems and are commonly included in the high intensity discharge (HID) family. LPS lamps are the most efficacious light sources, but they produce the poorest quality light of all the types of lamp. Being a monochromatic light source, all colours appear black, white, or shades of gray under an LPS source. LPS lamps are available in wattages ranging from 18–180. The use of LPS lamp has been limited to outdoor applications such as security or street lighting. Since the colour rendition is so poor, many municipalities do not allow them for roadway lighting. LPS are less effective in directing and controlling a light beam, compared with point sources like high pressure sodium and metal halide. Therefore, lower mounting heights will provide better results with LPS lamps. To compare a LPS installation with other alternatives, the installation efficacy is calculated as the average maintained foot candles divided by the input watts per square foot of illuminated area. The input wattage of an LPS system increases over time to maintain consistent light output over the lamp life. The low-pressure sodium lamp can explode if the sodium comes in contact with water.

Comparison of Sodium Vapour Lamp and Mercury Vapour Lamp

Table 4.2 list the difference between sodium vapour lamp and mercury vapour lamp.

Table 4.2 *Sodium Vapour Lamp vs Mercury Vapour Lamp*

Characteristics	Sodium Vapour Lamps	Mercury Vapour Lamp
Light source	Works by electric discharge (passage of electricity through sodium vapours at high/low pressure)	Works through the combined effect of electric discharge through mercury vapours and fluorescence from phosphors
Process of lighting	Filaments of the lamp sputter fast moving electrons, which hit the sodium atoms (vapour) causing the valence electrons of the sodium atoms to excite to higher energy levels.	The mechanism in mercury vapour lamp is more involved and sequential. The sputtered electrons from the filaments, after having been accelerated by high voltage, hit the mercury atoms.
Emitting characteristics	Monochromatic bright yellow light	Ultraviolet light. (Visible white light)
Application	Sodium vapour lamps produce much higher light output they cannot be used in lighting applications.	Mercury vapour lamp is quite suitable for lighting applications.
Colour-rendering property	Very crucial.	Not very crucial.
Visible region/spectrum	Most of the light emitted from a sodium vapour lamp is concentrated in the yellow part of the visible spectrum.	The mercury vapour lamp can feed almost the entire visible region of the human visual system.

4.6 Refrigeration

The term 'refrigeration' may be defined as the process of removing heat from a low temperature region to a high temperature region. This principle is based on the second law of thermodynamics. The second law of thermodynamics by Clausius states that heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time. From this, it is understood that for removing the heat from low temperature region to high temperature region, external work has to be done. This external work is done by compressor or condenser. The machine which performs the refrigeration process is called refrigerator. Refrigerant is a substance used as a working fluid in refrigeration process that removes the heat from a low temperature region. It has low boiling point and vapourises at low temperature and takes away the heat. The desirable properties of refrigerant are non-corrosive, non-toxic, non-flammable and free from objectionable odour. Commonly used refrigerants are Freon, ammonia, methylchloride, sulpur-dioxide and carbon dioxide. Among these refrigerants, Freons are being phased out because of their deleterious effect on ozone layer in the environment. Now-a-days, ammonia refrigerant is most widely used for commercial and industrial refrigeration due to its high efficiency, low cost and availability.

4.6.1 Main Components of a Refrigeration System

All refrigeration and air-conditioning systems have five basic parts: (i) throttling or expansion valve, (ii) the evaporator, (iii) thermostat, (iv) the compressor and (v) condenser. The refrigerant from the evaporator is compressed in to high pressure and high temperature gaseous state from its low pressure and low temperature state by the compressor. The size and power rating of the compressor depends on the size of the region in which refrigeration is required. Large size of the region needs large size and high power rating compressor. Figure 4.24 represents the main components of a vapour compression refrigeration system.

Thermostatic Expansion Valve (TXV)

The low pressure and low temperature liquid refrigerant enters the evaporator. The amount of refrigerant that flows to the evaporator is controlled by TXV. This TXV device also helps separate the high pressure and the low pressure sides of an air conditioning system. Through the system's liquid line, high pressured liquid refrigerant enters the valve but with the TVX's presence, the amount of liquid refrigerant entering the evaporator will be reduced.

Thermostatic expansion valves are used widely in medium and large sized refrigerating and air-conditioning systems.

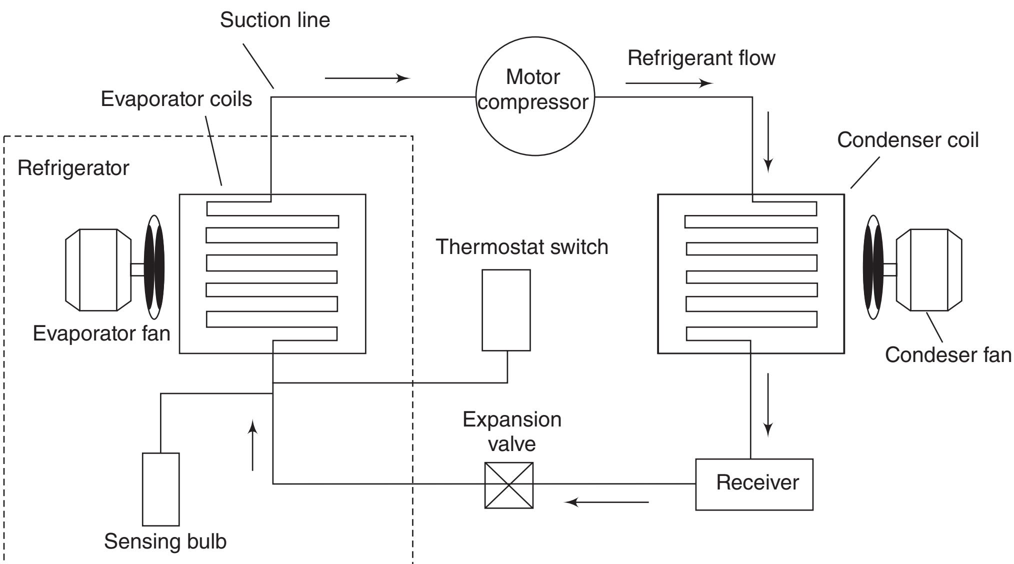


Figure 4.24 Main components of a refrigeration system

Evaporator

The refrigerant undergoes various changes throughout the vapour compression cycle and it is in the evaporator where it actually produces the cooling effect. The purpose of the evaporator is to remove the unwanted heat through liquid refrigerants. It is usually a closed insulated space where the refrigerant absorbs heat from the region to be cooled. Several types of evaporators are available. Evaporators used for industrial refrigeration and air-conditioning purposes are very large and also called chillers. They are usually made in the form of shell and tube types with two possible arrangements: namely, dry expansion evaporators and flooded evaporators.

Thermostat

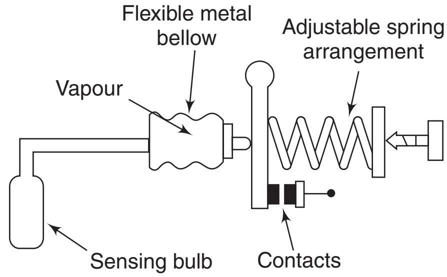


Figure 4.25 Thermostat

A temperature-sensitive control device is used in all refrigeration system. In order to maintain the temperature within predetermined condition, irrespective of the load on the system, is called thermostat. It is classified into two types, namely:
(i) Temperature-sensitive type and (ii) pressure-sensitive type

Figure 4.25 shows the temperature-sensitive thermostat arrangements.

It consists of a tube having sensing bulb at one end and flexible metal bellow at the other end. The sensing bulb is filled with a liquid partially and it is fixed to the evaporator. The temperature of the liquid in the bulb determines vapour pressure in the bellow. The bellow exerts pressure on a lever against the adjustable spring. When the evaporator temperature falls to the set value, the vapour pressure falls in the metal bellow and this will open the contacts of the thermostat. Similarly, when the vapour pressure rises with the temperature, contacts of the thermostat will re-close.

In pressure-sensitive type, the metal bellow is actuated directly by the suction pressure in the evaporator as the pressure and temperature are directly related.

Capacity Control Systems

The capacity control system is used to regulate the power and energy consumption, although it can also manage dehumidification or decrease compressor cycling. The ON or OFF cycling of the compressor is the simplest form of capacity control.

Compressor

A compressor converts low temperature to high temperature, which can be the cause of an increase in pressure. Heat is easily released through a compressor. There are two types of compressors used commonly in the refrigerating and air-conditioning unit, as discussed below:

(i) **Reciprocating Compressor:** The working of reciprocating compressor is similar to the reciprocating engine. The difference is that while the engine generates power, the compressor consumes power and compresses the refrigerant. The reciprocating compressor is comprised of the piston and the cylinder arrangement connected by the connecting rod to the motor shaft. When the shaft of the motor rotates, the piston performs the reciprocating motion inside the cylinder, absorbing and compressing the refrigerant. These compressors can be used for small as well as large refrigerating and air-conditioning units. It consumes more power when compared to rotary compressors and also it makes more noise.

(ii) **Rotary Compressors:** In this compressor, the compression of the refrigerant is achieved by the rotary motion of the rotors instead of the reciprocating motion of the piston. There are two commonly used types of rotary compressors: (i) Rolling piston type and (ii) Rolling vane type.

Condenser

The condenser is used to extract heat from the refrigerant. The refrigerant leaving the compressor is in the gaseous state with a high pressure and temperature. The refrigerant then enters the condenser where it loses the heat to the coolant air or water. It comes out in a partially liquid and gaseous state and then enters the throttling or expansion valve. The temperature of condensation should range from around –12°C to –1°C. Based on cooling medium, the condensers are classified as: air cooled, water cooled and evaporative.

Air Cooled Condensers: It is used where the cooling load is small and total quantity of the refrigeration cycle is small. For example, household refrigerators, water coolers, air-conditioners, etc.

Water Cooled Condensers: It is used where cooling loads are excessively high and a large quantity of refrigerant flows through the condenser. Examples are large refrigerating plants, central air-conditioning plants, etc.

Evaporative Condensers: Evaporative condenser is a combination of water cooled and air cooled condensers and it is usually used in ice plants.

Receiver

It is used as a temporary storage and surge tank for liquid refrigerant. The receiver acts as a vapour seal. Its purpose is to preserve the vapour moving down the liquid line. It is made for both horizontal and vertical installations.

The applications and the purpose of refrigeration are listed in the Table 4.3.

Table 4.3 *Applications and Purpose of Refrigeration*

S. No.	Application	Purpose
1	Chillers	To provide comfort cooling throughout a building or few location
2	Cold Storage Warehouses	Cold storage warehouses store meat, produce dairy products and other perishable goods
3	Commercial Ice Machines	To produce ice for consumer use
4	Household Refrigerators and Freezers	For residential use
5	Industrial Process Air Conditioning	To provide comfort cooling for operators and protect process equipment
6	Motor Vehicle Air Conditioning	To provide comfort cooling for passengers in cars, buses, trains and flight
7	Residential and Light Commercial Air Conditioning	To provide comfort cooling in residence and small commercial shops
8	Vending Machines	Vending machines are self-contained units that dispense goods that must be kept cold or frozen.
9	Water Coolers	Water coolers are self-contained units providing chilled water, and possibly heated water, for drinking.

4.7 Air Conditioning System

Air-conditioning is the process of changing the properties of air like temperature, humidity, pressure and its composites to desired conditions. The aim of air conditioning system is to maintain the properties of the air at a desired value by a program to improve the air quality and thermal comfort in the space considered. Air conditioners and refrigerators work in the same way where the refrigerator cools the small space inside it whereas air conditioner (AC) cools a room or entire house or commercial space. Air conditioners use refrigerant that can be easily converted from a gaseous state to a liquid state and can be reframed. This refrigerant is used to evacuate heat from the air inside of a house.

4.7.1 Classification of Air Condition System

The classification of air conditioning systems based on different parameters is as follows:

1. Based on the type of application

(i) **Air Conditioning System for Comfort:** It is used in homes, offices, stores, restaurants, theatres, schools to offer comfort to humans.

(ii) **Air Conditioning System for Industries:** It is used in industries which are required for manufacturing processes like candy industry, paper mills, textile mills, printing industries, control rooms of substations, etc.

2. Based on the Season of the Year

(i) **Winter Air Conditioning System:** It is used to produce warm air and maintain room temperature during winter season.

(ii) **Summer Air Conditioning System:** It is used to maintain the room temperature less than the atmospheric temperature during summer season.

(iii) **All Season Air Conditioning System:** This system includes the arrangements for both heating and cooling with suitable automatic controls.

2. Based on the Arrangement of the Equipment

(i) **Central Station System:** In this system, the various necessary components of air conditioning system are assembled in a central room. The air is circulated throughout the building through sheet metal ducts either naturally or by a fan.

(ii) **Unitary System or Compact Air Conditioning Set:** In this system, the compact air conditioning sets are used. A single unit of larger size can be used for many rooms whereas low capacity units are used for single room only.

Advantages

(a) Flexibility of operation
(b) No need of laying duct
(c) Initial cost required is low

(iii) **Combined System:** In this system, the features of both the central station and unitary system are employed.

4.7.2 Main Components of an Air Conditioning System

The air conditioning system consists of three main parts: (i) Compressor, (ii) Condenser and (iii) Evaporator. The compressor and condenser are usually installed outside the house whereas the evaporator is located inside the house. An air conditioning system performs the following functions:

(i) Supply the air at desired temperature (cool or hot) as fixed by the operator.
(ii) Condition the supply air and attenuate any objectionable noise produced by the machine.
(iii) Control and maintain the indoor environmental parameters such as temperature, humidity and pressure.

The functional block diagram of basic air conditioning unit is shown in Figure 4.26.

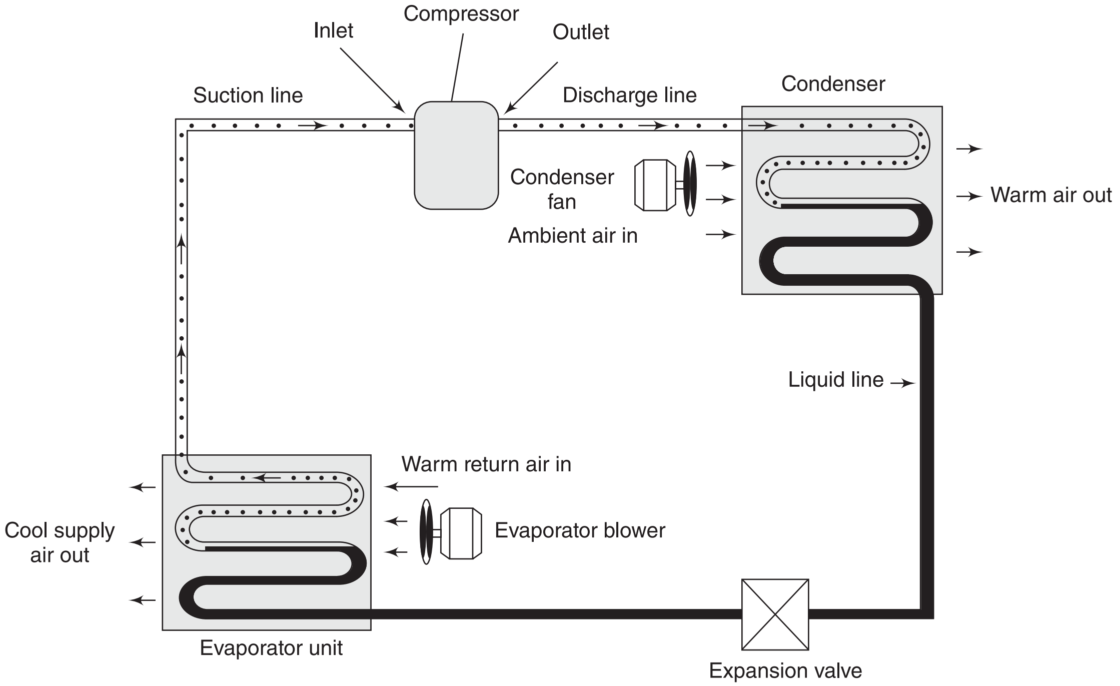


Figure 4.26 Basic air conditioning system

Compressor

The compressor is the heart of the system which assists the condenser where the refrigerant gas is pressurised to the specific level at a specific rate of flow. The compressor takes refrigerant vapour from the low pressure side of the circuit, and discharges it at a much higher pressure into the high side of the circuit.

Condenser

The condenser is a device used to condense the pressurised refrigerant from the compressor to a liquid state. The condenser uses heat transfer process in which heat will always move from a warmer to cooler substance. Most air cooled air conditioning and refrigeration systems are designed such that the refrigerant will condense at a temperature about 25° to 30° above outside ambient air temperature.

Evaporator

The evaporator is generally fitted inside the house which is paired with a fan that blows the cooled air. The evaporator receives the liquid refrigerant and converts it to gas with a drop in pressure. The air condition system is designed so that the refrigerant will evaporate in the evaporator at a temperature of about 40°, so that it will be cool compared to the warm air flowing over it.

Expansion Valve

The expansion valve is located between the evaporator and condenser coils. It is responsible for reducing pressure of the liquid refrigerant and allows the conversion of gas that occurs in the evaporator.

4.7.3 Central Air Conditioner Unit

Central air conditioner unit is designed to cool or heat the entire house which removes heat from one area, where it is undesirable, to an area where it is less significant. Central air conditioner has a centralised duct system. The duct system (air distribution system) has an air handler, air supply system, air return duct, grilles and register that circulates warm air from a furnace or cooled air from central air conditioning units to the room.

Working

The functional block diagram of central air conditioning unit is shown in Figure 4.27. It consists of compressor, condenser, fans and the blower. The compressor is located in outdoor which pulls in low pressure, low temperature refrigerant from the evaporator and compresses that gas to high pressure, high temperature. The condenser is a square (or round) metal box located in outdoor which receives the high pressure, high temperature vapour refrigerant from the compressor and rejects that heat to the surrounding air. As a result of condensing, the refrigerant is converted into liquid state. The evaporator located within the air handler or furnace is responsible for absorbing heat from places wherever cooling is required. The refrigeration cycle consists of the compressor, the condenser, the metering device and the evaporator with refrigerant. The expansion valve of air conditioner (meter devices) in the air handler or furnace is responsible for providing the correct amount of refrigerant to the evaporator coil.

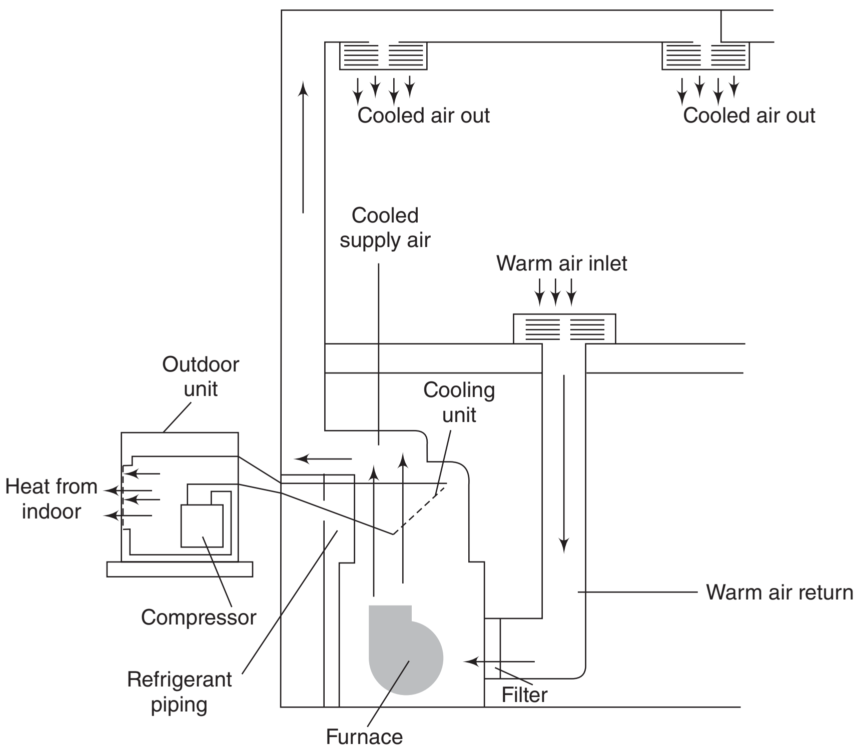


Figure 4.27 Functional block diagram of central air conditioning unit

4.7.4 Window Air Conditioner

Window air conditioner is referred to as room air conditioner. It is the simplest form of an air conditioning system and is mounted on windows or walls. It is a single unit assembled in a casing where all the components are located. This refrigeration unit has a double shaft fan motor with fans mounted on both sides of the motor. One is at the evaporator side and the other is at the condenser side. The evaporator is located facing the room for cooling the space. The condenser facing is located outdoor for heat rejection. The side view representation of window air conditioner is shown in Figure 4.28.

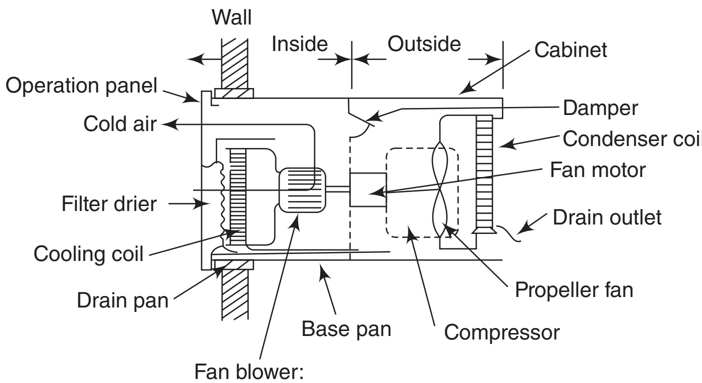


Figure 4.28 Side view representation of window air conditioner

Indoor Side Components

The indoor parts of a window air conditioner include:

(i) **Cooling Coil:** It is a coil where the heat exchange happens between the refrigerant in the system and the air in the room.

(ii) **Fan Blower:** It is a centrifugal evaporator blower to discharge the cool air to the room.

(iii) **Capillary Tube:** It is used as an expansion device. It can be noisy during operation if installed too near the evaporator.

(iv) **Operation Panel:** It is used to control the temperature and speed of the blower fan. A thermostat is used to sense the return air temperature and another one to monitor the temperature of the coil.

(v) **Filter Drier:** It is used to remove the moisture from the refrigerant.

(vi) **Drain Pan:** It is used to collect the water that condensates from the cooling coil and is discharged out to the outdoor by gravity.

Outdoor Side Components

The outdoor side parts include:

(i) **Compressor:** It is used to compress the refrigerant.

(ii) **Condenser Coil**: It is used to reject heat from the refrigeration to the outside air.

(iii) **Propeller Fan:** It is used in air-cooled condenser to help move the air molecules over the surface of the condensing coil.

(iv) **Fan Motor:** It has a double shaft where the indoor blower and outdoor propeller fan are connected together.

Working of Window Air Conditioner (AC)

The working of window air conditioner can be explained by separately considering the two cycles of air: (i) Room air cycle and (ii) Hot air cycle. The compartments of the room and hot air are separated by an insulated partition located inside the body of the air conditioner.

Room Air Cycle

The air moving inside the room and in the front part of the air conditioner where the cooling coil is located is considered to be the room air. When the window AC is switched on, the blower starts immediately and after a few seconds the compressor also starts. The evaporator coil or the cooling gets cooled as soon as the compressor is started. The blower behind the cooling coil starts sucking the room air, which is at high temperature and also carries the dirt and dust particles. On its path towards the blower, the room air first passes through the filter where the dirt and dust particles get removed. The air then passes over the cooling coil.

Since the temperature of the cooling coil is much lesser than the room air, the refrigerant inside the cooling coil absorbs the heat from the air. Due to this, the temperature of the room air becomes very low. Hence, the air becomes chilled.

Due to the reduction in temperature of the air, some dew is formed on the surface of the cooling coil. This is because the temperature of the cooling coil is lower than the dew point temperature of the air. Thus, the moisture from the air is removed so the relative humidity of the air reduces. This low temperature and low humidity air is sucked by the blower and blows at high pressure. This chilled air then passes through small duct inside the air conditioner and then thrown outside the air conditioner through the opening in the front panel or the grill. This chilled air then enters the room and cools the room thereby maintaining low temperature and low humidity inside the room.

Hot Air Cycle

The hot air cycle includes the atmospheric air used for cooling the condenser. The condenser of the window air conditioner is exposed to the external atmosphere. The propeller fan located behind the condenser sucks the atmospheric air at high temperature and blows over the condenser. The refrigerant inside the condenser is at very high temperature and it has to be cooled to produce the desired cooling effect. When the atmospheric air passes over the condenser, it absorbs the heat from the refrigerant and its temperature increases. The atmospheric air is already at high temperature and after absorbing the condenser heat, its temperature becomes even higher. The person standing behind the condenser of the window AC can clearly feel the heat of this hot air. Since the temperature of this air is very high, this is called as hot air cycle. The refrigerant after getting cooled enters the expansion valve and then the evaporator. On the other hand, the hot mixes with the atmosphere and then the fresh atmospheric air is absorbed by the propeller fan and blown over the condenser. This cycle of the hot air continues.

4.7.5 Split Systems

Split systems are made up of an outdoor unit and an indoor unit. The outdoor unit contains the condenser and compressor and an indoor unit is connected to a furnace or heat pump.

An outdoor compressor is heart of the system that circulates the refrigerant in a closed loop between the condenser and evaporator coils. The majority of homes and small commercial air conditioning systems circulate a compressed gas refrigerant, widely recognised as Freon, in a closed "split" system to cool and condition the inside air. The term "split" simply means that components are divided into inside and outside portions as opposed to being located together in a "window" unit.

An internal blower circulates the conditioned air through ducts to the rooms where the cool air is needed. The air ducts generally run either below the ceiling and inside the rooms or in the unconditioned air. An outside fan pulls air across the external parts of the system to cool and condense the refrigerant. The refrigerants, including Freon and Puron may never need recharging through the life of AC. The schematic diagram of split air conditioner is shown in Figure 4.29.

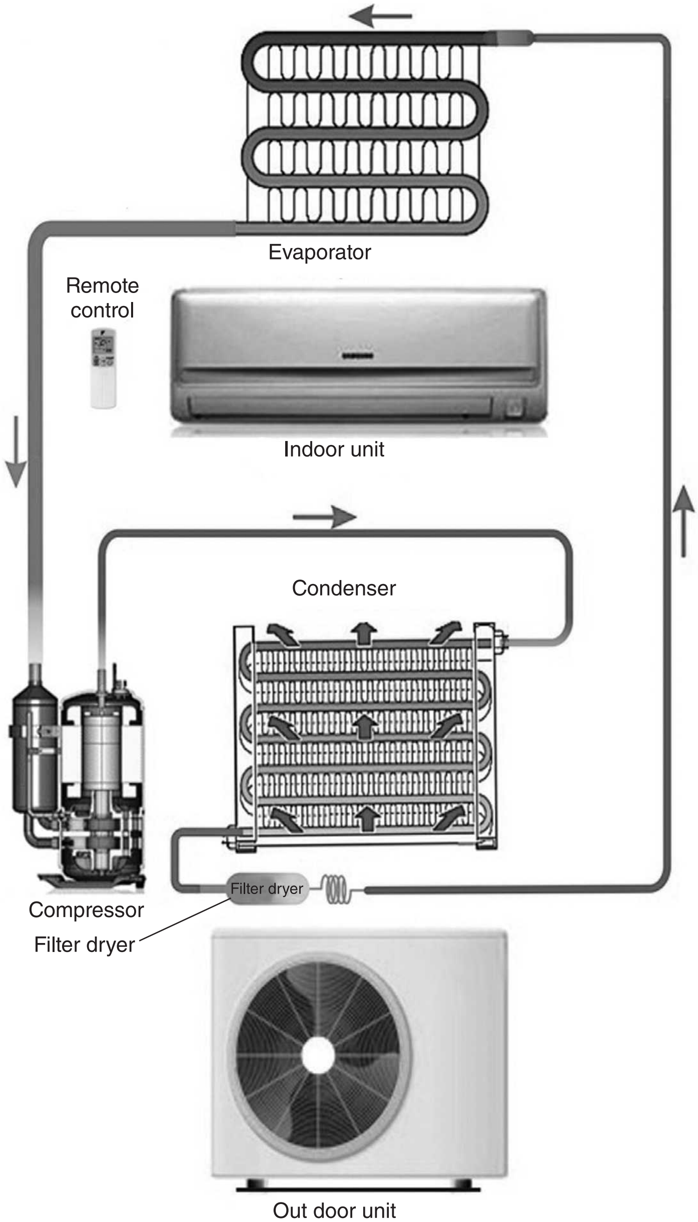


Figure 4.29 Schematic diagram of split air conditioner

Coolant in the systems travels through four components:

(i) An expansion valve to reduce the pressure of liquid coolant.
(ii) An evaporation coil to absorb heat from surrounding air.
(iii) A compressor to pressurise the gas.
(iv) A condenser coil in which the hot air becomes liquid, releasing its heat to the outside air.

Evaporator Coils are indoors and consist of a network of tubes filled with refrigerant that remove heat and moisture from the air as the refrigerant evaporates into a gas again.

Condenser Coils are outdoors and consist of a network of tubes filled with refrigerant that remove heat from the heated gas refrigerant and convert the refrigerant into a liquid form again. The excess heat escapes into the outside air.

4.7.6 Air Cleaning

Air cleaning can be carried out by the following filter types:

(i) Dry Unit Type Filters

In this type of filter, air is cleaned by straining action in passing through cloth or woven wire mesh. Proper maintenance is needed for this type of filters as they may clog and restrict air circulation. Therefore, plant operation is adversely affected.

(ii) In Viscous Filter

In this type of filter, the filtering medium is covered with an adhesive liquid in which the dust particles stay. It is also needs periodic maintenance.

4.8 Battery

A device that converts the stored chemical energy into electrical energy using chemical action is called battery. The chemical action that takes place in the battery is the movement of electrons from one terminal to another. Due to this chemical action, there exists a difference in charge between two terminals that creates an electrical energy between them. A cell is a device that consists of two electrodes and an electrolyte. But battery is a single unit which comprises of two or more cells which are connected together electrically. In day-to-day activities, battery is used as an energy source in many residential and industrial applications.

Construction

The component of a battery that participates actively in a chemical reaction to generate electrical energy is called the active component. The three main active components of a battery are:

(i) **Anode:** The electrode that oxidises and release electrons when an electrochemical reaction occurs is called anode. Since it releases electrons, it is also called negative electrode or reducing electrode. The common materials used as anode are zinc and lithium.
(ii) **Cathode:** The electrode that acquires electrons during electrochemical reaction is called cathode. Since it acquires electrons, it is also called positive electrode or oxidising electrode. The common materials used as cathode are metallic oxides. The anode and cathode materials are chosen such that the combination is light weight with high voltage and capacity.

(iii) **Electrolyte:** The medium through which electrons get transferred from anode to cathode is called electrolyte. In general, electrolytes are in liquid form like water or other solvents in which the material required for ionic conduction, i.e., salt, acid, or alkalis are dissolved. It is noted that solid electrolytes also exist in some conventional batteries. The schematic diagram representing the components of a battery is shown in Figure 4.30.

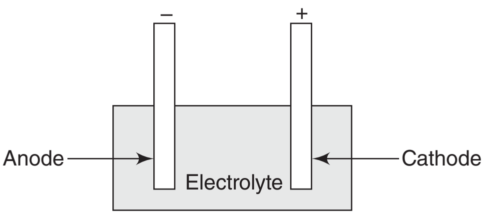


Figure 4.30 Components of a battery

Working

When a load is connected between the cathode and anode, due to electrochemical action, the electrons get transferred from anode and cathode. Due to this movement of electrons, the current starts flowing from cathode to anode through the connected load. The diagram explaining the working of a battery is shown in Figure 4.31.

Advantages of Batteries over Other Energy Sources

The advantages of using batteries as energy sources are:

(i) Energy can be stored for a long duration of time.
(ii) Delivers the energy effectively when compared to fossil fuels.
(iii) Response time of the battery is less when compared to other fossil fuels.
(iv) Since battery can handle both light and heavy loads effectively, it has wide power bandwidth.
(v) Operation of battery is environment friendly, i.e., it reduces the pollution in the environment.
(vi) Efficiency of the battery is high.
(vii) Battery can be operated at any place as it offers good tolerance to shock and vibrations.
(viii) Operating cost of the battery is cheap.
(ix) Low maintenance cost is required for the battery.

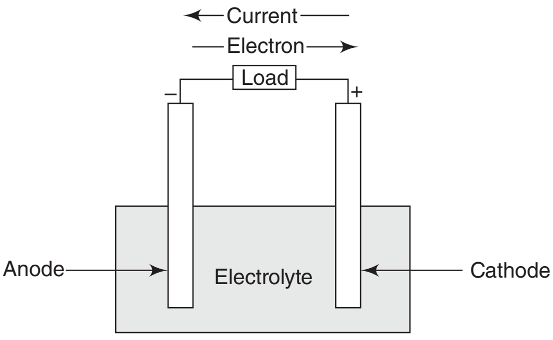


Figure 4.31 Working of a battery

4.8.1 Classification of Batteries

The two main categories of batteries are: (i) Primary batteries and (ii) Secondary batteries.

(i) Primary Batteries

It is also called single-use, or throw-away batteries as it cannot be recharged to reuse. It is discarded after complete depletion of charge in it. Generally, electrolytes inside the primary battery get utilised completely. Hence, these batteries are also called as dry cells. Examples of primary batteries are alkaline batteries, mercury batteries, silver-oxide batteries, and zinc carbon batteries.

(ii) Secondary Batteries

The batteries that can be electrically recharged again are called secondary batteries. By allowing the current in the opposite direction, these batteries can be recharged. Nickel Cadmium, Lead-Acid batteries and Lithium batteries fall into the secondary battery category. The comparison between primary and secondary batteries is listed in Table 4.4.

Table 4.4 *Primary battery versus Secondary battery*

Primary Battery	Secondary Battery
Initial cost is less.	Initial cost is high.
Cost per kWh is high.	Cost per kWh is less.
As these batteries are disposable, there is no requirement of maintenance.	As these batteries are rechargeable, regular maintenance is required.
Most suited for portable application since it is smaller and light weight in nature.	Less suited for portable applications.
Has good charge maintenance.	Has poor charge maintenance.
Not suitable for heavy load application since the discharge rate is poor.	Suitable for heavy load applications due to its superior discharge rate.
In general, these batteries are limited to specific applications.	Due to its inherent versatility, these batteries are used in most of the applications.
Examples: Alkaline batteries, mercury batteries, silver-oxide batteries, zinc carbon batteries etc.	Examples: Nickel Cadmium, Lead-Acid batteries , Lithium batteries etc.

4.8.2 Lead Acid Battery

Lead acid battery is the most commonly used secondary battery. It uses sponge lead and lead peroxide for the generation of electrical energy from chemical reaction. It is also denoted as Pb-Acid battery. This battery is used in the applications that demands high currents and storage of energy. A single lead acid cell is capable of producing 2.1 V. If the application demand higher voltages, more lead acid cell can be connected electrically. In general, a single Pb-Acid battery can consist of 3, 6 or 12 lead acid cells.

The active components of lead acid battery are:

(i) Cathode: Lead peroxide (PbO_2) and it is dark chocolate brown in colour when fully charged.
(ii) Anode: Sponge lead and is grey in colour when it is fully charged
(iii) Electrolyte: Dilute sulphuric acid, H_2SO_4, that contains 31 per cent of concentrated H_2SO_4.

Chemical Action in Pb-Acid Battery

Electrical energy is supplied to the load when it is connected to Pb-acid battery and the battery gets charged when it is connected to DC supply. The chemical action taking place in battery during charging and discharging are given below:

During Discharging

When Pb-acid battery supplies current, the electrolyte, i.e., H_2SO_4 gets splits into Hydrogen ions ($2H^+$) and Sulphate ions (SO_4^-).

At Anode:

$$Pb + SO_4 \rightarrow PbSO_4$$

At Cathode:

$$PbO_2 + 2H + H_2SO_4 \rightarrow PbSO_4 + 2H_2$$

The changes that take place during discharging of energy are:
(i) Cathode is covered with white colour $PbSO_4$.
(ii) Similarly, anode is covered with $PbSO_4$ and converts grey colour plate to white colour.
(iii) Concentration of electrolyte decreases due to formation of water.
(iv) Output voltage of a cell falls to 1.8 V at no load condition.
(v) Electrical energy is produced from chemical energy.

During Charging

Here, a DC supply voltage higher than battery voltage is connected to the electrodes of the battery such that positive electrode gets connected to positive terminal.
At anode:

$$PbSO_4 + 2H \rightarrow Pb + H_2SO_4$$

At cathode:

$$PbSO_4 + SO_4 + 2H_2O \rightarrow PbO_2 + 2H_2SO_4$$

The changes that take place during charging of energy are:
(i) Cathode gets converted into PbO_2 and makes it dark chocolate brown colour.
(ii) Similarly, anode changes into grey colour spongy lead.
(iii) Concentration of electrolyte increases.
(iv) Output voltage of a cell rises to 2.1 V at no load condition.
(v) Chemical energy is produced from electrical energy.

Advantages, Disadvantages and Applications of Pb-Acid Battery

The advantages of Pb-Acid Battery are:
(i) Efficiency of the battery is high, i.e., nearly 80 per cent.
(ii) Provides good service for long duration.
(iii) Number of times the battery can be recharged is 300 to 1500.
(iv) Self-discharging of the battery is low.
(v) It is environmental friendly.
(vi) Possesses good safety characteristics.
(vii) Cost of the battery is less.

The disadvantages of Pb-Acid Battery are:
(i) Effectiveness of the battery gets reduced at low temperature.
(ii) Due to overcharging, corrosion of battery occurs.
(iii) Water content in the electrolyte should be checked as it gets evaporated during the operation.
(iv) It is not possible to keep it in ideal position for long duration.

The major applications of Pb-Acid Battery are:
(i) Used in automobile applications for starting of internal combustion engines.
(ii) Used in emergency lighting and security alarm systems.
(iii) Used in heavy duty loads like trains, lift, truck, etc.
(iv) Used as an energy source in submarines.

4.8.3 Nickel Cadmium Battery

A secondary battery made of nickel and cadmium is called Nickel Cadmium battery and is denoted as Ni-Cd battery. When compared to other batteries, this battery with reasonable capacity offers good performance at low temperature with good duration of life. The most important characteristic of Ni-Cd battery is that it delivers all its stored capacity with high discharge rates. The active components of Ni-Cd battery are: (i) Anode: Cadmium, Cd (ii) Cathode: Nickel hydroxide, $NiOH_2$ and (iii) Electrolyte: Alkaline Potassium hydroxide, KOH.

In general, the number of cathode electrodes is one greater than the anode electrodes. The chemical action taking place in Ni-Cd battery during charging and discharging process is given as follows:

During Discharge

During the battery operation, the electrolyte KOH breaks into K and OH ions. Hence, the following action takes place at anode and cathode when the battery is connected to the load.
At Anode:

$$Cd + 2OH \rightarrow Cd(OH)_2$$

At Cathode:

$$NiO(OH) + H_2O \rightarrow Ni(OH)_2 + OH$$

During Charging

When the battery is connected to DC supply for charging purpose, the following action takes place at anode and cathode.
At Anode:

$$Cd(OH)_2 + 2K \rightarrow Cd + 2KOH$$

At Cathode:

$$Ni(OH)_2 + OH \rightarrow NiO(OH) + H_2O$$

Advantages, Disadvantages and Applications of Ni-Cd Battery

The advantages of Ni-Cd Battery are:

(i) It can be recharged many times.
(ii) During discharge, it maintains voltage at a constant level.
(iii) During charging and discharging operation, composition of electrolyte does not change.
(iv) It can be kept in an ideal position for long duration of time.
(v) At low temperature, the performance of the battery is good.
(vi) It is possible to produce large instantaneous current in the range of 1000-8000 A /s.
(vii) It is available in different configurations like button, cylindrical and rectangular.

The disadvantages of Ni-Cd battery are:

(i) Due to high toxicity level of cadmium, it creates environmental pollution.
(ii) Since cadmium is a heavy metal, the weight of the batteries is high.
(iii) Since cadmium metal is very costly, the initial cost of Ni-Cd battery is high.
(iv) The electrolyte used is in this battery is a corrosive hazardous chemical.

The major applications of Ni-Cd battery are:

(i) Used in flash lights, photoflash units and portable electronic equipment.
(ii) Used in emergency lighting and alarm systems.
(iii) Used in air-crafts, space satellite systems.
(iv) Used to start large diesel engines, gas turbines, etc.

4.8.4 Lithium ion Battery

The secondary battery that plays a major role in electric vehicles is Lithium-ion battery or Li-ion battery. The main characteristics of Li-ion battery are that it possesses higher charge and discharge efficiency, and its energy density is high. The active components of Li-ion battery are: (i) Anode: Lithiated carbon, (ii) Cathode: Lithium metal oxide, $LiMO_x$ where M is any metal, and (iii) Electrolyte: Non-aqueous electrolyte like Ethylene carbonate or Diethyl carbonate

Based on the metal, the Li-ion battery is classified as:

(i) Lithium Cobalt oxide battery
(ii) Lithium Manganese oxide battery
(iii) Lithium Nickel Manganese battery
(iv) Lithium Iron Phosphate battery
(v) Lithium Nickel Cobalt Aluminium Oxide battery
(vi) Lithium Titanate battery

The most commonly used material for negative electrode is Graphite and Lithium cobalt oxide. Lithium ion phosphate or Lithium manganese oxide is used as positive electrode. During discharging, this battery makes use of Li-ions obtained from negative electrode as charge carriers to move it to positive electrode. During charging, when an external supply is given to the battery, Li-ions move back to negative electrode. The movement of Li-ions between positive and negative electrodes takes place through the non-aqueous electrolyte. The charging and discharging equations of Li-ion battery depends on the type of material used in cathode.

Advantages, Disadvantages and Applications of Li-ion Battery

The advantages of Li-ion Battery are:

(i) Weight of the battery is less when compared to other batteries.
(ii) Li -ion battery is available in different shapes.
(iii) The open circuit voltage is high and it makes to discharge the power at less current.
(iv) It possesses very low self-discharge rate, i.e., 5–10 per cent per month.
(v) This battery does not pollute the environment i.e., it is eco-friendly.

The disadvantages of Li-ion Battery are:

(i) Flow of charge inside the battery gets affected due to deposition of ions.
(ii) The internal resistance of the battery gets increases gradually and hence, the output decreases.
(iii) Due to over-charging and high temperature, capacity of the battery decreases.
(iv) It suffers from thermal run away and corrosion due to overheating.
(v) It cannot be used to charge the normal charges.

The major applications of Ni-Cd Battery are:

(i) Used in Laptop computers and advanced cellular phones.
(ii) Used in military equipment like mine detectors, satellites, military radios, and thermal weapon sights.

The comparison between these batteries is given in Table 4.5.

Table 4.5 *Comparison between secondary batteries*

Lead Acid	Nickel Cadmium	Lithium Ion
It has very low internal resistance	It has very low internal resistance	It has medium internal resistance
Nominal battery voltage is 2 V	Nominal battery voltage is 1.2 V	Nominal battery voltage is 3.2–3.7 V
Charge and discharge cut off voltage are 2 V and 2.4 V respectively	Charge and discharge cut off voltage are 1.2 V and 1 V respectively	Charge and discharge cut off voltage are 4.2 V and 2.5 V respectively
Less maintenance is required	Moderate maintenance is required	Free from maintenance
Cost of the battery is less	Cost of the battery is moderate	Cost of the battery is high
Its efficiency is approximately 90 per cent	Its efficiency is approximately 70–90 per cent	Its efficiency is 99 per cent
Very high toxicity level	Very high toxicity level	Low toxicity level
It is thermally stable	It is thermally stable	Requires protection circuit for stability
Time taken to charge the battery is 8 to 16 hours	Time taken to charge the battery is 1 to 2 hours	Time taken to charge the battery is 1 to 4 hours
Self-discharge per month is 5 per cent of its total capacity	Self-discharge per month is 20 per cent of its total capacity	Self-discharge per month is less than 3 per cent of its total capacity

4.9 Power System Protection

In general, the operating voltage of electrical power system is from 415 V to 400 kV or even higher. The different equipment used in the electrical power system are machines, transformers, transmission lines, insulators, bus bars, cables and so on which may be placed in open or closed condition. Due to various reasons, all the equipment may undergo abnormal or faulty conditions in its life span. Some examples of faulty conditions are insulation failure in the cable due to lightning surges, increase in the voltage (i.e., overvoltage) of the alternator due to sudden loss of load, undesirable heating in the transformer due to open circuit condition, deterioration of insulation winding in transformer due to winding short-circuits and so on. Hence, it becomes necessary to protect these equipment from different faulty conditions like overvoltage, lightning surges, insulation failure, resonance, improper earthing, short circuit or open circuit condition, balanced and unbalanced faults which ensure the continuous working of the equipment. Also, it is important to safeguard the human personnel who get exposed to power system equipment under faulty or abnormal conditions. In general, the protection scheme is classified as: (i) Primary protection and (ii) Back-up protection. The primary protection is designed in such a way that the components of the power system are protected. The back-up protection is the secondary defense mechanism which gets operated when the primary protection scheme fails.

The different protective equipment or devices which are used in the power system to safeguard different equipment are: (i) Circuit breakers, (ii) Fuses and (iii) Protective relays. During normal operation of the power system, these protective devices allow to switch on or off the electrical equipment. On the other hand, if a fault occurs on any part of the power system, the protective device detects these faults and disconnects the unhealthy portion of the system thereby protects the power system from damage to ensure continuity of the supply.

4.9.1 Essential Features of Protective Devices **[May/June, 2014]**

The essential features of protective devices are:

(i) **Complete Reliability:** It is the most important feature which all the protective devices should have in the power system. If a fault occurs in any part of the power system, then the protective device should operate in such a way that the fault section gets isolated from the other part of the power system.

(ii) **Absolutely Certain Discrimination:** Clear and accurate discrimination between the faulty section and healthy section is required to isolate the faulty section. This feature will ensure the continuity of supply.
(iii) **Quick Operation:** The time taken by the protective device to isolate the faulty section must be very minimum so that the other part of the system does not get damaged.
(iv) **Provision for Manual Control:** Even if the protective device can be made automatic, there should be a provision for manual control to carry out the necessary operations when the automatic control fails.

4.10 EARTHING

Earthing refers to connecting the electrically conductive part of an electrical equipment to the ground through earth plate or electrode with negligible resistance for its safety. In general, galvanised iron is used as the material for earthing purpose. It helps in protecting the equipment and provides a return path for the leakage or fault current to pass to the ground through electrode. The two main functions of earthing are: (i) It ensures that electrical equipment through which the current flows does not rise to a potential greater than its designed value and (ii) It provide safety to human lives. It is noted that the neutral of the supply system is also earthed.

4.10.1 Need for Earthing

The reasons for the requirement of earthing are to:
(i) Protect the human lives and electrical equipment from fault current.
(ii) Maintain the voltage at a constant level even when a fault occurs in any system.
(iii) Protect the electrical equipment and buildings from over voltages occurring due to lighting.
(iv) Provide a return path for the fault current occurring in the system.
(v) Prevent fire in electrical systems

Terms Related to Earthing

(i) **Earth:** The connection between electrical systems to the buried plate in the ground is called Earth.
(ii) **Earthed:** The electrical system that is connected to the ground through an electrode is called earthed device or earthed.
(iii) **Solidly Earthed:** If there is no protection device like fuse, circuit breaker, etc., between an electrical system and the ground, then it is called solidly earthed.
(iv) **Earth Electrode:** The conductor that is buried inside the ground to provide safety for the electrical system is called Earth Electrode and it is available in different shapes.
(v) **Earthing Lead or Earth conductor:** The connection between the electrical system and the earth electrode is done with the help of a conductor wire or strip called Earthing lead.
(vi) **Earth Continuity Conductor:** The conductor wire or strip that is used to connect different electrical devices is called earth continuity conductor. It is also defined as the conductor wire used in connecting earthing lead and electrical devices and it is available in different shapes.
(vii) **Sub Main Earthing Conductor:** The conductor that helps in connecting switch board and distribution board is called sub main earthing conductor.
(viii) **Earth Resistance:** It is defined as the total resistance between earth electrode and earth. Also, it is defined as the algebraic sum of the resistances of earth continuity conductor, earthing lead, earth electrode and earth.

The three main components of earthing system are:
(i) Earthing continuity conductor
(ii) Earthing lead or earth conductor
(iii) Earth electrode

4.10.2 Methods of Earthing

The two different methods by which earthing is done are: (i) pipe earthing and (ii) plate earthing.

Pipe Earthing

In this method, a galvanised iron (GI) pipe is used as an earth electrode. The length and diameter of pipe depend upon the current to be carried and soil type to which earth electrode is buried. According to I.S.I standard, the diameter of the pipe used for earthing should be greater the 3.81 cm and length of the pipe used should be 2 m, 2.75 m and 1.75 m for ordinary, dry and rocky soils respectively.

The GI pipe is vertically placed and buried in the wet ground. The depth at which the GI pipe is to be buried depends on moisture level of the ground. According to I.S.I standard, the depth should be 4.75 m and it can be less if the moisture content of the ground is sufficient. In the underground, broken coke or charcoal piece is used to surround the GI pipe for a distance of 15 cm. It is necessary since coke along with salt helps in decreasing the earth resistance. Generally, alternate layers of coke and salt are used. In summer, to prevent the increase in earth resistance due to decrease in moisture level, buckets of water is poured to the funnel connected to GI pipe. Pipe earthing is shown in Figure 4.32.

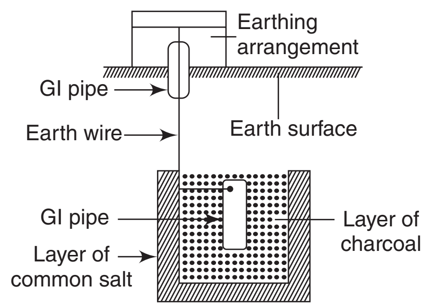


Figure 4.32 Pipe earthing

Plate Earthing

In this type of earthing, a GI or copper plate is used as an earth electrode. The size of GI plate should be greater than 60 cm × 60 cm × 6.35 mm and the size of copper plate should be 60 cm × 60 cm × 3.18 mm. These plates with their face vertical are buried inside the ground so that the distance between the ground level and plate is greater than 2 m. If the moisture content of the place is high and it should be 0.6 m away from building foundations. The plate shall be completely covered by 15 cm of coke and salt.

Factors to be Remembered in Providing Earthing

The different factors to be remembered while providing earthing are:
(i) **Distance:** The distance between earth electrode and electrical system should be greater than 1.5 m.
(ii) **Cross Section of the Earthing Lead:** The cross section of the earthing lead should be greater than half the cross section of the main wire or conductor. Its minimum size should not be less than 12.97 sq. mm.
(iii) **Cross Section of the Earth Continuity Conductor:** The size of the earth continuity conductor should be greater than 2.894 sq. mm.
(iv) **Electrode:** The material used for earth electrode and earth lead should be same and it should always be placed in vertical position. The size of the electrode varies with respect to load and insulation material.
(v) **Earth Resistance:** The maximum value of the earth resistance should be 5 Ω. The earth resistance depends on the electrode area in contact with the ground, coal, salt and quantity of earth.

4.11 Circuit Breaker [AU April/May, 2015]

A switching device, which can be used to make or break a circuit manually, automatically or with the help of remote control, under different conditions i.e., normal and under faulty conditions, is known as a circuit breaker. Special attention must be given while designing a circuit breaker, to safely interrupt the arc produced during its operation, as modern power systems deal with very high currents.

4.11.1 Working of a Circuit Breaker

The fixed and moving contacts, which are called electrodes, exist in a normal circuit breaker. The medium in which these contacts are placed could either be oil or air. When the power system is operating normally, these contacts will remain closed and will not open automatically until a faulty condition occurs in the system. Whenever a fault occurs in the system, these contacts can be opened either manually or automatically or by using a remote control. During a faulty condition, due to energization of trip coils of the circuit breaker, the moving contact is pulled apart, which opens the circuit and an arc is formed between these contacts. The schematic diagram of the circuit breaker under normal and fault conditions is shown in Figure 4.33.

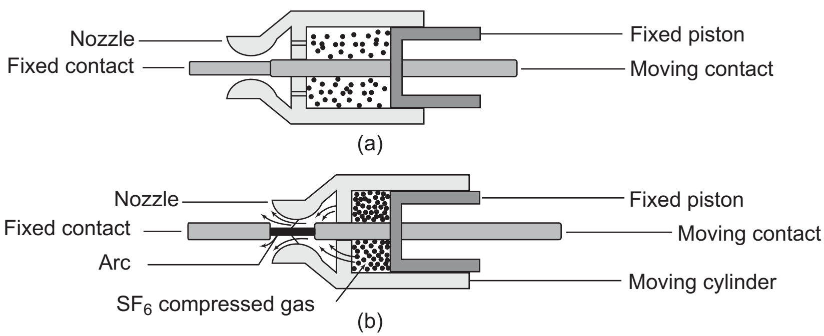


Figure 4.33 Circuit Breaker in (a) Normal and (b) Faulty Conditions

An arc develops between the fixed and moving contacts, when a fault occurs in the power system. The faulty current in the power system will continue to flow until this arc is extinguished or stopped. Therefore, the formation of the arc not only delays the interruption of faulty current, but also generates huge amounts of heat, which might cause damage to the power system or the circuit breaker itself. Hence, it is necessary to extinguish the arc developed in a short interval of time, so that the magnitude of heat generated will not exceed a maximum value.

4.11.2 Arc Phenomenon [AU May/June, 2014]

When a faulty condition occurs before the fixed and moving contacts are separated, a huge amount of current starts flowing in the power system. During the separation of contacts, due to the rapid decrease in the contact area and the large amount of fault current, the current density increases, which in turn increases the temperature. The increase in temperature in the medium either ionizes the air or vaporizes and ionizes the oil, which then act as conductors and help in striking an arc between the contacts. The developed arc is maintained, as there exists only a small potential difference between the contacts. Since the arc developed a low-resistance path, the fault current existing in the power system will remain uninterrupted till the persistence of arc between the contacts.

The period till which the arc exists between the contacts is known as arcing period. In arcing period, the arc resistance plays a vital role and is inversely proportional to the magnitude of current flowing through the contacts. The factors that affect the arc resistance are:

- **Degree of ionization:** An inverse relation exists between the arc resistance and the number of ionized particles in the medium.
- **Length of the arc:** When the length of the arc or distance between the contacts increases, the arc resistance increases.
- **Cross section of arc:** An inverse relation exists between the arc cross-section and the arc resistance.

It is very important to extinguish or quench the arc as early as possible, before it could cause serious damage to the system. As the arc developed forms a conductive path for electricity, the fault current flowing through the circuit breaker will not be interrupted till the arc is extinguished. The most important designing criteria of a circuit breaker is providing proper technique for quenching the arc, to ensure a quick and safe fault-current interruption.

4.11.3 Principles of Arc Extinction **[AU Nov/Dec, 2012]**

The principle factors responsible for arc maintenance between the contacts are:

- **Potential difference between the contacts:** In general, the potential difference between two points is inversely proportional to the distance between the points. Hence, one way of quenching the arc is to separate the contacts such that the potential difference between the contacts will be inadequate to maintain the arc. But, in practice, this method is not possible in a high-voltage system, where the required distance between the contacts is very high to quench the arc.
- **Ionized particles in the medium between the contacts:** The particles in the medium, which are ionized between the contacts, help maintain the arc. Therefore, by either cooling the arc or by removing the ionized particles existing between the contacts, the arc can be extinguished.

4.11.4 Methods of Arc Extinction **[AU Nov/Dec, 2012]**

The two different methods by which the arc can be extinguished are:

- High-resistance method and
- Low-resistance method or current zero method

High-Resistance Method

In this method, arc resistance is allowed to increase with time to a particular value, which will be insufficient to maintain the arc and consequently interrupts the fault current or extinguishes the arc. The main disadvantage of this method is the large amount of energy that is dissipated in the arc, as it is directly proportional to the arc resistance. Hence, this method is applicable only to DC and low-capacity circuit breakers. The different ways by which the arc resistance can be increased are:

- **Increasing the length of the arc:** When the gap between the contacts is increased to increase the length of the arc, the arc resistance increases, as there exists a direct relation between the arc resistance and the length of the arc.
- **Cooling the arc:** Efficient cooling of the medium, directed along the arc using gas blast, helps in deionization of the particles in the medium, which increases the arc resistance.

- **Reducing cross-sectional area of the arc:** Allowing the arc to pass through a small opening or by having a smaller contact area will reduce the cross-sectional area of the arc, which in turn increases the arc resistance, as the cross-sectional area is inversely proportional to the arc resistance.
- **Arc splitting:** Splitting of the arc into more number of smaller arcs, using conducting plates between the two contacts, will experience the length-decreasing effect and cooling effect, which helps in increasing the arc resistance.

Low-resistance or Current-Zero Method

The extinction of an arc in modern high power AC circuits is carried out using this method. In this method, the resistance of the arc is kept as low as possible till the fault current becomes zero. The arc is naturally extinguished and helps in preventing the re-strike of the arc, even when there is a rise in potential difference across the contacts. In an AC system, whenever the current drops to zero at every half-cycle, there will be a brief moment where the arc gets extinguished. Here, the dielectric strength of the ions and the electrons existing in the medium between the contacts is small, which can breakdown easily due to the rising contact voltage called as re-striking voltage. Due to such a breakdown, the arc developed between the contacts exists for another half cycle. If the dielectric strength of the medium is made to increase rapidly than the voltage across the contacts, the arc fails to re-strike near the current zero and hence the fault current gets interrupted. The different ways by which the dielectric strength can be increased rapidly are:

- Recombination of ionized particles with neutral molecules in the medium between the contacts.
- Swiping the ionized particles with the unionized particles.

Therefore, the rapid de-ionization of the particles of the medium as soon the current becomes zero is the most important problem in an AC arc interruption. The de-ionization of the particles in the medium can be achieved by using any one of the following methods:

- **Lengthening of the gap:** Rapid opening of the contacts increases the dielectric strength of the medium, as it depends on the length of the gap between the contacts and dielectric strength of the medium.
- **High pressure:** Increasing the pressure in the surrounding area of the arc increases the density of the discharging particles, which in turn increases the speed of de-ionization and helps in increasing the dielectric strength of the medium.
- **Cooling:** If the ionized particles are allowed to cool, there will be a natural combination of ionized particles and thereby, the dielectric strength of the medium increases.
- **Blast effect:** Complete removal of the ionized particles using gas blast or forced oil along the discharge and replacing it with unionized particles increases the dielectric strength of the medium.

4.11.5 Terms Associated with Circuit Breaker **[AU Nov/Dec, 2014]**

The terms which are associated with circuit breaker are: (i) arc voltage (ii) re-striking voltage and (iii) recovery voltage.

- **Arc voltage:** It is the voltage that is obtained across the contacts of the circuit breaker during the arcing period.
- **Re-striking voltage:** It is the voltage that appears across the contacts at the instant of arc extinction.
- **Recovery voltage:** It is the normal frequency RMS voltage that appears across the contacts of circuit breaker after the final arc extinction and it is approximately equal to the system voltage.

4.11.6 Classification of Circuit Breakers **[AU Nov/Dec, 2014]**

The different categories, based on which the circuit breaker is classified, are: (i) medium in which the circuit-breaker operates (ii) actuating signal in which it works (iii) construction type (iv) voltage level and so on.

Category 1: Working Medium

Based on the working medium, the circuit breaker is further classified as:

- Oil circuit breaker
- Air Blast circuit breaker
- SF6 circuit breaker
- Vacuum circuit breaker

Category 2: Actuating Signal

Based on the actuating signal, which is required to actuate the circuit breaker, it is further classified as:

- Spring-operated circuit breaker
- Pneumatic circuit breaker
- Hydraulic circuit breaker

Category 3: Voltage Level

Based on the voltage level in which the circuit breakers operate, it is further classified as:

- Low-voltage circuit breakers (< 1 kV)
- Medium-voltage circuit breakers (1-72 kV)
- High-voltage circuit breakers (> 72 kV)

Category 4: Location of Circuit Breaker

Based on the location of circuit breaker, it is further classified into:

- Outdoor circuit breaker
- Indoor circuit breaker

4.12 Fuse

A fuse is a short piece of wire or a thin strip of metal, which is inserted in series to the circuit. When the fault current flows through the fuse for a sufficient time, it melts the fuse, thus isolating the circuit. Under normal operation, the fuse is kept at a temperature below the melting point of the material used, which helps in carrying the normal current without any rise in temperature. But when fault occurs in the power system due to a short circuit or when an overload current, which is greater than the normal current, flows through the fuse, this fault current will increase the temperature above the melting point of the material used for the fuse. Hence, the material melts or blows, thereby isolating the healthy part and protects the circuit. The magnitude of excessive current flowing in the circuit is an important factor in deciding the time taken for melting or blowing out the fuse. Greater the fault current, lesser the time required to melt or blow out the fuse. The inverse time-current characteristics of a fuse are shown in Figure 4.34.

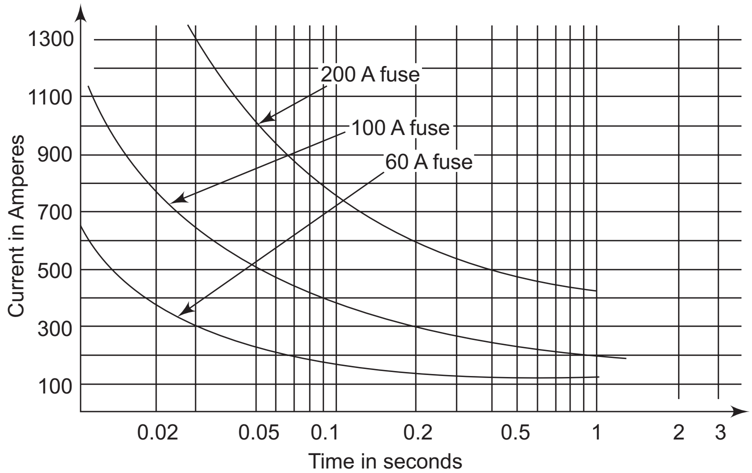


Figure 4.34 Inverse Time–Current Characteristics of a Fuse

4.12.1 Advantages and Disadvantages

The advantages and disadvantages of fuse are given as follows:

Advantages

1. Cheapest form of protection device.
2. Requires no maintenance.
3. Operation of fuse is completely automatic.
4. Easily breaks a large amount of fault current.
5. Pollution-free protection device i.e., does not create any smoke or noise.
6. Suitable for over-current conditions due to its inverse current–time characteristics.
7. Requires less time for isolating the faulty part of the circuit.

Disadvantages

1. Rewiring or replacing a fuse takes a considerable time.
2. Discrimination between fuses connected in series is not possible.
3. Correlation of the characteristics of fuse with the protected device is not always possible.

4.12.2 Desirable Characteristics of Fuse Element Materials

The desirable characteristics of the material used for the fuse to perform satisfactorily are:

- Low melting point, e.g., tin, lead
- High conductivity, e.g., silver, copper
- Least reactive to oxidation, e.g., silver
- Affordable, e.g., lead, tin, copper

It can be noted that no element possesses all the desirable characteristics of a fuse and hence a compromise is to be made in selecting a material for the fuse. Lead, tin, copper, zinc and silver are the most commonly used fuse materials. Tin, or an alloy of lead and tin (0.37 and 0.63 respectively) is used as a fuse element material, where the rating of current is up to 10 A. For larger currents, copper or silver is used as fuse element material. Usually, the copper is tinned to prevent oxidation effect. Zinc, in strip form, is used where a considerable time-delay is required. In day-to-day activities, silver is used as a fuse element due to the following characteristics:

- Does not get affected or deteriorate when used in dry air.
- As the expansion coefficient of silver is very small, it can carry the rated current continuously for a long time.
- Conductivity is very high.
- Instantaneous transition to vapour state from melting state, when compared to other materials, is possible due to low specific heat.
- Faster operation is possible at higher currents.
- Quick interruption of fault current is possible as the element vaporises at a temperature much lower than the temperature required to ionize it.

4.12.3 Important Terms **[AU Nov/Dec, 2014]**

Following are the terms required in fuse analysis:

- **Current rating of fuse element:** It is the amount of current that the fuse element can carry under normal operation, without overheating or melting. It depends on temperature rise in the fuse holder, the fuse material and the surroundings of the fuse.
- **Fusing current:** It is the minimum current at which the fuse element melts or blows away and isolates the healthy portion of the power system. It is higher than the current rating of the fuse element.
- **Fusing factor:** It is the ratio of the fusing current to the current rating of fuse element and its value is always greater than 1.
- **Prospective current:** It is the RMS value of the fault current, which is obtained by replacing the fuse with a conductor of negligible resistance.
- **Cut-off current:** It is the maximum value of fault current obtained before the fuse element melts.
- **Pre-arcing time:** The time taken to cut off the fault current from its commencement is known as pre-arcing time.
- **Arcing time:** The time taken to extinguish the arc after the pre-arcing time is known as arcing time.
- **Total operating time:** It is the summation of pre-arcing and arcing time.
- **Breaking capacity:** The RMS value of the maximum prospective current, which a fuse can deal at rated voltage, is known as breaking capacity.

4.12.4 Classification of Fuses

The general classification of fuses is given as follows:

1. Low-voltage fuses
   - Semi-enclosed re-wireable fuse
   - High rupturing capacity (HRC) cartridge fuse with and without tripping device
2. High-voltage fuses
   - Cartridge type
   - Liquid type
   - Metal-clad fuses

4.12.5 Comparison Between Circuit Breaker and Fuse [AU May/June, 2014]

Specification	Circuit Breaker	Fuse
Function	Performs interruption function	Performs detection and interruption functions
Operation	Requires more equipment for automatic operation.	Requires less equipment for full automatic operation
Breaking capacity	Very large	Small
Operating time	Large (0–1 or 0–2 sec)	Very small (0.002 sec)
Replacement	It need not be replaced after its operation.	Requires continuous replacement after every operation.

4.13 Protective Relays [AU April/May, 2015]

A device that detects the fault in the system and initiates the circuit breaker operation and helps in isolating the faulty element from the healthy portion of the system is known as a protective relay. Electrical quantities whose values get changed during normal and abnormal conditions are: voltage, current, frequency and phase angle. Protective relay constantly measures these electrical quantities. If any one of the electrical quantities changes, the protective relay detects the abnormal condition and operates in such a way that the circuit breaker isolates the faulty portion from the healthy portion. A typical relay circuit is shown in Figure 4.35.

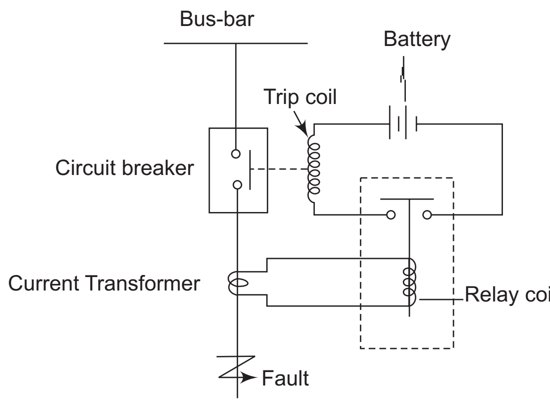


Figure 4.35 Simple Protective Relay Circuit With Circuit Breaker

The three different parts of a simple relay circuit are: (i) primary winding of the current transformer (CT), which is placed in series with the transmission line to be protected, (ii) secondary winding of CT and operating coil of relay and (iii) tripping circuit, which has an AC or DC source, trip coil of the circuit breaker and relay contacts. When a fault occurs at point F, the current through the transmission line increases and it starts flowing through the relay coil. Then, the increased current causes the relay contact to close the tripping circuit of the circuit breaker, which makes the circuit breaker to open, thereby isolating the faulty section from rest of the system.

4.13.1 Fundamental Requirements of Protective Relay [AU Nov/Dec, 2014]

The fundamental requirements of a protective relay to detect the fault and trigger the circuit breaker are given below:

- **Selectivity:** It is the ability of the protective relay to select the exact location of the faulty system and disconnect it without affecting the other parts of the system.
- **Speed:** The disconnection of the faulty section using protective relay should be as fast as possible. Otherwise, (i) the electrical equipment may get damaged (ii) the system voltage may get reduced and (iii) one type of fault may develop into other types of faults.

- **Sensitivity:** The minimum value of the actuating quantity, which is required to operate the protective relay system.
- **Reliability:** It is the ability of the protective relay system to operate under predetermined conditions.
- **Simplicity:** Maintenance of the protective relay system should be simple.
- **Economy:** Economically, a particular type of protective relay system should be selected.

4.13.2 Operating Principle of Protective Relays

The most commonly used protective relay in the power system is an electro-mechanical type relay. The operating principles on which these protective relays work are: electromagnetic attraction and electromagnetic induction.

Electromagnetic Attraction Relays

The basic concept of armature attraction by poles of an electromagnet or a plunger attraction using solenoid is implemented in electromagnetic attraction relays. These relays are actuated either by DC or AC sources.

Electromagnetic Induction Relays

In the electromagnetic induction relays, the initial force is developed on the moving element due to the interaction of electromagnetic fluxes with eddy current. The electromagnetic induction relays are widely used in applications involving AC quantities and not preferred with DC quantities.

4.13.3 Relay Timing

The operation time is an important characteristic of the relay. Operation time is the time taken between the instant at which the actuating signal is energized and the instant at which the relay contacts are closed. Based on the relay timing or the operation time of the relay, protective relays are classified as:

- **Instantaneous relay:** The contact in the relay circuit gets closed immediately when the electrical quantity in the relay coil exceeds the maximum limit, without any intentional time delay.
- **Inverse-time relay:** The operating time of the inverse-time relay is inversely proportional to the magnitude of the electrical quantity.
- **Definite time-lag relay:** There exists a definite time lag between the instant at which the electrical quantity exceeds the maximum value and the instant at which the relay contacts are closed. This type of time setting is independent of the magnitude of the electrical quantity flowing through the relay coil.

4.13.4 Important Terms **[AU Nov/Dec, 2014]**

- **Pick-up current:** The minimum value of current required to flow through the relay coil to make the relay operational is known as pick-up current. When the current flowing through the relay coil is less than this value, the relay does not operate. The magnitude of pick-up current is the product of the rated secondary current of CT and the current setting.
- **Current setting:** The current setting is the setting of pick-up current to any required value and is achieved using tappings in the operating coil of the relay.
- **Plug-setting multiplier (PSM):** It is given by the ratio of the magnitude of fault current in the relay coil to the pick-up current value.
- **Time-setting multiplier:** The adjustment provided in the relay to control or adjust the operation time of the relay is known as time-setting multiplier.

The operating time vs. P.S.M of a protective relay is shown in Figure 4.36. The operating time of the relay is calculated if all the above terms are known.

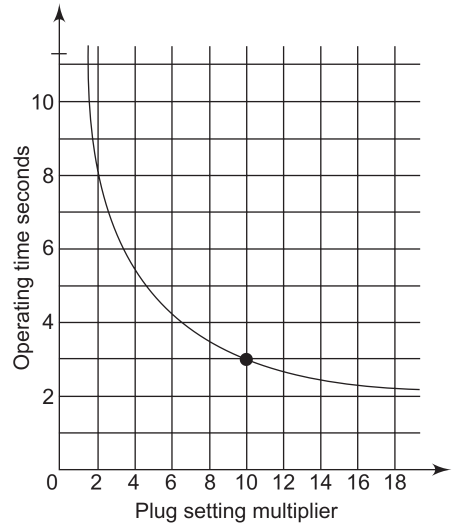


Figure 4.36 Operating Time vs. Plug-Setting Multiplier

4.13.5 Functional Relays

The most important functional relays are:

- **Induction type over-current relay (non-directional):** It works on the induction principle and operates the circuit breaker when the current in the circuit exceeds a predetermined value.
- **Induction type directional power relay:** It works on the induction principle and operates when the power in the circuit flows in a particular direction.
- **Distance or impedance relay:** Its operation is based on the ratio of applied voltage to the current flowing in the circuit. Two types of distance or impedance relays are: (a) definite distance-type impedance relay and (b) time-distance impedance relay.
- **Differential relay:** When the phasor difference of two or more similar electrical quantities exceeds a maximum value, the relay becomes operational. Two types of differential relays are: (a) current differential relay and (b) voltage-balance differential relay.
- **Transley scheme:** It is a balanced voltage scheme with the addition of a directional feature.

4.14 Tariff

A large number of consumers can be influenced to use electrical energy produced by numerous power stations, if it is sold at a reasonable price or rate. The reasonable price or rate at which the produced electrical energy is supplied to the consumer is defined as tariff. In other words, the price charged by the supplier to supply electrical energy to various types of consumers is known as tariff. Based on tariff, there will be a healthy competition between the companies that supply electrical energy. Each supply company will fix the tariff for the produced electric energy in such a way that it earns profit on its capital investment, in addition to the total cost spent in producing electrical energy. The tariff, at which the electrical energy is charged, is not uniform for all the consumers (domestic, commercial, and industrial), as it depends on the magnitude of consumption and its load condition. Therefore, in fixing the tariff, due consideration has to be given to the type of consumer, which leads to more complications.

4.14.1 Objectives of Tariff

Like in the case of other commodities, electricity tariff covers the production and supply cost of electrical energy with a reasonable profit. Therefore, certain objectives and requirements are to be satisfied before fixing a tariff for the electrical energy. They are:

- Cost of producing electrical energy at a place should be recovered.
- Cost of the capital investment for generation, transmission and distribution of electrical energy should be recovered.

- Cost of the operation, raw materials, maintenance and losses of electrical energy must be recovered.
- Cost of miscellaneous services like metering, billing, collection and so on, should be recovered.
- Tariff should ensure the satisfactory net return or profit on the capital investment.
- Tariff should be uniform for large number of population.
- Tariff should be simple and easily understandable to the consumers.
- Proper advantage must be given to the consumers using electrical energy during off-peak hours.
- Proper charges must be provided to the consumers demanding more electrical energy during peak hours.
- Penalty must be imposed on the consumers for low power factors.

4.14.2 Factors Affecting the Tariff

The factors for fixing the tariff for electrical energy are:

Nature of Load

The three different types of loads are: domestic, commercial and industrial. The tariff allocated for electrical energy varies for these different types of loads. The industrial load consumes more energy for a longer time when compared to domestic and commercial loads. Hence, the tariff must be decided based on the nature of load and it should not be uniform for all loads.

Maximum Demand

The maximum of all the demands that a particular station supplies in a given period is called as maximum demand. It depends on the maximum installed-capacity of the station and the maximum kWh generated from the station. The tariff allocated for the electrical energy generated from a particular generation station is directly proportional to the maximum demand it supplies. Hence, increase in maximum demand increases the installed capacity of the generating station, which increases the capital investment cost and as a result, the tariff increases.

Load Requirement Time

In general, the time of consumption of electrical energy is classified as peak and off-peak times. The time at which the maximum demand is consumed is called peak time. If the consumer demands the power in peak time, tariff will be higher and if the consumer demands the same power in off-peak time, the tariff will be less.

Load Power Factor

The power factor of the load is inversely proportional to the tariff of the electrical energy consumed. If the power factor is low, additional devices are required to correct the power factor and hence the tariff will be high.

4.14.3 Characteristics of Tariff

The desirable characteristics of a tariff are given as follows:

Proper Return

Proper return from each consumer must be ensured by the tariff i.e., the total cost obtained from the consumers must match the production and supply cost of electrical energy incurred, along with a reasonable profit to the generation station. This will ensure continuous and reliable service from the generation company to the consumers.

Fairness

Fairness of tariff to different consumers depends on: (i) the amount of energy consumed and (ii) the deviation in the load pattern. Tariff should be low for consumers who consume more amount of energy when compared to those consuming small amounts of energy. Similarly, the consumer whose load pattern does not deviate much should be charged low when compared to other consumers.

Simplicity

Tariff calculation of the electrical energy consumed should be easily understandable by any consumer. It helps in maintaining a smooth relation between the consumer and the supplier. Otherwise, it will make the consumers distrust the supply companies.

Reasonable Profit

The profit that the supplier receives through the tariff charged should be reasonable. To maintain a good relation between the supplier and the consumer, the profit should be restricted to 8 per cent per annum.

Attractive

Tariff charged for the electrical energy should attract more number of consumers, encouraging them to utilize the services provided by the company. It can be achieved by fixing the tariff such that the consumer can easily afford it.

4.14.4 Types of Tariff

The different forms by which the tariff for electrical energy consumed is determined are:

- Simple tariff
- Flat-rate tariff
- Block-rate tariff
- Two-part tariff
- Maximum-demand tariff
- Power-factor tariff
- Three-part tariff

They are explained in detail as follows.

Simple Tariff

When a fixed rate is applied to each unit of consumed energy, it is known as simple tariff or uniform tariff. In this type, the rate per unit of energy consumed is constant and it does not depend on the quantity of energy consumed by the consumer. The electrical energy consumed by the consumer is recorded using an energy metre. This is the simplest of all tariffs and is represented graphically as shown in Figure 4.37.

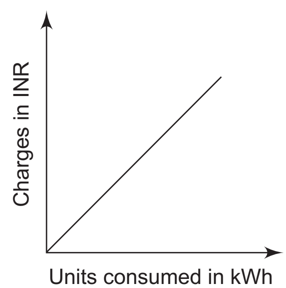


Figure 4.37 Simple Tariff

Advantages

1. Simple method
2. Easy to understand and to apply
3. Consumers pay according to their usage of electrical energy.

Disadvantages

1. There is no discrimination between different types of consumers.
2. Higher cost per unit of energy consumed
3. As it does not provide any incentives, this type of tariff does not encourage the use of more electrical energy.
4. Even though there exists a connection, the supplier cannot charge any amount when the consumer does not consume any energy.

Flat-rate Tariff

In flat-rate tariff, different types of consumers are charged at different fixed costs, per unit of electrical energy consumed. Here, the consumers are grouped into different categories and each category is charged a fixed rate, similar to the simple tariff method. The fixed rate for each category of consumers is decided based on their loads and the power factor of the loads. The flat-rate tariff for different categories is represented in Figure 4.38.

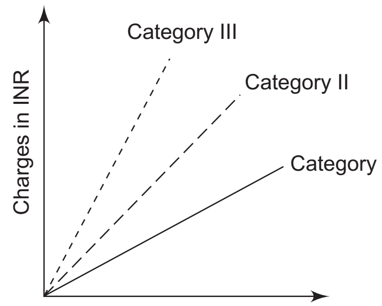


Figure 4.38 Flat-rate Tariff

Advantages

1. It is fair to the consumers.
2. Calculation is simple.

Disadvantages

1. Though the consumers are categorized, incentives are not provided to them.
2. Since separate metres are required for different loads like lighting load, power load and so on, it makes the whole arrangement complex and expensive.
3. Each category of consumer is charged a fixed rate, irrespective of the amount of energy consumed by the consumer.

Block-rate Tariff

In this type of tariff, the energy consumed by the consumers is divided into different blocks and then each block will be charged a particular fixed rate. It is to be noted that if a block of energy is charged at a specified rate, then the succeeding blocks will be charged at a progressively reduced rate i.e., the rate per unit in the first block is the highest and it is progressively reduced for the succeeding blocks of energy. Graphically, it is represented as shown in Figure 4.39.

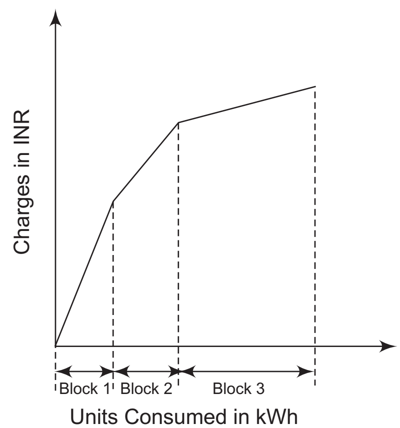


Figure 4.39 Block-rate Tariff

Advantages

1. Requires only one energy-metre.
2. Incentives are provided to the consumers in terms of reduced tariff, which attract the consumers to use more energy.
3. Increased load factor of the system.
4. Reduced cost of electrical energy generation.

Disadvantage

Even though there exists a connection, the supplier cannot charge any amount when the consumer does not consume any energy.

Two-part Tariff

In this type of tariff, the total cost that the consumer is charged by the supplier consists of two components: fixed charges and running charges. It depends on maximum demand and number of units consumed by the consumer respectively. Mathematically, it is expressed as

$$\text{Total cost} = [a \times MD + b \times UC]$$

where MD is the maximum demand of the consumer in kW, UC is the number of units consumed by the consumer in kWh, a is the charge per kW, which when multiplied by MD gives the fixed charge and b is the charge per kWh, which when multiplied by UC gives the total running charges.

The capital cost of investment, taxes and some part of operating cost, which is independent of total energy generated, will be covered by the fixed charges. The running charge covers the operating cost, which varies with respect to the variation in energy generated or supplied.

Advantages

1. Consumers can easily understand this type of tariff.
2. As the fixed charge is independent of the units consumed by the consumer, the supplier will receive the fixed charges, even when the consumer does not consume any energy.

Disadvantages

1. Irrespective of the usage of electrical energy, the consumer has to pay the fixed charges.
2. Error exists in assessing the maximum demand of the consumer.

Maximum-demand Tariff

The only difference between the two-part tariff and maximum-demand tariff is that the maximum demand of a particular consumer is determined by installing a maximum-demand metre in the premises of the consumer. This type of tariff eliminates the error that exists in assessing the maximum demand.

It is not suitable for small residential consumers, as a separate maximum-demand metre is required.

Power-factor Tariff **[AU May/June, 2014]**

In this type of tariff, the power factor of the consumers' load is taken into consideration. The power factor plays an important role in an AC system. High power factor helps in an optimal operation of the system. Low power factor of the system causes more line losses and imbalance to the system and hence, the consumer will have to be penalized. The power-factor tariff is further classified into:

- **kVA maximum-demand tariff:** It is also known as a modified form of two-part tariff, where the fixed charges are calculated based on the maximum demand in kVA instead of kW, since the power factor is inversely proportional to kVA demand. Therefore, a consumer having low power factor has to pay more fixed charges. This type of tariff encourages the consumers to operate their load at an improved power factor.
- **Sliding-scale tariff:** It is also known as average power-factor tariff, where a power factor of 0.8 lagging is considered as the reference, which helps in penalizing the consumer whose power factor is less than the reference. On the other hand, if the power factor of the consumer is greater than the reference power factor, an incentive in the form of a discount will be provided to the consumer.
- **kW and kVAR tariff:** In this type of tariff, both active power (kW) and reactive power (kVAR) consumptions are measured and charged separately. As there is an inverse relation between kVAR and power factor, low power factor consumes more reactive power and as a result, the consumer will be charged heavily.

Three-part Tariff

In this type of tariff, the total cost that the consumer is charged by the supplier is split into three parts: fixed charge, semi-fixed charge and running charge, and is expressed mathematically as

$$\text{Total cost} = [c + a \times MD + b \times UC]$$

where c is the fixed charge for every billing period, which includes the capital investment cost of secondary distribution and labour cost of collecting revenues.

Applications of different types of tariff are listed in Table 4.6.

Table 4.6 *Applications of Different Types of Tariff*

Tariff Type	Application
Simple tariff	Tube wells used for irrigation purposes.
Flat-rate tariff	Domestic consumers
Block-rate tariff	Major residential and small commercial consumers
Two-part tariff	Industrial consumers whose maximum demand is significant.
Maximum demand tariff	Large industrial consumers
Three-part tariff	Big consumers

Example 4.7

An industrial consumer has a single-phase 230 V supply. The monthly energy consumption is 2020 kWh. A maximum demand indicator installed at the consumer premises indicates the total energy consumption per month for maximum demand is 552 kWh which is charged at INR 3.50 per kWh. The remaining units are charged at INR 1.80 per kWh. Determine the monthly bill and the average tariff for the consumer.

Solution

Maximum demand energy consumption = 552 kWh

Charge per kWh of maximum demand = INR 3.50

Total energy consumption of the consumer = 2020 kWh

Therefore, the monthly bill for the consumer = (552 × 3.5) + ((2020 – 552) × 1.8)

= INR 4574.40

$$\text{Average tariff for the consumer} = \frac{4574.4}{2020} = \text{INR } 2.2645 \text{ per kWh}$$

Example 4.8

An electrical supply company offers two tariffs: (i) (30 + (0.03 per kWh)) and (ii) 0.06 per kWh for the first 400 units and 0.05 per kWh for the remaining units. If the total bill obtained using the two tariffs is same, determine the energy consumption per month for the consumer.

Solution

Let x be the total energy in kWh consumed by the consumer per month.

Then, the monthly bill using first tariff = $30 + (0.03 \times x)$ and the monthly bill using second tariff = $(0.06 \times 400) + (0.05 \times (x - 400))$

Since the monthly bill obtained using two tariffs are equal, we get

$$30 + (0.03 \times x) = (0.06 \times 400) + (0.05 \times (x - 400))$$

Solving the above equation, we get $x = 1300$

Hence, the total monthly energy consumption of the consumer is 1300 kWh.

Two Mark Questions and Answers

1. **List the disadvantages of conventional energy sources.** **[AU Nov/Dec, 2009]**

   Refer to section 4.1 for the disadvantages of conventional energy sources.

2. **Mention the advantages and disadvantages of solar energy.** **[AU April/May, 2010]**

   Refer to section 4.2.1 for the advantages and disadvantages of solar energy.

3. **What are the components of solar PV power generation system?** **[AU April/May, 2008]**

   The components of solar PV power generation system are:
   (i) Solar PV panels
   (ii) Batteries
   (iii) Controller
   (iv) Inverter

4. **List the applications of solar energy.** **[AU Nov/Dec, 2011]**

   Refer to section 4.2.1 for the applications of solar energy.

5. **Mention the factors that determine the wind power.** **[AU Nov/Dec, 2012]**

   The factors that determine wind power are:
   (i) Density of the air
   (ii) Swept area of the rotor and
   (iii) Velocity of the wind speed

6. **List the advantages and disadvantages of wind energy.** **[AU April/May, 2011]**

   Refer to section 4.2.2 for the advantages and disadvantages of wind energy.

7. **Draw the block diagram of wind energy generation system.** **[AU April/May, 2010]**

   The block diagram of wind energy generation system is shown in Figure 4.5.

8. **List the applications of wind energy.** **[AU April/May, 2009]**

   Refer to section 4.2.2 for the applications of wind energy.

9. **Define the term MSCP and MHCP.** **[AU April/May, 2012]**

   Mean Spherical Candle Power (MSCP): Mean spherical candle power (MSCP) is the mean or average of candle power of a source of light in all directions in all the planes.

   Mean horizontal candle power (MHCP): Mean horizontal candle power (MHCP) is the mean of the candle power in all directions in the horizontal plane containing source of light.

10. Define luminous intensity. **[AU Nov/Dec, 2012]**

The amount of light power emitting from a point source within a solid angle of one steradian is called luminous intensity.

11. State laws of illumination and its limitation in actual practice. **[AU Nov/Dec, 2012]**

Refer section 4.3.2 for the laws of illumination and its limitation.

12. List the various factors that affect the design of lighting system. **[AU April/May, 2011]**

Refer to section 4.4.1 for the factors affecting the design of lighting system.

13. What are the requirements of good lighting? **[AU April/May, 2012]**

Refer to section 4.4.1 for the factors affecting the design of lighting system.

14. Define solid angle. **[AU Nov/Dec, 2009]**

The angle subtended at a point in space by an area is called solid angle. The solid angle is expressed in steradians as shown in Figure UQ4.14.

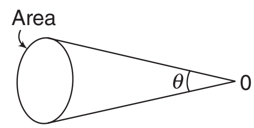


Figure UQ4.14 Illustration of solid angle

$$\text{Solid angle} = \frac{\text{area}}{(\text{radius})^2} = \frac{A}{r^2}$$

The solid angle subtended by a point in all directions in space is given by

$$\text{Solid angle} = \frac{\text{area of sphere}}{(\text{radius})^2} = \frac{4\pi r^2}{r^2} = 4\pi$$

15. Distinguish between direct and indirect lighting. **[AU Nov/Dec, 2009]**

Refer to section 4.4 for the comparison between direct and indirect lightings.

16. List the different types of lamps. **[AU Nov/Dec, 2010]**

Refer to section 4.5 for the different types of lamps.

17. What are the components of refrigeration system? **[AU April/May, 2010]**

Refer to section 4.6.1 for the components of refrigeration system.

18. What are the types of air condition system? **[AU Nov/Dec, 2011]**

Refer to section 4.7.1 for the types of air condition system.

19. Write the charge and discharge equations of Li-ion battery. **[AU Nov/Dec, 2008]**

Refer to section 4.8.4 for the charge and discharge equation of Li-ion battery.

20. Compare primary and secondary batteries. **[AU Nov/Dec, 2010]**

Refer to section 4.8.1 for the comparison between primary and secondary batteries.

21. Write the charge and discharge equation of Ni-Cd battery. **[AU April/May, 2010]**

Refer to section 4.8.3 for the charge and discharge equation of Ni-Cd battery.

22. What are the requirements of protection? [AU April/May, 2010]

Refer to section 4.9.1 for the requirements of protection.

23. What is the necessity of earthing? [AU April/May, 2011]

Refer to section 4.10.1 for the necessity of earthing.

24. Define: (i) Earth electrode and (ii) Earth continuity conductor. [AU April/May, 2012]

(i) **Earth Electrode:** The conductor that is buried inside the ground to provide safety for the electrical system is called earth electrode and it is available in different shapes.

(ii) **Earth Continuity Conductor:** The conductor wire or strip that is used to connect different electrical devices is called earth continuity conductor. It is also defined as the conductor wire used in connecting earthing lead and electrical devices and it is available in different shapes.

25. What is the importance of arc resistance? On which factor does it depend? [AU April/May, 2010]

Refer to section 4.11.2 for the importance of arc resistance and the factors to which it depends on.

26. Distinguish between recovery voltage and re-striking voltage. [AU Nov/Dec, 2011]

The recovery voltage is the normal frequency RMS voltage which appears across the contacts of circuit breaker after the final arc extinction and it is approximately equal to the system voltage, whereas the re-striking voltage is the transient voltage which is obtained across the contacts when the current becomes zero during the arcing period.

27. What are the basic requirements of a circuit breaker? [AU Nov/Dec, 2011]

A switching device which can be used to make or open a circuit either manually or automatically or using remote control under different conditions like normal and under fault conditions is known as circuit breaker. A special attention must be given in designing a circuit breaker as the modern power system deals with very high currents to safely interrupt the arc produced during its operation.

28. Write the effects of arc resistance. [AU Nov/Dec, 2012]

It is very important to extinguish the arc as early as possible before it causes serious damage to the system. As the arc developed forms a conductive path for electricity, the fault current flowing through the circuit breaker will not be interrupted till the arc is extinguished. The most important designing criteria of a circuit breaker are to provide proper technique in quenching the arc for a quick and safe fault current interruption.

29. What is meant by relay operating time? [AU Nov/Dec, 2012]

The relay operation time is the time taken between the instant at which the actuating signal is energised and the instant at which the relay contacts are closed.

30. List the methods of arc interruption. [AU Nov/Dec, 2012; April/May, 2015]

The two different methods by which the arc can be extinguished are:

(i) High resistance method and

(ii) Low resistance method or current zero method.

31. How do you classify the circuit breaker? **[AU Nov/Dec, 2012]**

The circuit breaker is classified as: (i) medium in which the circuit breaker operates (ii) actuating signal in which it works (iii) construction type and (iv) voltage level.

32. Write the operational difference between a fuse and a circuit breaker. **[AU Nov/Dec, 2012]**

Refer to section 4.12.5 for the difference between fuse and circuit breaker.

33. What are the functions of protective relays? **[AU April/May, 2015]**

Refer to section 4.13 for the functions of protective relays.

34. Differentiate between a fuse and a protective relay. **[AU April/May, 2010]**

Refer to section 4.12 and 4.13 for the comparison between fuse and protective relay.

35. Define the term "maximum demand". **[AU Nov/Dec, 2012]**

The maximum of all the demands that a particular station supplies in a given period is called as maximum demand. It depends on the maximum installed capacity of the station and maximum kWh generated from the station.

36. Write the effect of power factor in energy consumption billing. **[AU April/May, 2014]**

The power factor of the load is inversely proportional to the tariff of the electrical energy consumed. If the power factor is low, additional devices are required to correct the power factor and hence the tariff will be high.

37. What is tariff? What are its objectives? **[AU Nov/Dec, 2013]**

Refer to section 4.14.1 for the tariff and its objectives.

38. Name the different types of tariffs. **[AU April/May, 2014]**

Refer to section 4.14.4 for the different types of tariff.

39. What are the factors affecting the tariff? **[AU Nov/Dec, 2013]**

Refer to section 4.14.2 for the factors affecting the tariff.

40. What is two-part tariff? **[AU Nov/Dec, 2012]**

Refer to section 4.14.4 for the two-part tariff.

Review Questions

1. List the disadvantages of conventional energy sources.
2. Explain the generation of electrical power from solar energy.
3. Explain the components of solar PV power system.
4. Explain the different types of solar PV power generation systems.
5. List the advantages, disadvantages and applications of solar energy.
6. Explain the power curve of the wind turbine.
7. Define start up, cut in, cut out and rated speed.

8. With a neat diagram, explain the components of wind turbine.
9. How are wind turbines classified? Explain.
10. List the advantages, disadvantages and applications of wind energy.
11. Explain the different terms related to illumination.
12. With neat diagram, explain the laws of illumination.
13. Discuss the different lighting schemes.
14. Explain the factors required in designing of lighting scheme.
15. With neat diagram, explain the construction and working of incandescent and fluorescent lamp.
16. With neat diagram, explain the construction and working of sodium vapour lamp.
17. Explain the working of low pressure mercury discharge lamp with neat circuit diagram. Describe the construction and principle of operation of mercury vapour lamp.
18. Explain refrigeration and air-conditioning with necessary diagrams.
19. Discuss the construction and working of a battery.
20. Differentiate primary and secondary batteries.
21. Explain the charging and discharging equations of lead acid batteries.
22. How Nickel cadmium battery charges and discharges the energy.
23. With necessary equations, explain lithium ion battery.
24. Compare Pb-acid, Ni-Cd and Li-ion batteries.
25. List the advantages, disadvantages and applications of Pb-acid, Ni-Cd, and Li-ion batteries.
26. Discuss the terms related to earthing.
27. Explain the different methods of earthing.
28. With necessary diagram explain the operation of circuit breaker.
29. Discuss the different methods available to extinguish the arc.
30. Explain the operation of fuse and protective relays.
31. What is tariff? Discuss and compare various tariffs used in practice.
32. Explain two part tariff. Also explain how the total cost is reduced in this type.
33. A lamp giving 300 *CP* in all directions below horizontal is suspended 2 m above the centre of square table 1m aside. Calculate the maximum and minimum illuminations. [*Ans.* 75 lux and 62.9 lux]
34. An incandescent lamp rate 230 V takes 1.2 A and emits 8000 lumens. Calculate the efficiency in lumens/watt. [*Ans.* 28.99 per cent]
35. Two lamp posts of 200 CP each at height of 15 m and 30 m above the ground. Calculate illumination midway between them. [*Ans.* 0.0817 lux]

CHAPTER 5

Electronic Devices and Circuits

5.1 Introduction

Electronics has been defined as that branch of science and technology which relates to the conduction of electricity through vacuum by electrons alone or through gases by electrons and ions. Basically, it is a study of electron devices and their utilisation. An electron device is that in which electrons flow through a vacuum or gas or semiconductor. Electronics has a wide range of applications, such as rectification, amplification, power generation, industrial control, photo-electricity, communications and so on. The electronic industry turns out a variety of items in the range of consumer electronics, control and industrial electronics, communication and broadcasting equipment, biomedical equipment, calculators, computers, microprocessors, aerospace and defense equipment and components.

This chapter deals with the construction, operation and characteristics of some conventional semiconductor devices like PN junction diode, Bipolar Junction Transistor (BJT), Junction Field Effect Transistor (JFET), Metal Oxide Semiconductor Field Effect Transistor (MOSFET) and Zener diode.

This chapter also discusses about the characteristics and applications of operational amplifier (op-amp) which is a fundamental building block of analogue circuit design. The op-amp can be used in amplifiers and signal processing applications involving DC to several MHz of frequency ranges. The circuits using op-amp, namely, inverting and non-inverting amplifier, RC and LC sine-wave oscillator, precision half-wave and full-wave rectifiers, integrator, differentiator, digital to analogue converter (DAC) and analogue to digital converter (ADC) are discussed in this chapter. The operation of non-sinusoidal oscillators like multi-vibrators is also explained using 555 Timer IC. All electronic circuits need DC power supply either from battery or power pack units and it is not economical to depend upon battery power supply. Hence, the DC voltage regulated power supplies are employed to provide a stable DC voltage, independent of the load current and temperature. Linear regulator ICs are available for fixed positive and negative output voltages, and variable positive and negative output voltages. The linear voltage regulator ICs such as LM317/337 and IC 723 are also discussed in this chapter.

5.2 Types of Materials—Silicon and Germanium

All semiconductors have crystalline structure. The most commonly used semiconductor materials, germanium, silicon and gallium arsenide have practical applications in Electronics. The most frequently used semiconductors are germanium and silicon because the energy required to break their covalent bonds and release a free electron from their valence bands is lesser than that required for gallium arsenide. The energy required for releasing an electron from the valence band is 0.66 eV for germanium, 1.08 eV for silicon and 1.58 eV for gallium arsenide.

Germanium can be purified relatively well and crystallised easily. Germanium is an earth element and it is obtained from the ash of the certain coals or from the flue dust of the zinc smelters. The recovered germanium is in the form of germanium dioxide powder which is then reduced to pure germanium. Germanium diodes are used as infrared detectors in fibre-optic communication system because of narrower energy gap.

Silicon is an element found in most of the common rocks. Sand is silicon dioxide which is then reduced to 100% pure silicon. Silicon dioxide is a natural insulator which is useful in the fabrication of semiconductor devices and integrated circuits. Silicon is largely preferred to germanium because of its large gap energy, which produces improved device properties at high temperatures. Silicon is a better thermal conductor and is required to remove unavoidable heat developed in the device.

Gallium arsenide has higher electron mobility, μ_n which leads to faster switching capabilities. It has high temperature operating capabilities because of its larger energy gap.

5.3 *N*-TYPE AND *P*-TYPE MATERIALS

Semiconductors are classified as (i) intrinsic (pure), and (ii) extrinsic (impure) types. The extrinsic semiconductors are of *N*-type and *P*-type.

Intrinsic Semiconductor

A pure semiconductor is called an intrinsic semiconductor. As already explained in the first chapter, even at room temperature, some of the valence electrons may acquire sufficient energy to enter the conduction band to form free electrons. Under the influence of an electric field, these electrons constitute electric current. A missing electron in the valence band leaves a vacant space there, which is known as a hole, as shown in Figure 5.1. Holes also contribute to electric current.

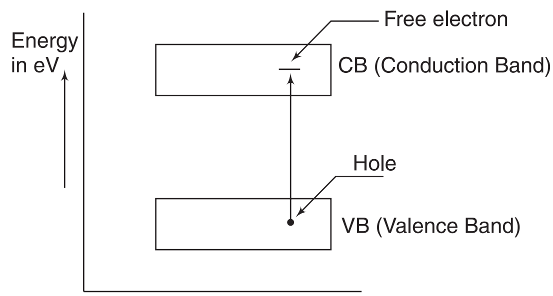


Figure 5.1 Creation of electron-hole pair in a semiconductor

In an intrinsic semiconductor, even at room temperature, electron-hole pairs are created. When an electric field is applied across an intrinsic semiconductor, the current conduction takes place due to free electrons and holes. Under the influence of an electric field, the total current through the semiconductor is the sum of currents due to free electrons and holes.

Though the total current inside the semiconductor is due to free electrons and holes, the current in the external wire is fully by electrons. In Figure 5.2, holes being positively charged move towards the negative terminal of the battery. As the holes reach the negative terminal of the battery, electrons enter the semiconductor near the terminal (X) and combine with the holes. At the same time, the loosely held electrons near the positive

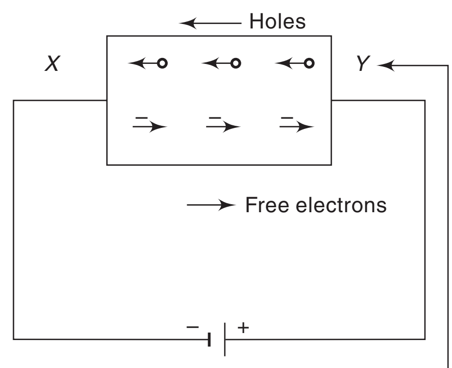


Figure 5.2 Current conduction in semiconductor

terminal (Y) are attracted towards the positive terminal. This creates new holes near the positive terminal which again drift towards the negative terminal.

Extrinsic Semiconductor

Due to the poor conduction at room temperature, the intrinsic semiconductor as such, is not useful in the electronic devices. Hence, the current conduction capability of the intrinsic semiconductor should be increased. This can be achieved by adding a small amount of impurity to the intrinsic semiconductor, so that it becomes impure or extrinsic semiconductor. This process of adding impurity is known as doping.

The amount of impurity added is extremely small, say 1 to 2 atoms of impurity for 10^6 intrinsic atoms.

N-type Semiconductor

A small amount of pentavalent impurity such as arsenic, antimony, or phosphorus is added to the pure semiconductor (germanium or silicon crystal) to get an N-type semiconductor.

The germanium atom has four valence electrons and antimony has five valence electrons. As shown in Figure 5.3, each antimony atom forms a covalent bond with surrounding four germanium atoms. Thus, four valence electrons of the antimony atom form a covalent bond with four valence electrons of an individual germanium atom and the fifth valence electron is left free which is loosely bound to the antimony atom.

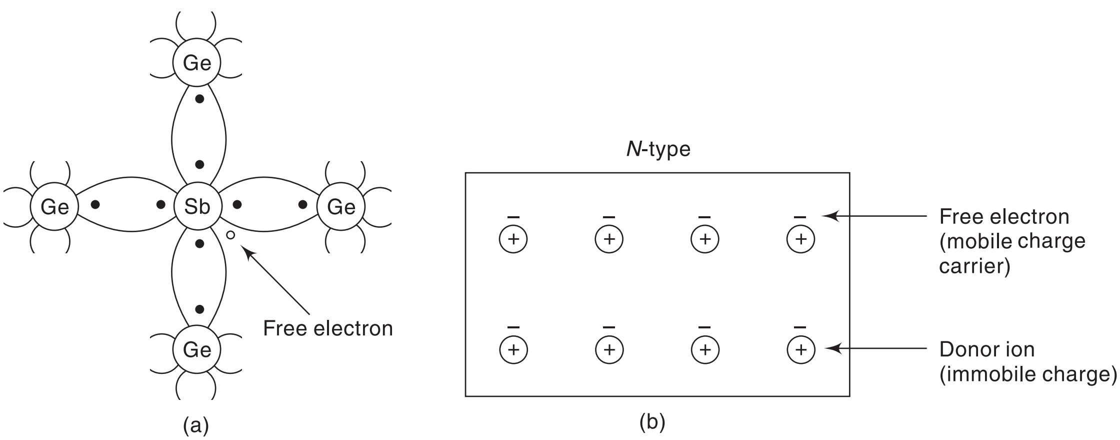


Figure 5.3 N-type semiconductor: (a) Formation of covalent bonds (b) Charged carriers

This loosely bound electron can be easily excited from the valence band to the conduction band by the application of electric field or increasing the thermal energy. Thus, every antimony atom contributes one conduction electron without creating a hole. Such a pentavalent impurity is called a donor impurity because it donates one electron for conduction. On giving an electron for conduction, the donor atom becomes a positively charged ion because it loses one electron. But it cannot take part in conduction because it is firmly fixed in the crystal lattice.

Thus, the addition of a pentavalent impurity (antimony) increases the number of electrons in the conduction band, thereby increasing the conductivity of an N-type semiconductor. As a result of doping, the number of free electrons far exceeds the number of holes in an N-type semiconductor. So electrons are called majority carriers and holes are called minority carriers.

P-type Semiconductor

A small amount of trivalent impurity such as aluminium or boron is added to the pure semiconductor to get the *P*-type semiconductor. The germanium (Ge) atom has four valence electrons and boron has three valence electrons as shown in Figure 5.4. Three valence electrons in boron form a covalent bond with four surrounding atoms of Ge leaving one bond incomplete which gives rise to a hole. Thus, the trivalent impurity (boron) when added to the intrinsic semiconductor (germanium) introduces a large number of holes in the valence band. These positively charged holes increase the conductivity of the P-type semiconductor. A trivalent impurity such as boron is called acceptor impurity because it accepts free electrons in the place of holes. As each boron atom donates a hole for conduction, it becomes a negatively charged ion. As the number of holes is very much greater than the number of free electrons in a *P*-type material, holes are termed *majority carriers* and electrons *minority carriers*.

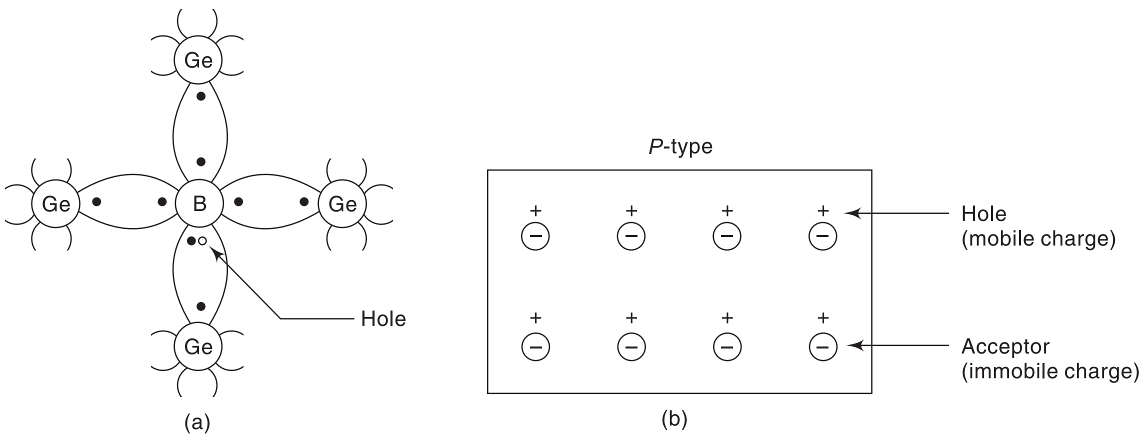


Figure 5.4 P-type semiconductor: (a) Formation of covalent bonds (b) Charged carriers

5.4 *PN* JUNCTION DIODE

5.4.1 Forward Bias with *V–I* Characteristics

When the positive terminal of the battery is connected to the *P*-type and negative terminal to the *N*-type of the *PN* junction diode, the bias applied is known as forward bias.

Operation

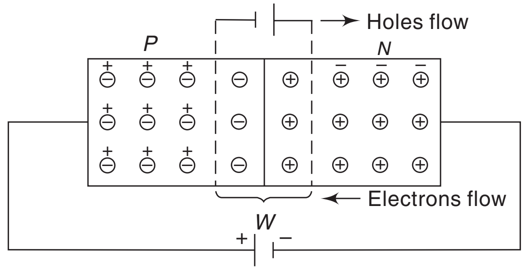


Figure 5.5 PN junction diode under forward bias

As shown in Figure 5.5, the applied potential with external battery acts in opposition to the internal potential barrier and disturbs the equilibrium. As soon as equilibrium is disturbed by the application of an external voltage, the Fermi level is no longer continuous across the junction. Under the forward-bias condition, the applied positive potential repels the holes in the *P*-type region so that the holes move towards the junction and the applied negative potential repels the electrons in the *N*-type region and the electrons move towards the junction. Eventually, when the applied potential is more than the internal barrier potential, the depletion region and internal potential barrier disappear.

V–I Characteristics of a Diode under Forward Bias

Under forward-bias condition, the *V–I* characteristics of a *PN* Junction diode are shown in Figure 5.6. As the forward voltage (V_F) is increased, for $V_F < V_O$, the forward current I_F is almost zero (region OA) because the potential barrier prevents the holes from *P*-region and electrons from *N*-region to flow across the depletion region in the opposite direction.

For $V_F > V_O$, the potential barrier at the junction completely disappears and hence, the holes cross the junction from *P*-type to *N*-type and the electrons cross the junction in the opposite direction, resulting in relatively large current flow in the external circuit.

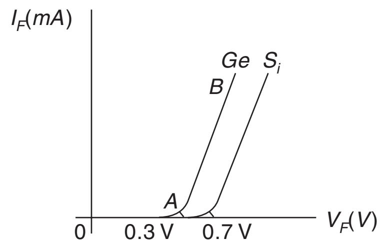


Figure 5.6 V–I characteristics of a diode under forward-bias condition

A feature worth to be noted in the forward characteristics shown in Figure 5.6 is the cut in or threshold voltage (V_r) below which the current is very small. It is 0.3 V and 0.7 V for germanium and silicon, respectively. At the cut-in voltage, the potential barrier is overcome and the current through the junction starts to increase rapidly.

5.4.2 Reverse Bias with *V–I* Characteristics

When the negative terminal of the battery is connected to the *P*-type and positive terminal of the battery is connected to the *N*-type of the *PN* junction diode, the bias applied is known as reverse bias.

Operation

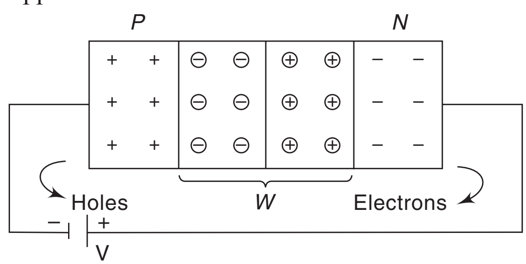


Figure 5.7 PN junction under reverse bias

Under applied reverse bias as shown in Figure 5.7, holes which form the majority carriers of the *P*-side move towards the negative terminal of the battery and electrons which form the majority carrier of the *N*-side are attracted towards the positive terminal of the battery. Hence, the width of the depletion region which is depleted of mobile charge carriers increases. Thus, the electric field produced by applied reverse bias, is in the same direction as the electric field of the potential barrier. Hence, the resultant potential barrier is increased which prevents the flow of majority carriers in both directions; the depletion width, *W*, is proportional to $\sqrt{V_o}$ under reverse bias. Therefore, theoretically, no current should flow in the external circuit. But in practice, a very small current of the order of a few microampere flows under reverse bias as shown in Figure 5.8. Electrons forming covalent bonds of the semiconductor atoms in the P- and N-type regions may absorb sufficient energy from heat and light to cause breaking of some covalent bonds. Hence, electron-hole pairs are continually produced in both the regions. Under the reverse-bias condition, the thermally generated holes in the *P*-region are attracted towards the negative terminal of the battery and the electrons in the *N*-region are

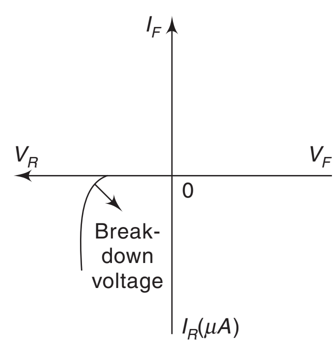


Figure 5.8 V–I characteristics under reverse bias

attracted towards the positive terminal of the battery. Consequently, the minority carriers, electrons in the *P*-region and holes in the *N*-region, wander over to the junction and flow towards their majority carrier side giving rise to a small reverse current. This current is known as reverse saturation current, I_o. The magnitude of the reverse saturation current mainly depends upon junction temperature because the major source of minority carriers is thermally broken covalent bonds.

For large applied reverse bias, the free electrons from the *N*-type moving towards the positive terminal of the battery acquire sufficient energy to move with high velocity to dislodge valence electrons from semiconductor atoms in the crystal. These newly liberated electrons, in turn, acquire sufficient energy to dislodge other parent electrons. Thus, a large number of free electrons are formed which is commonly called an avalanche of free electrons. This leads to the breakdown of the junction leading to very large reverse current. The reverse voltage at which the junction breakdown occurs is known as *breakdown voltage*, V_{BD}.

5.4.3 *PN* Junction as a Diode

Figure 5.9 shows the current-voltage characteristics of *PN* junction. The characteristics of the *PN* junction vary enormously depending upon the polarity of the applied voltage. For a forward-bias voltage, the current increases exponentially with the increase of voltage. A small change in the forward-bias voltage increases the corresponding forward-bias current by orders of magnitude and hence, the forward-bias *PN* junction will have a very small resistance. The level of current flowing across a forward-biased *PN* junction largely depends upon the junction area. In the reverse-bias direction, the current remains small, i.e., almost zero, irrespective of the magnitude of the applied voltage and hence the reverse-bias *PN* junction will have a high resistance. The reverse-bias current depends on the area, temperature and type of semiconductor material.

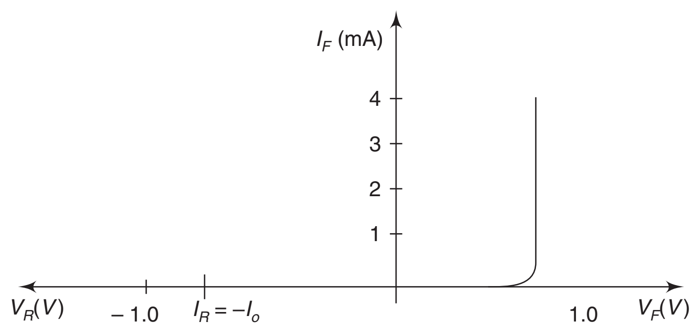


Figure 5.9 Ideal *I–V* characteristics of a *PN* junction diode

The semiconductor device that displays these *I–V* characteristics is called a *PN* junction diode. Figure 5.10 shows the *PN* junction diode with forward-bias and reverse-bias and their circuit symbols. The metal contacts are indicated with which the homogeneous *P*-type and *N*-type materials are provided. Thus, two metal-semiconductor junctions, one at each end of the diode, are introduced. The contact potential across these junctions is approximately independent of the direction and magnitude of the current. A contact of this type is called an *ohmic contact*, which has low resistance. In the forward bias, a relatively large current is produced by a fairly small applied voltage. In the reverse bias, only a very small current, ranging from nanoamps to microamps is produced. The diode can be used as a voltage controlled switch, i.e., OFF for a reverse-bias voltage and ON for a forward-bias voltage.

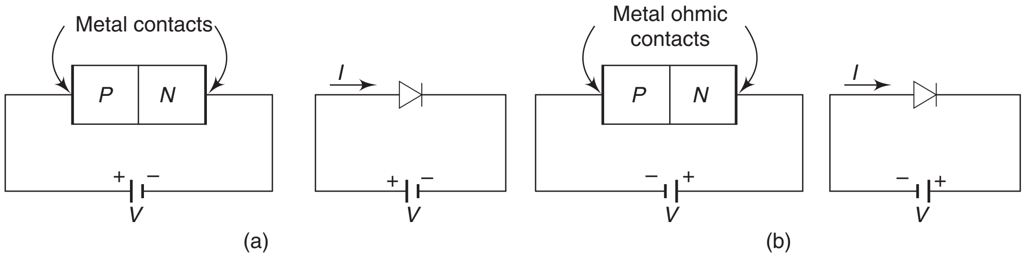


Figure 5.10 (a) Forward-biased *PN* junction diode and its circuit symbol (b) Reverse-biased *PN* junction diode and its circuit symbol

When a diode is reverse-biased by at least 0.1 V, the diode current is $I_R = -I_o$. As the current is in the reverse direction and is a constant, it is called the diode *reverse saturation current.* Real diodes exhibit reverse-bias current that are considerably larger than I_o. This additional current is called a *generation current* which is due to electrons and holes being generated within the space-charge region. A typical value of I_0 may be 10^{-14} A and a typical value of reverse-bias current may be 10^{-9} A.

5.5 Semiconductor Diode

In addition to the *PN* junction diode, other types of diodes are also manufactured for specific applications. These special diodes are two-terminal devices with their doping levels carefully selected to give the desired characteristics.

Zener Diode

When the reverse voltage reaches breakdown voltage in a normal *PN* junction diode, the current through the junction and the power dissipated at the junction will be high. Such an operation is destructive and the diode gets damaged. Whereas diodes can be designed with adequate power dissipation capabilities to operate in the breakdown region. One such diode is known as the Zener diode. The Zener diode is heavily doped than the ordinary diode.

From the *V–I* characteristics of the Zener diode, shown in Figure 5.11, it is found that the operation of the Zener diode is same as that of an ordinary *PN* diode under forward-biased condition. Whereas under reverse-baised condition, breakdown of the junction occurs. The breakdown voltage depends upon the amount of doping. If the diode is heavily doped, the depletion layer will be thin and, consequently, breakdown occurs at lower reverse voltage and further, the breakdown voltage is sharp. Whereas a lightly doped diode has a higher breakdown voltage. Thus, breakdown voltage can be selected with the amount of doping.

The sharp increasing currents under breakdown conditions are due to the following two mechanisms.

1. Avalanche breakdown
2. Zener breakdown

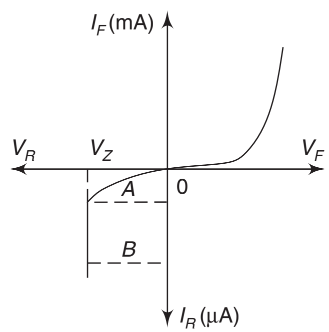


Figure 5.11 *V–I* characteristics of a Zener diode

Avalanche Breakdown

As the applied reverse bias increases, the field across the junction increases correspondingly. Thermally generated carriers, while traversing the junction, acquire a large amount of kinetic energy from this field. As a result, the velocity of these carriers increases. These electrons disrupt covalent bond by colliding with immobile ions and create new electron-hole pairs. These new carriers again acquire sufficient energy from the field and collide with other immobile ions thereby generating further electron-hole pairs. This process is cumulative in nature and results in generation of avalanche of charge carriers within a short time. This mechanism of carrier generation is known as *avalanche multiplication*. This process results in flow of large amount of current at the same value of reverse bias.

Zener Breakdown

When the *P*- and *N*-regions are heavily doped, direct rupture of covalent bonds takes place because of the strong electric fields, at the junction of the *PN* diode. The new electron-hole pairs so created increase the reverse current in a reverse-biased *PN* diode. The increase in current takes place at a constant value of reverse bias typically below 6 V for heavily doped diodes. As a result of heavy doping of *P*- and *N*-regions, the depletion-region width becomes very small and for an applied voltage of 6 V or less, the field across the depletion region becomes very high, of the order of 10^7 V/m, making conditions suitable for Zener breakdown. For lightly doped diodes, Zener breakdown voltage becomes high and breakdown is then predominantly by avalanche multiplication. Though Zener breakdown occurs for lower breakdown voltage and avalanche breakdown occurs for higher breakdown voltage, such diodes are normally called Zener diodes.

Applications

From the Zener characteristics shown in Figure 5.11, under the reverse-bias condition, the voltage across the diode remains almost constant although the current through the diode increases as shown in region *AB*. Thus, the voltage across the Zener diode serves as a reference voltage. Hence, the diode can be used as a voltage regulator.

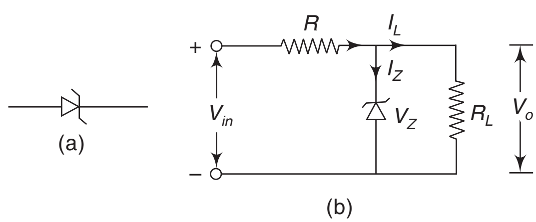


Figure 5.12 Zener diode: (a) Circuit symbol (b) As a voltage regulator

In Figure 5.12, it is required to provide constant voltage across load resistance R_L, whereas the input voltage may be varying over a range. As shown, Zener diode is reverse biased and as long as the input voltage does not fall below V_Z (Zener breakdown voltage), the voltage across the diode will be constant and hence the load voltage will also be constant.

5.6 Bipolar Junction Transistor (BJT)

A Bipolar Junction Transistor (BJT) is a three-terminal semiconductor device in which the operation depends on the interaction of both majority and minority carriers and hence, the name *bipolar*. The BJT is analogous to a vacuum triode and is comparatively smaller in size. It is used in amplifier and oscillator circuits, and as a switch in digital circuits. It has wide applications in computers, satellites, and other modern communication systems.

5.6.1 Construction of BJT

The BJT consists of a silicon (or germanium) crystal in which a thin layer of *N*-type silicon is sandwiched between two layers of *P*-type silicon. This transistor is referred to as *PNP*. Alternatively, in an *NPN* transistor, a layer of *P*-type material is sandwiched between two layers of *N*-type material. The two types of the BJT are represented in Figure 5.13.

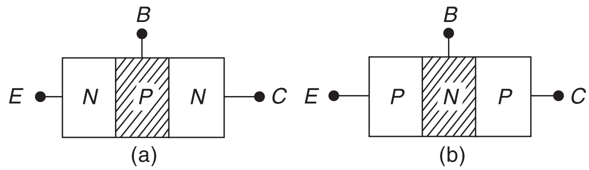


Figure 5.13 Transistor: (a) *NPN* (b) *PNP*

The symbolic representation of the two types of the BJT is shown in Figure 5.14. The three portions of the transistor are emitter, base, and collector, shown as *E*, *B*, and *C*, respectively. The arrow on the emitter specifies the direction of current flow when the *EB* junction is forward biased.

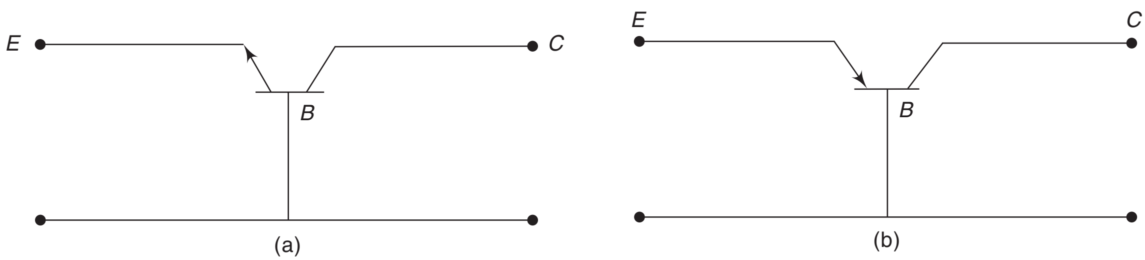


Figure 5.14 Circuit symbol: (a) *NPN* transistor (b) *PNP* transistor

The emitter is heavily doped so that it can inject a large number of charge carriers into the base. The base is lightly doped and very thin. It passes most of the injected charge carriers from the emitter into the collector. The collector is moderately doped.

5.6.2 Transistor Biasing

As shown in Figure 5.15, usually the emitter-base junction is forward biased and the collector-base junction is reverse biased. Due to the forward bias on the emitter-base junction, an emitter current flows through the base into the collector. Though the collector-base junction is reverse biased, almost the entire emitter current flows through the collector circuit.

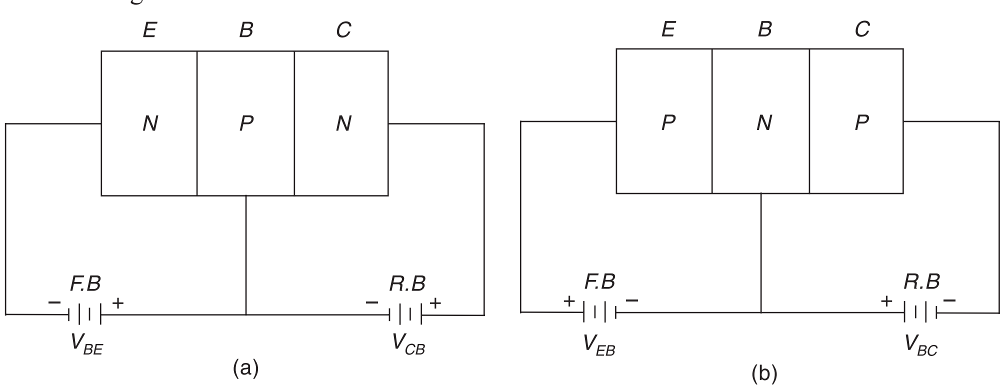


Figure 5.15 Transistor biasing: (a) *NPN* transistor (b) *PNP* transistor

5.6.3 Operation of an *NPN* Transistor

As shown in Figure 5.16, the forward bias applied to the emitter base junction of an *NPN* transistor causes a lot of electrons from the emitter region to cross over to the base region. As the base is lightly doped with *P*-type impurity, the number of holes in the base region is very small and hence, the number of electrons that combine with holes in the *P*-type base region is also very small. Hence, a few electrons combine with holes to constitute a base current I_B. The remaining electrons (more than 95%) cross over into the collector region to constitute a collector current I_C. Thus, the base and collector current summed up gives the emitter current, i.e., $I_E = -(I_C + I_B)$.

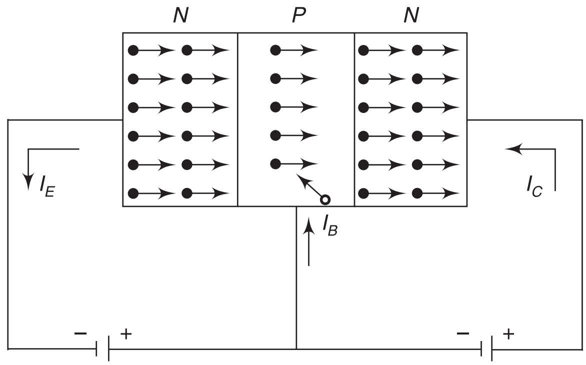


Figure 5.16 Current in an *NPN* transistor

In the external circuit of the *NPN* bipolar junction transistor, the magnitudes of the emitter current I_E, the base current I_B, and the collector current I_C are related by $I_E = I_C + I_B$.

5.6.4 Operation of a *PNP* Transistor

As shown in Figure 5.17, the forward bias applied to the emitter-base junction of a *PNP* transistor causes a lot of holes from the emitter region to cross over to the base region as the base is lightly doped with *N*-type impurity. The number of electrons in the base region is very small and hence, the number of holes combined with electrons in the *N*-type base region is also very small. Hence, a few holes combined with electrons to constitute a base current I_B. The remaining holes (more than 95%) cross over into the collector region to constitute a collector current I_C. Thus, the collector and base current when summed up gives the emitter current, i.e., $I_E = -(I_C + I_B)$.

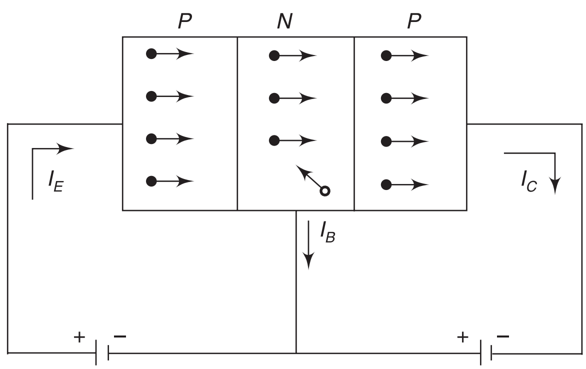


Figure 5.17 Current in a *PNP* transistor

In the external circuit of the *PNP* bipolar junction transistor, the magnitudes of the emitter current I_E, the base current I_B and the collector current I_C are related by

$$I_E = I_C + I_B \tag{5.1}$$

This equation gives the fundamental relationship between the currents in a bipolar transistor circuit. Also, this fundamental equation shows that there are current amplification factors α and β in common-base transistor configuration and common-emitter transistor configuration respectively for the static (DC) currents, and for small changes in the currents.

Large-Signal Current Gain (α)

The large-signal current gain of a common-base transistor is defined as the ratio of the negative of the collector-current increment to the emitter-current change from cut-off ($I_E = 0$) to I_E, i.e.,

$$\alpha = \frac{(I_C - I_{CBO})}{I_E - 0} \tag{5.2}$$

where I_{CBO} (or I_{CO}) is the reverse saturation current flowing through the reverse-biased collector-base junction, i.e., the collector-to-base leakage current with the emitter open. As the magnitude of I_{CBO} is negligible when compared to I_E, the above expression can be written as

$$\alpha = \frac{I_C}{I_E} \tag{5.3}$$

Since I_C and I_E are flowing in opposite directions, α is always positive. The typical value of α ranges from 0.90 to 0.995. Also, α is not a constant but varies with emitter current I_E, collector voltage V_{CB}, and temperature.

General Transistor Equation

In the active region of the transistor, the emitter is forward biased and the collector is reverse biased. The generalized expression for collector current I_C for collector junction voltage V_C and emitter current I_E is given by

$$I_C = -\alpha I_E + I_{CBO}\left(1 - e^{V_C/V_T}\right) \tag{5.4}$$

If V_C is negative and $|V_C|$ is very large compared with V_T, then the above equation reduces to

$$I_C = -\alpha I_E + I_{CBO} \tag{5.5}$$

If V_C, i.e., V_{CB}, is a few volts, then I_C is independent of V_C. Hence, the collector current I_C is determined only by the fraction α of the current I_E flowing in the emitter.

Relation Among I_C, I_B, and I_{CBO}

From Eqn. (5.5), we have

$$I_C = -\alpha I_E + I_{CBO}$$

Since I_C and I_E are flowing in opposite directions,

$$I_E = -(I_C + I_B)$$

Therefore,

$$I_C = -\alpha[-(I_C + I_B)] + I_{CBO}$$

$$I_C - \alpha I_C = \alpha I_B + I_{CBO}$$

$$I_C(1 - \alpha) = \alpha I_B + I_{CBO}$$

$$I_C = \frac{\alpha}{1-\alpha} I_B + \frac{I_{CBO}}{1-\alpha}$$

Since

$$\beta = \frac{\alpha}{1-\alpha} \tag{5.6}$$

the above expression becomes

$$I_C = (1 + \beta) I_{CBO} + \beta I_B \tag{5.7}$$

Relation Among I_C, I_B, and I_{CEO}

In the common-emitter (CE) transistor circuit, I_B is the input current and I_C is the output current. If the base circuit is open, i.e., $I_B = 0$, then a small collector current flows from the collector to emitter. This is denoted as I_{CEO}, the collector-emitter current with base open. This current I_{CEO} is also called the collector-to-emitter leakage current.

In this CE configuration of the transistor, the emitter-base junction is forward-biased and collector-base junction is reverse-biased and hence, the collector current I_C is the sum of the part of the emitter current I_E that reaches the collector, and the collector-emitter leakage current I_{CEO}. Therefore, the part of I_E, which reaches collector is equal to $(I_C - I_{CEO})$.

Hence, the *large-signal current gain* (β) is defined as,

$$\beta = \frac{(I_C - I_{CEO})}{I_B} \tag{5.8}$$

From the equation, we have

$$I_C = \beta I_B + I_{CEO} \tag{5.9}$$

Relation Between I_{CBO} and I_{CEO}

Comparing Eqs (5.7) and (5.9), we get the relationship between the leakage currents of transistor common-base (CB) and common-emitter (CE) configurations as

$$I_{CEO} = (1 + \beta)\, I_{CBO} \tag{5.10}$$

From this equation, it is evident that the collector-emitter leakage current (I_{CEO}) in CE configuration is $(1 + \beta)$ times larger than that in CB configuration. As I_{CBO} is temperature-dependent, I_{CEO} varies by large amount when temperature of the junctions changes.

Expression for Emitter Current

The magnitude of emitter current is

$$I_E = I_C + I_B$$

Substituting Eqn. (5.7) in the above equation, we get

$$I_E = (1 + \beta)\, I_{CBO} + (1 + \beta)\, I_B \tag{5.11}$$

Substituting Eqn. (5.6) into Eqn. (5.11), we have

$$I_E = \frac{1}{1-\alpha} I_{CBO} + \frac{1}{1-\alpha} I_B \tag{5.12}$$

DC Current Gain (β_{dc} or h_{FE})

The DC current gain is defined as the ratio of the collector current I_C to the base current I_B. That is,

$$\beta_{\text{dc}} = h_{FE} = \frac{I_C}{I_B} \tag{5.13}$$

As I_C is large compared with I_{CEO}, the large-signal current gain (β) and the DC current gain (h_{FE}) are approximately equal.

5.6.5 Transistor Configurations and Input/Output Characteristics

When a transistor is to be connected in a circuit, one terminal is used as an input terminal, the other terminal is used as an output terminal, and the third terminal is common to the input and output. Depending upon the input, output, and common terminals, a transistor can be connected in three configurations. They are (i) Common Base (CB) configuration, (ii) Common Emitter (CE) configuration, and (iii) Common Collector (CC) configuration.

CB Configuration This is also called *grounded-base configuration*. In this configuration, the emitter is the input terminal, the collector is the output terminal, and the base is the common terminal.

CE Configuration This is also called *grounded-emitter configuration*. In this configuration, the base is the input terminal, the collector is the output terminal, and the emitter is the common terminal.

CC Configuration This is also called *grounded-collector configuration*. In this configuration, the base is the input terminal, the emitter is the output terminal, and the collector is the common terminal.

The supply voltage connections for normal operation of an *NPN* transistor in the three configurations are shown in Figure 5.18.

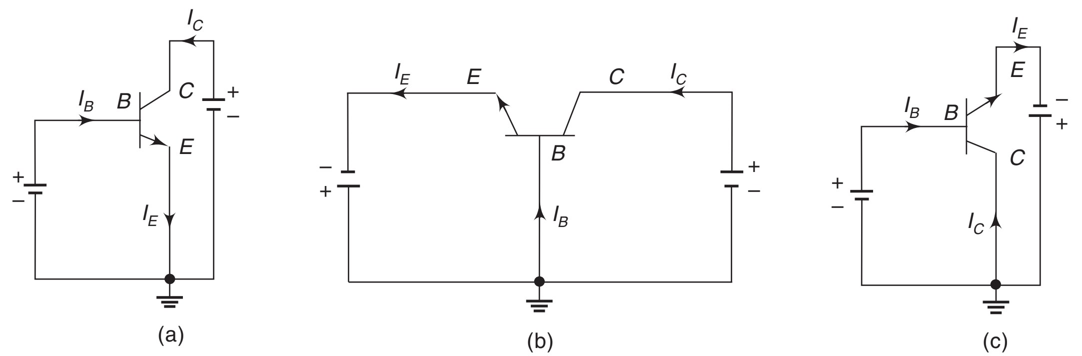


Figure 5.18 Transistor configuration: (a) Common emitter (b) Common base (c) Common collector

CB Configuration

The circuit diagram for determining the static characteristics curves of an *NPN* transistor in the common-base configuration is shown in Figure 5.19.

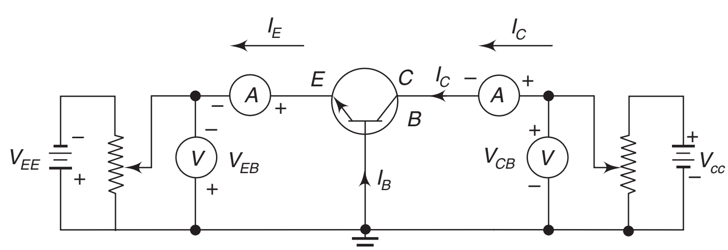


Figure 5.19 Circuit to determine CB static characteristics

Input Characteristics

To determine the input characteristics, the collector-base voltage V_{CB} is kept constant at zero volt and the emitter current I_E is increased from zero in suitable equal steps by increasing V_{EB}. This is repeated for higher fixed values of V_{CB}. A curve is drawn between emitter current I_E and emitter-base voltage V_{EB} at constant collector-base voltage V_{CB}. The input characteristics thus obtained are shown in Figure 5.20.

When V_{CB} is equal to zero and the emitter-base junction is forward biased as shown in the characteristics, the junction behaves as a forward-biased diode so that emitter current I_E increases rapidly with small increase in emitter-base voltage V_{EB}. When V_{CB} is increased keeping V_{EB} constant, the width of the base region will decrease. This effect results in an increase of I_E. Therefore, the curves shift towards the left as V_{CB} is increased.

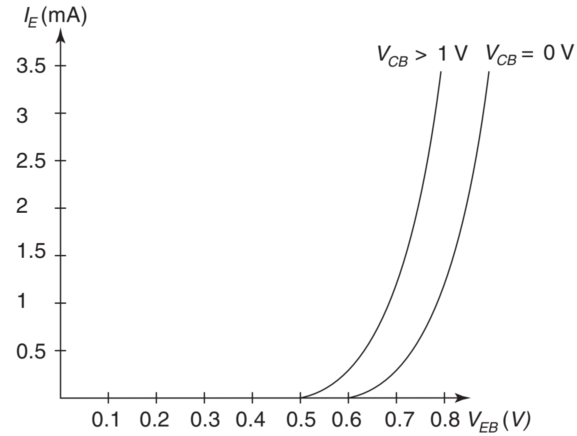


Figure 5.20 CB input characteristics

Output Characteristics

To determine the output characteristics, the emitter current I_E is kept constant at a suitable value by adjusting the emitter-base voltage V_{EB}. Then V_{CB} is increased in suitable equal steps and the collector current I_C is noted for each value of I_E. This is repeated for different fixed values of I_E. Now the curves of I_C versus V_{CB} are plotted for constant values of I_E and the output characteristics thus obtained is shown in Figure 5.21.

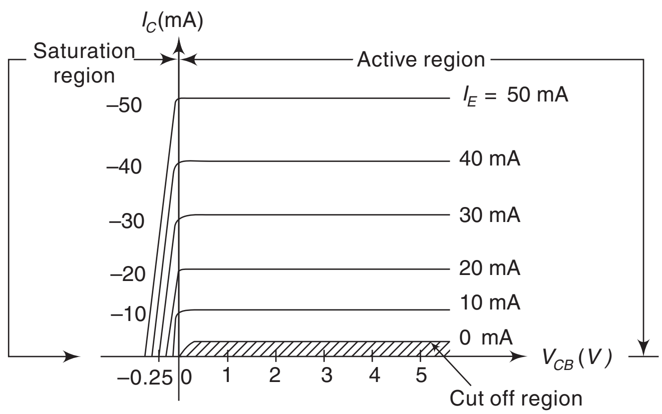


Figure 5.21 CB output characteristics

From the characteristics, it is seen that for a constant value of I_E, I_C is independent of V_{CB} and the curves are parallel to the axis of V_{CB}. Further, I_C flows even when V_{CB} is equal to zero. As the emitter-base junction is forward biased, the majority carriers, i.e., electrons, from the emitter are injected into the base region. Due to the action of the internal potential barrier at the reverse-biased collector-base junction, they flow to the collector region and give rise to I_C even when V_{CB} is equal to zero.

Early Effect or Base Width Modulation

As the collector voltage V_{CC} is made to increase the reverse bias, the space charge width between collector and base tends to increase, with the result that the effective width of the base decreases. This dependency of base width on collector-to-emitter voltage is known as the *Early effect*. This decrease in effective base width has three consequences:

(i) There is less chance for recombination within the base region. Hence, α increases with increasing $|V_{CB}|$.

(ii) The charge gradient is increased within the base, and consequently, the current of minority carriers injected across the emitter junction increases.

(iii) For extremely large voltages, the effective base width may be reduced to zero, causing voltage breakdown in the transistor. This phenomenon is called the *punch-through*.

For higher values of V_{CB}, due to Early effect, the value of α increases. For example, α changes, say from 0.98 to 0.985. Hence, there is a very small positive slope in the CB output characteristics and hence, the output resistance is not zero.

CE Configuration

Input Characteristics

To determine the input characteristics, the collector-to-emitter voltage is kept constant at zero volt, and the base current is increased from zero in equal steps by increasing V_{BE} in the circuit shown in Figure 5.22.

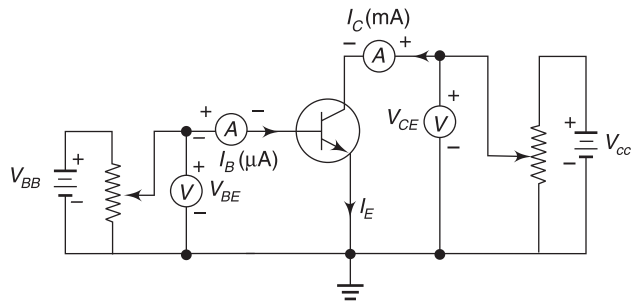


Figure 5.22 Circuit to determine CE static characteristics

The value of V_{BE} is noted for each setting of I_B. This procedure is repeated for higher fixed values of V_{CE}, and the curves of I_B vs. V_{BE} are drawn. The input characteristics thus obtained are shown in Figure 5.23.

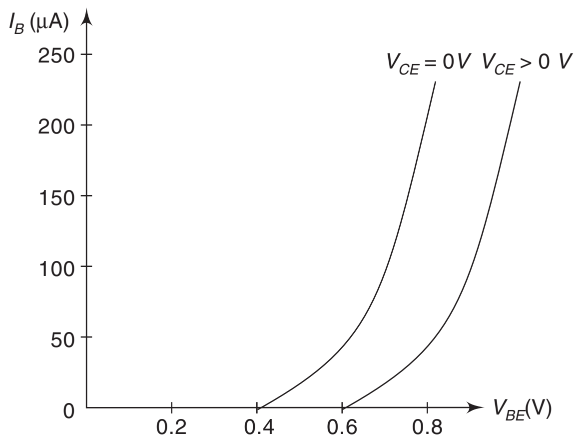


Figure 5.23 CE input characteristics

When $V_{CE} = 0$, the emitter-base junction is forward biased and the junction behaves as a forward biased diode. Hence, the input characteristic for $V_{CE} = 0$ is similar to that of a forward-biased diode. When V_{CE} is increased, the width of the depletion region at the reverse-biased collector-base junction will increase. Hence, the effective width of the base will decrease. This effect causes a decrease in the base current I_B. Hence, to get the same value of I_B as that for $V_{CE} = 0$, V_{BE} should be increased. Therefore, the curve shifts to the right as V_{CE} increases.

Output Characteristics

To determine the output characteristics, the base current I_B is kept constant at a suitable value by adjusting the base-emitter voltage, V_{BE}. The magnitude of the collector-emitter voltage V_{CE} is increased in suitable equal steps from zero and the collector current I_C is noted for each setting V_{CE}. Now, the curves of I_C versus V_{CE} are plotted for different constant values of I_B. The output characteristics thus obtained are shown in Figure 5.24.

From Eqs. (5.6) and (5.7), we have

$$\beta = \frac{\alpha}{1-\alpha} \text{ and } I_C = (1+\beta)\, I_{CBO} + \beta I_B$$

For larger values of V_{CE}, due to Early effect, a very small change in α is reflected in a very large change in β. For example, when $\alpha = 0.98$, $\beta = \frac{0.98}{1-0.98} = 49$. If α increases to 0.985, then $\beta = \frac{0.985}{1-0.985} = 66$. Here, a slight increase in α by about 0.5% results in an increase in β by about 34%. Hence, the output characteristics of CE configuration show a larger slope when compared with CB configuration.

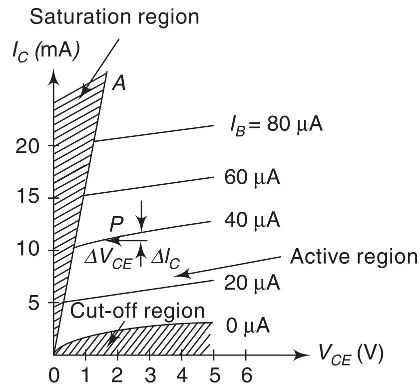


Figure 5.24 CE output characteristics

The output characteristics have three regions, namely, saturation region, cut-off region, and active region. The region of curves to the left of the line *OA* is called the *saturation region* (hatched), and the line *OA* is called the saturation line. In this region, both junctions are forward biased and an increase in the base current does not cause a corresponding large change in I_C. The ratio of $V_{CE(\text{sat})}$ to I_C in this region is called saturation resistance.

The region below the curve for $I_B = 0$ is called the *cut-off region* (hatched). In this region, both junctions are reverse biased. When the operating point for the transistor enters the cut-off region, the transistor is OFF. Hence, the collector current becomes almost zero and the collector voltage almost equals V_{CC}, the collector-supply voltage. The transistor is virtually an open circuit between collector and emitter.

The central region where the curves are uniform in spacing and slope is called the *active region* (unhatched). In this region, emitter-base junction is forward biased and the collector-base junction is reverse biased. If the transistor is to be used as a linear amplifier, it should be operated in the active region.

If the base current is subsequently driven large and positive, the transistor switches into the saturation region via the active region, which is traversed at a rate that is dependent on factors such as gain and frequency response. In this ON condition, large collector current flows and collector voltage falls to a very low value, called $V_{CE\text{sat}}$, typically around 0.2 V for a silicon transistor. The transistor is virtually a short circuit in this state.

High-speed switching circuits are designed in such a way that transistors are not allowed to saturate, thus reducing switching times between ON and OFF times.

CC Configuration

The circuit diagram for determining the static characteristics of an *NPN* transistor in the common collector configuration is shown in Figure 5.25.

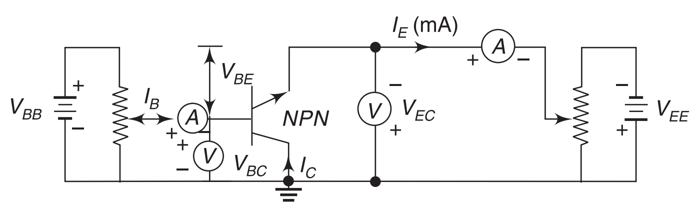


Figure 5.25 Circuit to determine CC static characteristics

Input Characteristics

To determine the input characteristics, V_{EC} is kept at a suitable fixed value. The base-collector voltage V_{BC} is increased in equal steps and the corresponding increase in I_B is noted. This is repeated for different fixed values of V_{EC}. Plots of V_{BC} versus I_B for different values of V_{EC} shown in Figure 5.26 are the input characteristics.

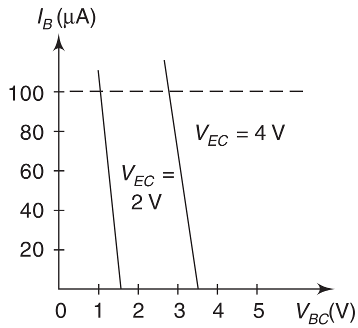


Figure 5.26 CC input characteristics

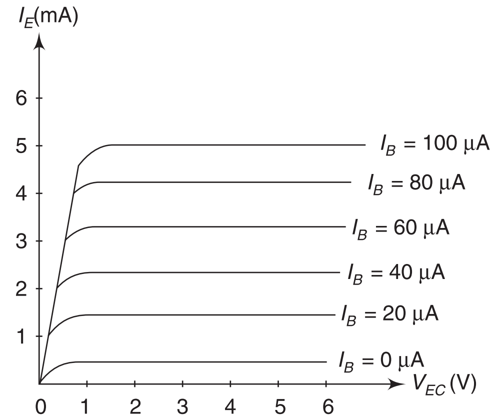


Figure 5.27 CC output characteristics

Output Characteristics

The output characteristics shown in Figure 5.27 are the same as those of the common emitter configuration.

Comparison

Table 5.1 *A comparison of CB, CE, and CC configurations*

Property	CB	CE	CC
Input resistance	Low (about 100 Ω)	Moderate (about 750 Ω)	High (about 750 kΩ)
Output resistance	High (about 450 kΩ)	Moderate (about 45 kΩ)	Low (about 25 Ω)
Current gain	1	High	High
Voltage gain	About 150	About 500	Less than 1
Phase shift between input and output voltages	0 or 360°	180°	0 or 360°
Applications	used in high frequency circuits	used in audio frequency circuits	used in impedance matching

Current Amplification Factor

In a transistor amplifier with ac input signal, the ratio of change in output current to the change in input current is known as the current amplification factor.

In the CB configuration, the current amplification factor,

$$\alpha = \frac{\Delta I_C}{\Delta I_E} \tag{5.14}$$

In the CE configuration, the current amplification factor, $\beta = \dfrac{\Delta I_C}{\Delta I_B}$ (5.15)

In the CC configuration, the current amplification factor, $\gamma = \dfrac{\Delta I_E}{\Delta I_B}$ (5.16)

Relationship between α and β

We know that $\Delta I_E = \Delta I_C + \Delta I_B$

By definition, $\Delta I_C = \alpha \Delta I_E$

Therefore, $\Delta I_E = \alpha \Delta I_E + \Delta I_B$

i.e., $\Delta I_B = \Delta I_E (1 - \alpha)$

Dividing both sides by ΔI_C, we get

$$\frac{\Delta I_B}{\Delta I_C} = \frac{\Delta I_E}{\Delta I_C}(1-\alpha)$$

Therefore, $$\frac{1}{\beta} = \frac{1}{\alpha}(1-\alpha)$$

$$\beta = \frac{\alpha}{(1-\alpha)}$$

Rearranging, we also get $\alpha = \dfrac{\beta}{(1+\beta)}$, or $\dfrac{1}{\alpha} - \dfrac{1}{\beta} = 1$ (5.17)

From this relationship, it is clear that as α approaches unity, β approaches infinity. The CE configuration is used for almost all transistor applications because of its high current gain, β.

Relation among α, β, and γ

In the CC transistor amplifier circuit, I_B is the input current and I_E is the output current.

From Eqn. (5.16), $\gamma = \dfrac{\Delta I_E}{\Delta I_B}$

Substituting $\Delta I_B = \Delta I_E - \Delta I_C$, we get $\gamma = \dfrac{\Delta I_E}{\Delta I_E - \Delta I_C}$

Dividing the numerator and denominator on RHS by ΔI_E, we get

$$\gamma = \frac{\dfrac{\Delta I_E}{\Delta I_E}}{\dfrac{\Delta I_E}{\Delta I_E} - \dfrac{\Delta I_C}{\Delta I_E}} = \frac{1}{1-\alpha}$$

Therefore, $$\gamma = \frac{1}{1-\alpha} = (\beta + 1)$$ (5.18)

5.6.6 Transistor as an Amplifier

CE Transistor as an Amplifier

Figure 5.28(a) shows an amplifier circuit using CE transistor configuration. In this circuit, an NPN transistor is used in CE configuration. Here, V_{BB} supply will forward bias the emitter-base junction and V_{CC} supply will reverse bias the collector-base junction. This biasing arrangement makes the transistor to operate in the active region. The magnitude of the input ac signal v_i always forward bias the emitter-base junction regardless of the polarity of the signal.

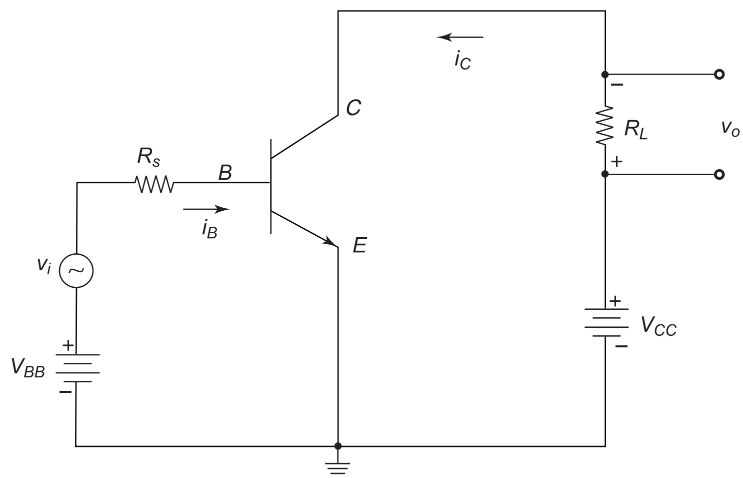


Figure 5.28(a) CE transistor as an amplifier

During the positive half cycle of the input signal v_i, the forward bias across the emitter-base junction is increased. As a result, more electrons are injected into the base and reaches the collector, resulting in an increase in collector current i_c. This increase in collector current produces a greater voltage drop across the load resistance R_L.

However, during the negative half cycle of the input signal v_i, the forward bias across the emitter-base junction is decreased, resulting in a decrease in collector current i_c. This decrease in collector current produces a smaller voltage drop across the load resistance R_L. Hence, it is clear that a small change in the input ac signal in CE transistor amplifier produces a large change at the output with a voltage gain of around 500 and a phase shift of 180°. Here, the voltage gain is the ratio of output voltage to input voltage. Comparing to *CB* and *CC* transistor configurations, this *CE* transistor configuration is widely used in amplifier circuits due to its high voltage gain.

CB Transistor as an Amplifier

A load resistor R_L is connected in series with the collector supply voltage V_{CC} of the CB transistor configuration as shown in Figure 5.28(b).

A small change in the input voltage between emitter and base, say ΔV_i, causes a relatively larger change in the emitter current, say ΔI_E. A fraction of this change in current is collected and passed through R_L and is denoted by the symbol α'. Therefore, the corresponding change in voltage across the load resistor R_L due to this current is $\Delta V_0 = \alpha' R_L \Delta I_E$.

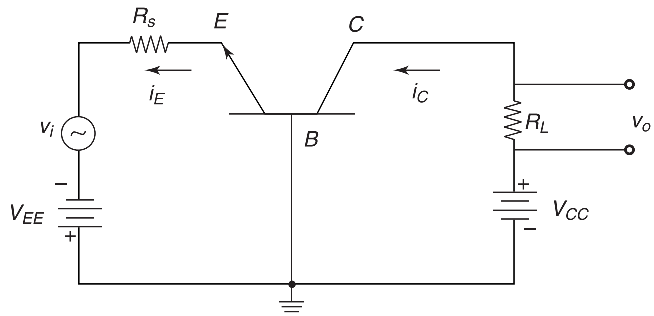


Figure 5.28(b) CB transistor as an amplifier

Here, the voltage amplification $A_v = \dfrac{\Delta V_0}{\Delta V_i}$ is around 150 without any phase shift and thus, the transistor acts as an amplifier.

Example 5.1

In a common-base transistor circuit, the emitter current I_E is 10 mA and the collector current I_C is 9.8 mA. Find the value of the base current I_B.

Solution

Given $I_E = 10$ mA and $I_C = 9.8$ mA

We know that emitter current is

$$I_E = I_B + I_C$$

i.e., $$10 \times 10^{-3} = I_B + 9.8 \times 10^{-3}$$

Therefore, $$I_B = 0.2 \text{ mA}$$

Example 5.2

The transistor has $I_E = 10$ mA and $\alpha = 0.98$. Determine the values of I_C and I_B.

Solution

Given $I_E = 10$ mA and $\alpha = 0.98$

The common-base DC current gain, $\alpha = \dfrac{I_C}{I_E}$

i.e., $$0.98 = \frac{I_C}{10 \times 10^{-3}}$$

Therefore, $$I_C = 0.98 \times 10 \times 10^{-3} = 9.8 \text{ mA}$$

The emitter current $$I_E = I_B + I_C$$

i.e., $$10 \times 10^{-3} = I_B + 9.8 \times 10^{-3}$$

Therefore, $$I_B = 0.2 \text{ mA}$$

Example 5.3

If a transistor has a α of 0.97, find the value of β. If $\beta = 200$, find the value of α.

Solution

If $\alpha = 0.97$, $$\beta = \frac{\alpha}{1-\alpha} = \frac{0.97}{1-0.97} = 32.33$$

If $\beta = 200$, $$\alpha = \frac{\beta}{\beta+1} = \frac{200}{200+1} = 0.995$$

Example 5.4

A transistor has $\beta = 150$. Find the collector and base currents, if $I_E = 10$ mA.

Solution

Given $\beta = 150$ and $I_E = 10$ mA

The common-base current gain, $$\alpha = \frac{\beta}{\beta+1} = \frac{150}{150+1} = 0.993$$

Also, $$\alpha = \frac{I_C}{I_E}$$

i.e., $$0.993 = \frac{I_C}{10}$$

Therefore, $$I_C = 0.993 \times 10 \times 10^{-3} = 9.93 \text{ mA}$$

The emitter current $$I_E = I_B + I_C$$

i.e., $$10 \times 10^{-3} = I_B + 9.93 \times 10^{-3}$$

Therefore, $$I_B = (10 - 9.93) \times 10^{-3} = 0.07 \text{ mA}$$

Example 5.5

A transistor has $I_B = 100$ µA and $I_C = 2$ µA. Find (a) β of the transistor, (b) α of the transistor, (c) emitter current I_E, and (d) if I_B changes by + 25 µA and I_C changes by + 0.6 mA, find the new value of β.

Solution

Given $I_B = 100\ \mu\text{A} = 100 \times 10^{-6}$ A and $I_C = 2\text{ mA} = 2 \times 10^{-3}$ A.

(a) To find β of the transistor

$$\beta = \frac{I_C}{I_B} = \frac{2 \times 10^{-3}}{100 \times 10^{-6}} = 20$$

(b) To find α of the transistor

$$\alpha = \frac{\beta}{\beta+1} = \frac{20}{1+20} = 0.952$$

(c) To find emitter current, I_E

$$I_E = I_B + I_C = 100 \times 10^{-6} + 2 \times 10^{-3}\ \text{A}$$
$$= (0.01 + 2) \times 10^{-3} = 2.01 \times 10^{-3}\ \text{A} = 2.01\ \text{mA}$$

(d) To find the new value of β when $\Delta I_B = 25\ \mu\text{A}$ and $\Delta I_C = 0.6$ mA

Therefore, $I_B = (100 + 25)\ \mu\text{A} = 125\ \mu\text{A}$

$$I_C = (2 + 0.6)\ \text{mA} = 2.6\ \text{mA}$$

New value of β of the transistor,

$$\beta = \frac{I_C}{I_B} = \frac{2.6 \times 10^{-3}}{125 \times 10^{-6}} = 20.8$$

5.6.7 BJT Biasing

The quiescent operating point of a transistor amplifier should be established in the active region of its characteristics. Since the transistor parameters such as β, I_{CO}, and V_{BE} are functions of temperature, the operating point shifts with changes in temperature.

Need for Biasing

In order to produce distortion-free output in amplifier circuits, the supply voltages and resistances in the circuit must be suitably chosen. These voltages and resistances establish a set of DC voltage V_{CEQ} and current I_{CQ} to operate the transistor in the active region. These voltages and currents are called *quiescent values* which determine the *operating point* or Q-point for the transistor. The process of giving proper supply voltages and resistances for obtaining the desired Q-point is called *biasing*. The circuits used for getting the desired and proper operating point are known as *biasing circuits*.

The collector current for a common-emitter amplifier is expressed by

$$I_C = \beta I_B + I_{CEO} = \beta I_B + (1 + \beta) I_{CO}$$

Here, the three variables h_{FE}, i.e., β, I_B, and I_{CO} are found to increase with temperature. For every 10°C rise in temperature, I_{CO} doubles itself. When I_{CO} increases, I_C increases significantly. This causes power dissipation to increase and hence, to make I_{CO} increase. This will cause I_C to increase further and the process becomes cumulative which will lead to thermal runaway that will destroy the transistor. In addition, the quiescent operating point can shift due to temperature changes and the transistor can be driven into the region of saturation. The effect of β on the Q-point is shown in Figure 5.29. One more source of bias instability to be considered is due to the variation of V_{BE} with temperature. V_{BE} is about 0.6 V for a silicon transistor and 0.2 V for a germanium transistor at room temperature. As the temperature increases, $|V_{BE}|$ decreases at the rate of 2.5 mV/°C for both silicon and germanium transistors. The transfer-characteristic curve shifts to the left at the rate of 2.5 mV/°C (at constant I_C) for increasing temperature and hence, the operating point shifts accordingly. To establish the operating point in the active region, compensation techniques are needed.

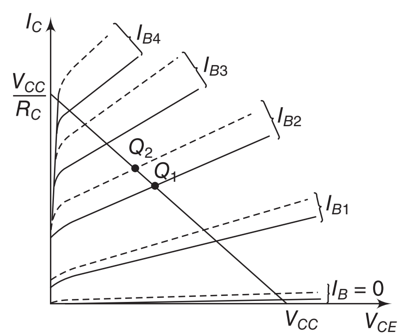


Figure 5.29 Effect of β on Q-point

DC Load Line

Referring to the biasing circuit of Figure 5.30(a), the values of V_{CC} and R_C are fixed and I_C and V_{CE} are dependent on R_B.

Applying Kirchhoff's voltage law to the collector circuit in Figure 5.30(a), we get

$$V_{CC} = I_C R_C + V_{CE}$$

The straight line represented by AB in Figure 5.30(b) is called the DC *load line.* The coordinates of the end point A are obtained by substituting $V_{CE} = 0$ in the above equation. Then $I_C = \dfrac{V_{CC}}{R_C}$. Therefore, the coordinates of A are $V_{CE} = 0$ and $I_C = \dfrac{V_{CC}}{R_C}$.

The coordinates of B are obtained by substituting $I_C = 0$ in the above equation. Then $V_{CE} = V_{CC}$. Therefore, the coordinates of B are $V_{CE} = V_{CC}$ and $I_C = 0$. Thus, the DC load line AB can be drawn if the values of R_C and V_{CC} are known.

As shown in Figure 5.30(b), the optimum Q-point is located at the midpoint of the DC load line AB between the saturation and cut-off regions, i.e., Q is exactly midway between A and B. In order to get faithful amplification, the Q-point must be well within the active region of the transistor.

Even though the Q-point is fixed properly, it is very important to ensure that the operating point remains stable where it is originally fixed. If the Q-point shifts nearer to either A or B, the output voltage and current get clipped, thereby output signal is distorted.

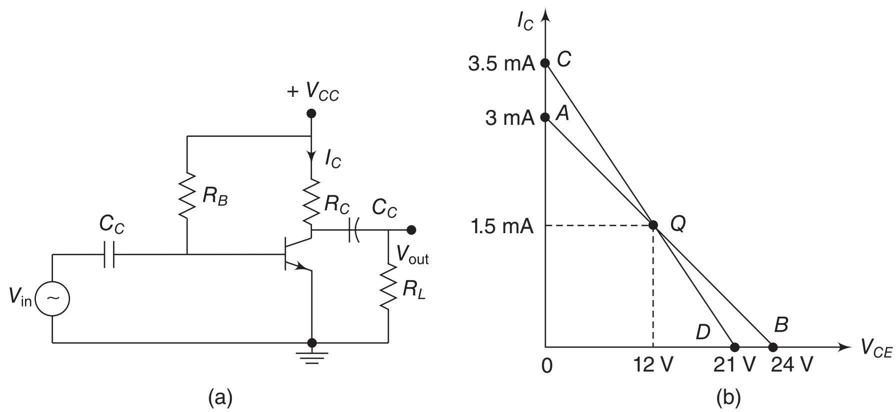


Figure 5.30 (a) Biasing circuit (b) CE output characteristics and load line

In practice, the Q-point tends to shift its position due to any or all of the following three main factors:

(i) Reverse saturation current, I_{CO}, which doubles for every 10°C increase in temperature.

(ii) Base-emitter voltage, V_{BE}, which decreases by 2.5 mV per °C.

(iii) Transistor current gain, β, i.e., h_{FE} which increases with temperature.

Referring to Figure 5.30(a), the base current I_B is kept constant since I_B is approximately equal to V_{CC}/R_B. If the transistor is replaced by another one of the same type, one cannot ensure that the new transistor will have identical parameters as that of the first one. Parameters such as β vary over a range. This results in the variation of collector current I_C for a given I_B. Hence, in the output characteristics, the spacing between the curves might increase or decrease which leads to the shifting of the Q-point to a location which might be completely unsatisfactory.

AC Load Line

After drawing the DC load line, the operating point Q is properly located at the center of the DC load line. This operating point is chosen under zero input signal condition of the circuit. Hence, the AC load line should also pass through the operating point Q. The effective ac load resistance, R_{ac}, is the combination of R_C parallel to R_L, i.e.,

$R_{ac} = R_C \| R_L$. So the slope of the ac load line CQD will be $\left(-\dfrac{1}{R_{ac}}\right)$.

To draw an AC load line, two end points, viz., maximum V_{CE} and maximum I_C when the signal is applied are required.

Maximum $V_{CE} = V_{CEQ} + I_{CQ}R_{ac}$, which locates the point D (OD) on the V_{CE} axis.

Maximum $I_C = I_{CQ} + \dfrac{V_{CEQ}}{R_{ac}}$, which locates the point C (OC) on the I_C axis.

By joining points C and D, ac load line CD is constructed. As $R_C > R_{ac}$, the DC load line is less steeper than the AC load line.

When the signal is zero, we have the exact DC conditions. From Figure 5.30(b), it is clear that the intersection of DC and AC load lines is the operating point Q.

Example 5.6

In the transistor amplifier shown in Figure 5.30(a), $R_C = 8\ \text{k}\Omega$, $R_L = 24\ \text{k}\Omega$ and $V_{CC} = 24$ V. Draw the DC load line and determine the optimum operating point. Also draw the AC load line.

Solution

(a) *DC load line:* Referring to Figure 5.30(a), we have $V_{CC} = V_{CE} + I_C R_C$.
For drawing the DC load line, the two end points, viz., maximum V_{CE} point (at $I_C = 0$) and maximum I_C point (at $V_{CE} = 0$) are required.

Maximum $V_{CE} = V_{CC} = 24$ V

Maximum $I_C = \dfrac{V_{CC}}{R_C} = \dfrac{24}{8 \times 10^3} = 3$ mA

Therefore, the DC load line AB is drawn with the point B (OB = 24 V) on the V_{CE} axis and the point A (OA = 3 mA) on the I_C axis, as shown in Figure 5.30(b).

(b) For fixing the optimum operating point Q, mark the middle of the DC load line AB and the corresponding V_{CE} and I_C values can be found.

Here, $V_{CEQ} = \dfrac{V_{CC}}{2} = 12$ V and $I_{CQ} = 1.5$ mA

(c) *AC load line:* To draw an AC load line, two end points, viz., maximum V_{CE} and maximum I_C when the signal is applied, are required.

The ac load, $R_{ac} = R_C \| R_L = \dfrac{8 \times 24}{8 + 24} = 6\ \text{k}\Omega$

This locates the point D (OD = 21 V) on the V_{CE} axis.

$$\text{Maximum collector current} = I_{CQ} + \frac{V_{CEQ}}{R_{ac}} = 1.5 \times 10^{-3} + \frac{12}{6 \times 10^3} = 3.5 \text{ mA}$$

This locates the point C (OC = 3.5 mA) on the I_C axis. By joining points C and D, the AC load line CD is constructed.

Methods of Transistor Biasing

There are three different biasing methods such as (a) Fixed bias (b) Collector to base bias and (c) Voltage divider bias as discussed next.

(a) Fixed Bias or Base Resistor Method

A common-emitter amplifier using a fixed-bias circuit is shown in Figure 5.31. The DC analysis of the circuit yields the following equation.

$$V_{CC} = I_B R_B + V_{BE}$$

Therefore,
$$I_B = \frac{V_{CC} - V_{BE}}{R_B}$$

Operating point Q can be fixed at

$$I_{CQ} = \beta I_B$$

and
$$V_{CEQ} = V_{CC} - I_{CQ} R_C$$

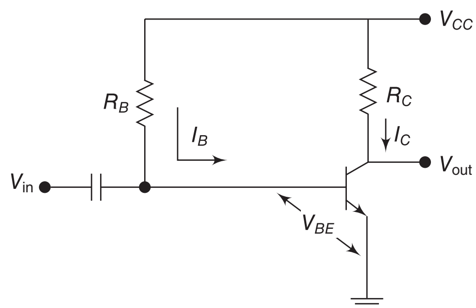


Figure 5.31 Fixed bias circuit

(b) Collector-to-Base Bias or Collector-Feedback Bias

A common-emitter amplifier using collector-to-base bias circuit is shown in Figure 5.32. This circuit is the simplest way to provide some degree of stabilization to the amplifier operating point.

The loop equation for this circuit is

$$V_{CC} = (I_B + I_C)R_C + I_B R_B + V_{BE}$$

i.e.,
$$I_B = \frac{V_{CC} - V_{BE} - I_C R_C}{R_C + R_B}$$

Operating point Q can be fixed at

$$I_{CQ} = \beta I_B$$

and
$$V_{CEQ} = V_{CC} - I_{CQ} R_C$$

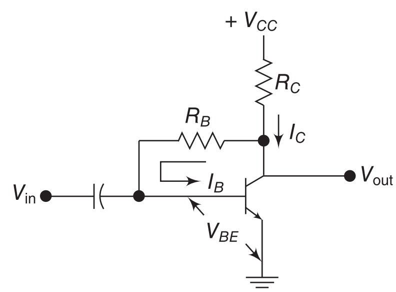


Figure 5.32 Collector-to-base bias circuit

(c) Voltage-Divider Bias, Self-Bias, or Emitter Bias

A simple circuit used to establish a stable operating point is the self-biasing configuration. The self-bias, also called emitter bias, or emitter resistor, and potential divider circuit that can be used for low collector resistance, is shown in Figure 5.33. The current in the emitter resistor R_E causes a voltage drop which is in the direction to reverse bias the emitter junction. For the transistor to remain in the active region, the base-emitter junction has to be forward biased. The required base bias is obtained from the power supply through the potential divider network of the resistances R_1 and R_2.

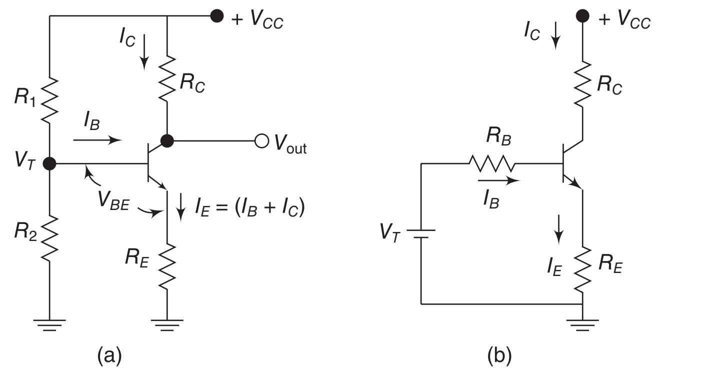


Figure 5.33 (a) Self-bias circuit and (b) Thevenin's equivalent circuit

To find the coordinates of the operating point

Thevenin's voltage, $V_T = \dfrac{R_2}{R_1 + R_2} V_{CC}$

Thevenin's resistance, $R_B = \dfrac{R_1 R_2}{R_1 + R_2}$.

The loop equation around the base circuit is

$$V_T = I_B R_B + V_{BE} + (I_B + I_C)R_E = \frac{I_C}{\beta} R_B + V_{BE} + \left(\frac{I_C}{\beta} + I_C\right) R_E$$

Since I_B is very small,

$$I_C \approx I_E$$

$$V_{CE} = V_{CC} - I_C R_C - I_E R_E = V_{CC} - I_C(R_C + R_E)$$

5.7 Field Effect Transistors

The FET is a device in which the flow of current through the conducting region is controlled by an electric field. Hence, the name Field Effect Transistor (FET). As current conduction is only by majority carriers, FET is said to be a unipolar device.

Based on the construction, the FET can be classified into two types as *Junction FET (JFET)* and *Metal Oxide Semiconductor FET (MOSFET)* or *Insulated Gate FET (IGFET)* or *Metal Oxide Silicon Transistor (MOST)*.

Depending upon the majority carriers, JFET has been classified into two types, namely, (i) *N-channel JFET* with electrons as the majority carriers, and (ii) *P-channel JFET* with holes as the majority carriers.

5.7.1 Construction of *N*-Channel JFET

It consists of an *N*-type bar which is made of silicon. Ohmic contacts (terminals), made at the two ends of the bar, are called source and drain.

Source (*S*) This terminal is connected to the negative pole of the battery. Electrons which are the majority carriers in the *N*-type bar enter the bar through this terminal.

Drain (*D*) This terminal is connected to the positive pole of the battery. The majority carriers leave the bar through this terminal.

Gate (*G*) Heavily doped *P*-type silicon is diffused on both sides of the *N*-type silicon bar by which *PN* junctions are formed. These layers are joined together and called the gate *G*.

Channel The region *BC* of the *N*-type bar between the depletion region is called the channel. Majority carriers move from the source to drain when a potential difference V_{DS} is applied between the source and drain.

5.7.2 Operation of *N*-Channel JFET

When $V_{GS} = 0$ and $V_{DS} = 0$

When no voltage is applied between drain and source, and gate and source, the thickness of the depletion regions around the PN junction is uniform as shown in Figure 5.34.

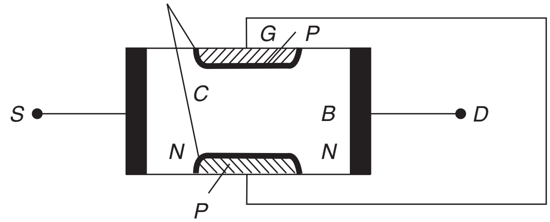


Figure 5.34 JFET construction

When $V_{DS} = 0$ and V_{GS} is Decreased from Zero

In this case, the PN junctions are reverse biased and hence, the thickness of the depletion region increases. As V_{GS} is decreased from zero, the reverse-bias voltage across the PN junction is increased and hence, the thickness of the depletion region in the channel also increases until the two depletion regions make contact with each other. In this condition, the channel is said to be cut-off. The value of V_{GS} which is required to cut off the channel is called the cut off voltage V_C.

When $V_{GS} = 0$ and V_{DS} is Increased from Zero

Drain is positive with respect to the source with $V_{GS} = 0$. Now the majority carriers (electrons) flow through the *N*-channel from source to drain. Therefore, the conventional current I_D flows from drain to source. The magnitude of the current will depend upon the following factors:

1. The number of majority carriers (electrons) available in the channel, i.e., the conductivity of the channel.
2. The length *L* of the channel.
3. The cross-sectional area *A* of the channel at *B*.
4. The magnitude of the applied voltage V_{DS}. Thus, the channel acts as a resistor of resistance *R* given by

$$R = \frac{\rho L}{A} \tag{5.19}$$

$$I_D = \frac{V_{DS}}{R} = \frac{AV_{DS}}{\rho L} \tag{5.20}$$

where ρ is the resistivity of the channel. Because of the resistance of the channel and the applied voltage V_{DS}, there is a gradual increase of positive potential along the channel from

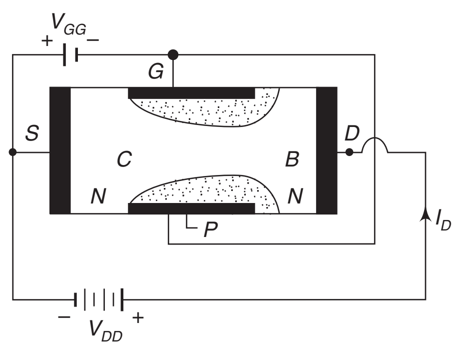


Figure 5.35 JFET under applied bias

source to drain. Thus, the reverse voltage across the *PN* junctions increases and hence the thickness of the depletion regions also increases. Therefore, the channel is wedge-shaped as shown in Figure 5.35.

As V_{DS} is increased, the cross-sectional area of the channel will be reduced. At a certain value V_P of V_{DS}, the cross-sectional area at *B* becomes minimum. At this voltage, the channel is said to be pinched off and the drain voltage V_P is called the pinch-off voltage.

As a result of the decreasing cross section of the channel with the increase of V_{DS}, the following results are obtained.

(i) As V_{DS} is increased from zero, I_D increases along *OP*, and the rate of increase of I_D with V_{DS} decreases as shown in Figure 5.36. The region from $V_{DS} = 0$ V to $V_{DS} = V_P$ is called the ohmic region. In the ohmic region, the drain-to-source resistance $\frac{V_{DS}}{I_D}$ is related to the gate voltage V_{GS}, in an almost linear manner. This is useful as a Voltage Variable Resistor (VVR) or Voltage Dependent Resistor (VDR).

(ii) When $V_{DS} = V_P$, I_D becomes maximum. When V_{DS} is increased beyond V_P, the length of the pinch-off or saturation region increases. Hence, there is no further increase of I_D.

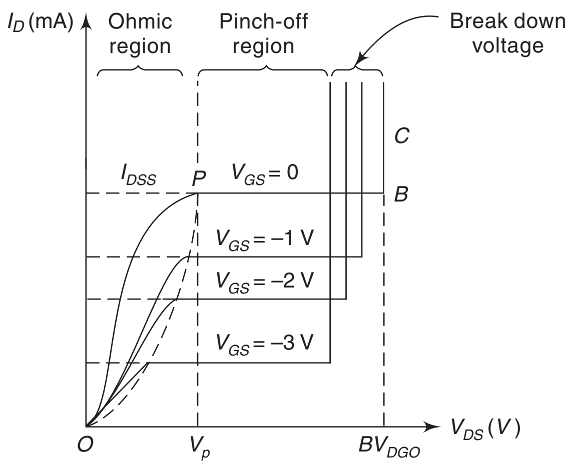


Figure 5.36 Drain characteristics

(iii) At a certain voltage corresponding to the point *B*, I_D suddenly increases. This effect is due to the avalanche multiplication of electrons caused by breaking of covalent bonds of silicon atoms in the depletion region between the gate and the drain. The drain voltage at which the breakdown occurs is denoted by BV_{DGO}. The variation of I_D with V_{DS} when $V_{GS} = 0$ is shown in Figure 5.36 by the curve *OPBC*.

When V_{GS} is Negative and V_{DS} is Increased When the gate is maintained at a negative voltage less than the negative cut-off voltage, the reverse voltage across the junction is further increased. Hence, for a negative value of V_{GS}, the curve of I_D versus V_{DS} is similar to that for $V_{GS} = 0$, but the values of V_P and BV_{DGO} are lower, as shown in Figure 5.36.

From the curves, it is seen that above the pinch-off voltage, at a constant value of V_{DS}, I_D increases with an increase of V_{GS}. Hence, a JFET is suitable for use as a voltage amplifier, similar to a transistor amplifier.

It can be seen from the curve that for voltage $V_{DS} = V_P$, the drain current is not reduced to zero. If the drain current is to be reduced to zero, then the ohmic voltage drop along the channel should also be reduced to zero. Further, the reverse biasing to the gate-source *PN* junction essential for pinching off the channel would also be absent.

The drain current I_D is controlled by the electric field that extends into the channel due to reverse-biased voltage applied to the gate; hence, this device has been given the name *Field Effect Transistor*.

In a bar of *P*-type semiconductor, the gate is formed due to *N*-type semiconductor. The working of the *P*-channel JFET will be similar to that of *N*-channel JFET with proper alterations in the biasing circuits; in this case, holes will be the current carriers instead of electrons. The circuit symbols for *N*-channel and *P*-channel JFETs are shown in Figure 5.37. It should be noted that the direction of the arrow points in the direction of conventional current which would flow into the gate if the *PN* junction was forward biased.

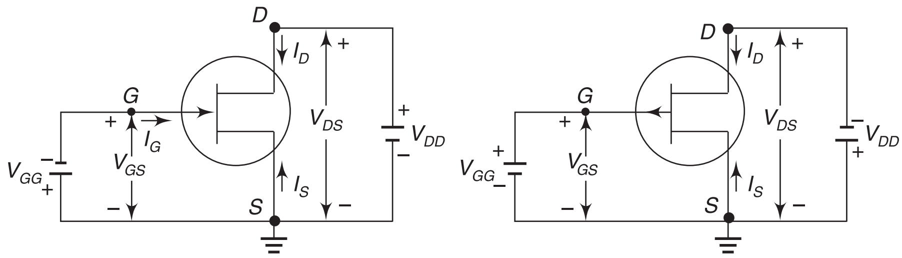


Figure 5.37 Circuit symbols for *N*- and *P*-channel JFET

5.7.3 Comparison of JFET with BJT

1. FET operation depends only on the flow of majority carriers-holes for *P*-channel FETs and electrons for *N*-channel FETs. Therefore, they are called *unipolar devices*. Bipolar transistor (BJT) operation depends on both minority and majority current carriers.
2. As FET has no junctions and the conduction is through an *N*-type or *P*-type semiconductor material, FET is less noisy than BJT.
3. As the input circuit of FET is reverse biased, FET exhibits a much higher input impedance (in the order of 100 Ω) and lower output impedance and there will be a high degree of isolation between input and output. So, FET can act as an excellent buffer amplifier but the BJT has low input impedance because its input circuit is forward biased.
4. FET is a voltage controlled device, i.e., voltage at the input terminal controls the output current, whereas BJT is a current controlled device, i.e., the input current controls the output current.
5. FETs are much easier to fabricate and are particularly suitable for ICs because they occupy less space than BJTs.
6. The performance of a BJT is degraded by neutron radiation because of the reduction in minority-carrier lifetime, whereas FETs can tolerate a much higher level of radiation since they do not rely on minority carriers for their operation.
7. The performance of an FET is relatively unaffected by ambient temperature changes. As it has a negative temperature coefficient at high current levels, it prevents the FET from thermal breakdown. The BJT has a positive temperature co-efficient at high current levels which leads to thermal breakdown.
8. Since the FET does not suffer from minority carrier storage effects, it has higher switching speeds and cut-off frequencies. BJT suffers from minority carrier storage effects and, therefore, has lower switching speed and cut-off frequencies.
9. FET amplifiers have low gain-bandwidth product due to the junction capacitive effects and produce more signal distortion except for small-signal operation.
10. BJTs are cheaper to produce than FETs.

5.7.4 Applications of JFET

1. FETs are used as a buffer in measuring instruments and receivers since they have high input impedance and low output impedance.
2. FETs are used in RF amplifiers in FM tuners, and in communication equipment for its low noise level.
3. Since the input capacitance is low, FETs are used in cascade amplifiers in measuring and test equipments.
4. Since the device is voltage controlled, FETs are used as a voltage variable resistors in operational amplifiers and tone controls.

5. FETs are used in mixer circuits in FM and TV receivers, and in communication equipment because their intermodulation distortion is low.
6. They are used in oscillator circuits because frequency drift is low.
7. As the coupling capacitor is small, FETs are used in low-frequency amplifiers in hearing aids and in inductive transducers.
8. FETs are used in digital circuits in computers, LSD, and memory circuits because of their small size.

5.7.5 FET Biasing

For the proper functioning of a linear FET amplifier, it is necessary to maintain the operating point Q stable in the central portion of the pinch-off region. The Q-point should be independent of device-parameter variations and ambient temperature changes. This can be achieved by suitably selecting the gate to source voltage (V_{GS}) and drain current (I_D) which is referred to as biasing.

Fixing the Q-point

The Q-point, the quiescent point or operating point for a self-biased JFET, is established by determining the value of drain current I_D for a desired value of gate-to-source voltage, V_{GS}, or vice versa. However, if the data sheet of JFET includes a transfer characteristics curve, then the Q-point may be determined by using the procedure given below.

(i) Select a convenient value of drain current whose value is generally taken half of the maximum possible value of drain current, I_{DSS}. Then find the voltage drop across source resistor, R_s, by

$$V_s = I_D R_s$$

and the gate-to-source voltage from the equation

$$V_{GS} = -V_s$$

(ii) Plot the assumed value of drain current, I_D, and the corresponding gate-to-source voltage, V_{GS}, on the transfer characteristics curve.

(iii) Draw a line through the plotted point and the origin. The point of intersection of the line and the curve gives the desired Q-point. Then, read the coordinates of the Q-point.

It is necessary to fix the Q-point near the midpoint of the transfer characteristic curve of a JFET. The midpoint bias allows a maximum amount of drain current swing between the values of I_{DSS} and the origin.

The following analytical method or graphical method can be used for the design of self-bias circuit.

Analytical Method

The values of the maximum drain current, I_{DSS}, and the gate-to-source cut-off voltage, $V_{GS(off)}$ are noted down from the data sheets of JFET.

The value of the drain current is determined by

$$I_D = \left[1 - \frac{V_{GS}}{V_{GS(\text{off})}}\right]^2$$

For example, if we select the gate-to-source voltage, $V_{GS} = \dfrac{V_{GS(\text{off})}}{4}$ then the value of the drain current will be

$$I_D = I_{DSS}\,\{1 - 0.25\}^2 = I_{DSS}\,(0.75)^2 = 0.56\,I_{DSS}$$

Here, the drain current is slightly more than one-half of I_{DSS}. But it will bias the JFET close to the mid-point of the curve. The value of the drain resistor, R_D, is selected in such a way that the drain voltage, V_D, is equal to half the drain supply voltage, R_D. The value of gate resistor, R_G, is chosen arbitrarily large, so that it prevents loading on the driving stages.

Graphical Method

A self-bias line is drawn such that it intersects the transfer characteristic curve near its midpoint giving the required Q-point. Then the coordinates of the Q-point are obtained. The value of source resistance, R_s, is expressed by the ratio of gate-to-source voltage, V_{GS}, to the drain current, I_D.

Therefore, the source resistance is given by

$$R_s = \frac{V_{GS}}{I_D}$$

However, a more accurate method is to draw a self-bias line through the coordinates of I_{DSS} and $V_{GS(\text{off})}$ as shown in Figure 5.38. Then the point of intersection of self-bias line and the transfer characteristic curve locates the Q-point. The value of the source resistor is expressed by the relation

$$R_s = \frac{V_{GS(\text{off})}}{I_{DSS}}$$

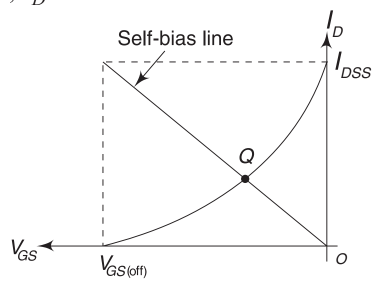


Figure 5.38 Self-bias line through I_{DSS} and $V_{GS\,(off)}$

The value of drain resistor, R_D, and the gate resistor, R_G, are selected in the same way as discussed above for the analytical method.

An FET may have a combination of self-bias and fixed bias to provide stability of the quiescent drain current against device and temperature variations.

Self-bias

Figure 5.39 shows the self-bias circuit for an N-channel FET. When the drain voltage V_{DD} is applied, a drain current I_D flows even in the absence of gate voltage (V_G). The voltage drop across the resistor R_s produced by the drain current is given by $V_s = I_D R_s$. This voltage drop reduces the gate -to-source reverse voltage required for FET operation. The feedback resistor R_s prevents any variation in FET drain current.

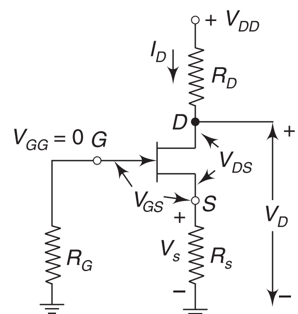


Figure 5.39 Self-bias circuit for an N-channel JFET

The drain voltage, $V_D = V_{DD} - I_D R_D$

The drain-to-source voltage,

$$\begin{aligned} V_{DS} &= V_D - V_s = (V_{DD} - I_D R_D) - I_D R_s \\ &= V_{DD} - I_D(R_D + R_s) \end{aligned}$$

The gate-to-source voltage

$$V_{GS} = V_{GG} - V_s = 0 - I_D R_s = -I_D R_s$$

When drain current increases, the voltage drop across R_s increases. The increased voltage drop increases the reverse gate-to-source voltage, which decreases the effective width of the channel and hence, reduces the drain current. Now, the reduced drain current decreases the gate-to-source voltage which, in turn, increases the effective width of the channel thereby increasing the value of drain current.

Voltage-Divider Bias

Figure 5.40(a) shows the voltage-divider bias circuit and its Thevenin's equivalent is shown in Figure 5.40(b). Resistors R_1 and R_2 connected on the gate side form a voltage divider. The gate voltage,

$$V_{GG} = \left(\frac{R_2}{R_1 + R_2}\right) V_{DD} \quad \text{and} \quad R_G = \frac{R_1 R_2}{R_1 + R_2}$$

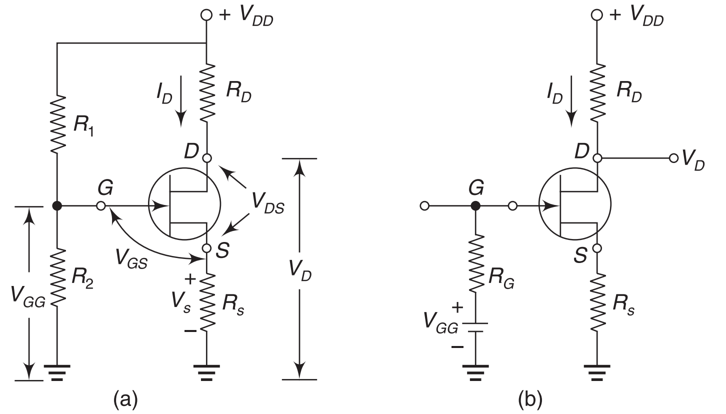


Figure 5.40 (a) Voltage-divider bias circuit (b) Thevenin's equivalent circuit

The bias satisfies the equation $V_{GS} = V_{GG} - I_D R_s$.

The drain-to-ground voltage, $V_D = V_{DD} - I_D R_D$. If the gate voltage V_{GG} is very large compared to gate-to-source V_{GS}, the drain current is approximately constant. In practice, the voltage-divider bias is less effective with JFET than BJT.

This is because, in a BJT, $V_{BE} \approx 0.7$ (silicon) with only minor variations from one transistor to another. But in a JFET, the V_{GS} can vary several volt from one JFET to another.

Fixed Bias

The FET device needs DC bias for setting the gate-to-source voltage V_{GS} to give desired drain-current I_D. For a JFET, the drain current is limited by I_{DSS}. Since the FET has a high input impedance, it does not allow the gate current to flow and the DC voltage of the gate set by a voltage divider or a fixed battery is not affected or loaded by the FET.

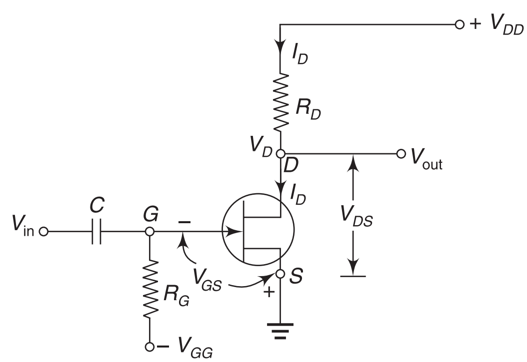


Figure 5.41 Fixed bias circuit for an *N*-Channel JFET

The fixed bias circuit for an *N*-channel JFET shown in Figure 5.41 is obtained by using a supply V_{GG}. This supply ensures that the gate is always negative with respect to source and no current flows through resistor R_G and gate terminal, i.e., $I_G = 0$. The V_{GG} supply provides a voltage V_{GS} to bias the *N*-channel JFET, but no resulting current is drawn from the battery V_{GG}. Resistor R_G is included to allow any ac signal applied through capacitor C to develop across R_G. While any AC signal will develop across R_G, the DC voltage drop across R_G is equal to $I_G R_G$ which is equal to zero volt.

Then, the gate to source voltage V_{GS} is

$$V_{GS} = V_G - V_s = -V_{GG} - 0 = -V_{GG}$$

The drain-source current I_D is then fixed by the gate-source voltage. This current will cause a voltage drop the drain resistor R_D and is given as

$$V_{DD} = I_D R_D + V_{DS}$$

$$I_D = \frac{V_{DD} - V_{DS}}{R_D}$$

5.8 Metal Oxide Semiconductor Field Effect Transistor (MOSFET)

MOSFET is the common term for the Insulated Gate Field Effect Transistor (IGFET). There are two basic forms of MOSFET: (i) Enhancement MOSFET, and (ii) Depletion MOSFET.

Principle

By applying a transverse electric field across an insulator deposited on the semiconducting material, the thickness and hence, the resistance of a conducting channel of a semiconducting material can be controlled.

In a depletion MOSFET, the controlling electric field reduces the number of majority carriers available for conduction, whereas in the enhancement MOSFET, application of electric field causes an increase in the majority carrier density in the conducting regions of the transistor.

5.8.1 Enhancement MOSFET

Construction

The construction of an *N*-channel enhancement MOSFET is shown in Figure 5.42(a), and the circuit symbols for an N-channel and a *P*-channel enhancement MOSFET are shown in Figures 5.42(b) and (c), respectively. As there is no continuous channel in an enhancement MOSFET, this condition is represented by the broken line in the symbols.

Two highly doped N^+ regions are diffused in a lightly doped substrate of *P*-type silicon substrate. One N^+ region is called the source *S* and the other one is called the drain *D*. They are separated by 1 mil (10^{-3} inch).

A thin insulating layer of SiO_2 is grown over the surface of the structure and holes are cut into the oxide layer, allowing contact with source and drain. Then a thin layer of metal aluminium is formed over the layer of SiO_2. This metal layer covers the entire channel region and it forms the gate *G*.

The metal area of the gate, in conjunction with the insulating oxide layer of SiO_2 and the semiconductor channel forms a parallel-plate capacitor. This device is called the insulated gate FET because of the insulating layer of SiO_2. This layer gives an extremely high input impedance for the MOSFET.

Operation

If the substrate is grounded and a positive voltage is applied at the gate, the positive charge on G induces an equal negative charge on the substrate side between the source and drain regions. Thus, an electric field is produced between the source and drain regions. The direction of the electric field is perpendicular to the plates of the capacitor through the oxide. The negative charge of electrons which are minority carriers in the P-type substrate forms an inversion layer. As the positive voltage on the gate increases, the induced negative charge in the semiconductor increases. Hence, the conductivity increases and current flows from source to drain through the induced channel. Thus, the drain current is enhanced by the positive gate voltage as shown in Figure 5.43.

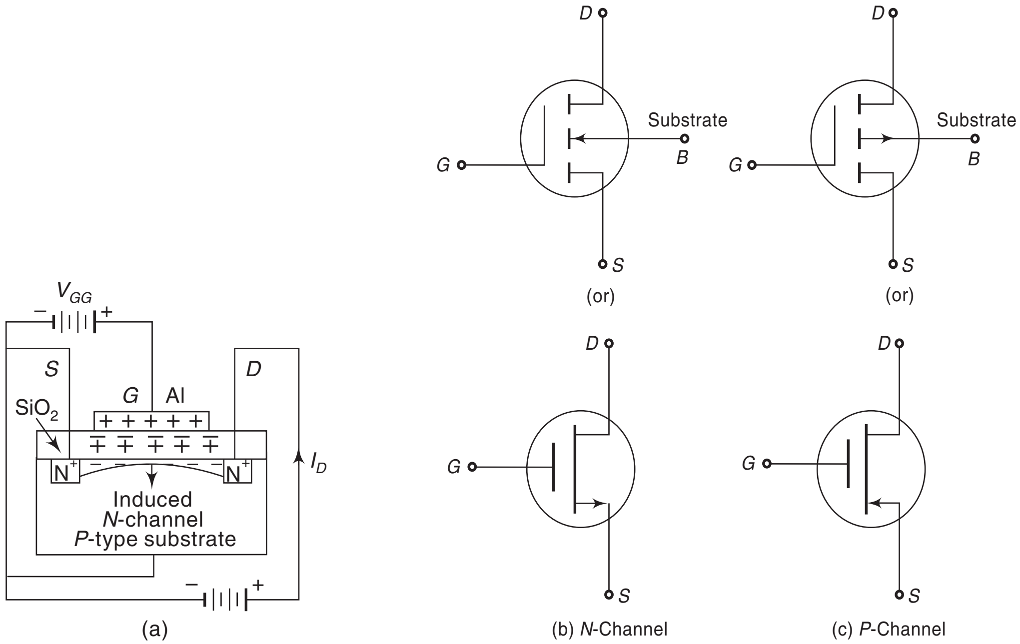


Figure 5.42 (a) N-channel enhancement MOSFET (b) and (c) Circuit symbols for enhancement MOSFET

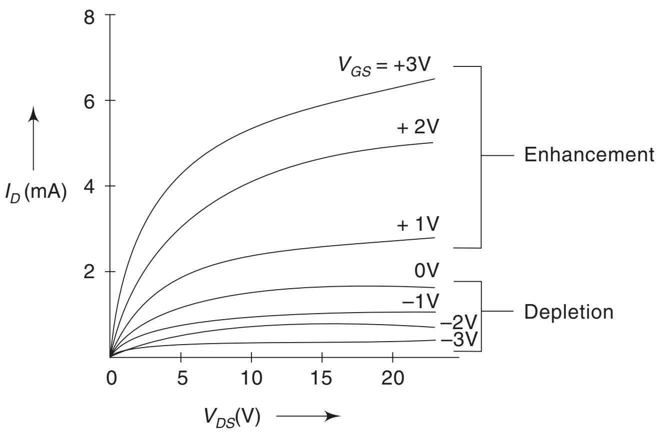


Figure 5.43 Volt-ampere characteristics of MOSFET

5.8.2 Depletion MOSFET

The construction of an *N*-channel depletion MOSFET is shown in Figure 5.44(a) where an *N*-channel is diffused between the source and drain to the basic structure of MOSFET. The circuit symbols for an *N*-channel and a *P*-channel depletion MOSFET are shown in Figures 5.44(b) and (c), respectively.

With $V_{GS} = 0$ and the drain *D* at a positive potential with respect to the source, the electrons (majority carriers) flow through the *N*-channel from *S* to *D*. Therefore, the conventional current I_D flows through the

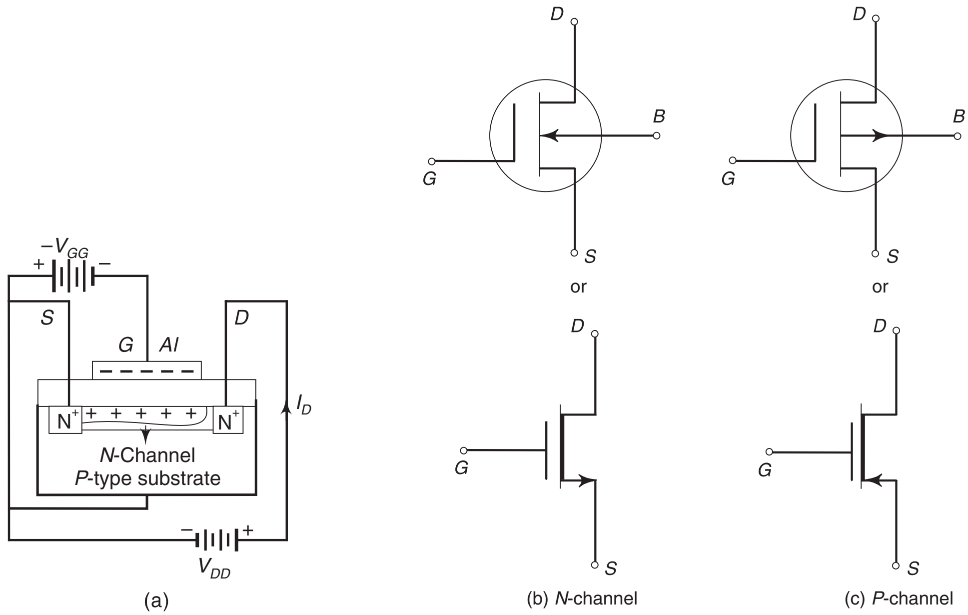


Figure 5.44 (a) N-channel depletion MOSFET, (b) and (c) Circuit symbols for depletion MOSFETs

channel D to S. If the gate voltage is made negative, positive charge consisting of holes is induced in the channel through SiO_2 of the gate-channel capacitor. The introduction of the positive charge causes depletion of mobile electrons in the channel. Thus, a depletion region is produced in the channel. The shape of the depletion region depends on V_{GS} and V_{DS}. Hence, the channel will be wedge shaped as shown in Figure 5.44(a). When V_{DS} is increased, I_D increases and it becomes practically constant at a certain value of V_{DS}, called the pinch-off voltage. The drain current I_D almost gets saturated beyond the pinch-off voltage.

Since the current in an FET is due to majority carriers (electrons for an N-type material), the induced positive charges make the channel less conductive, and I_D drops as V_{GS} is made negative.

The depletion MOSFET may also be operated in an enhancement mode. It is only necessary to apply a positive gate voltage so that negative charges are induced into the N-type channel. Hence, the conductivity of the channel increases and I_D increases. As the depletion MOSFET can be operated with bipolar input signals irrespective of doping of the channel, it is also called *dual-mode MOSFET*. The volt-ampere characteristics are indicated in Figure 5.43.

The curve of I_D versus V_{GS} for constant V_{DS} is called the transfer characteristics of the MOSFET and is shown in Figure 5.45.

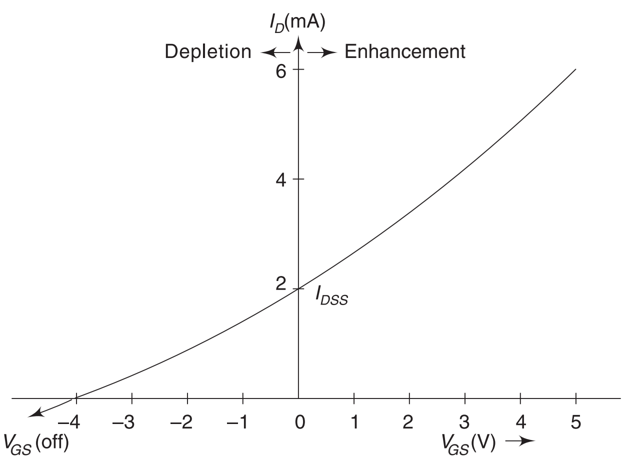


Figure 5.45 Transfer characteristics of MOSFET

5.9 Introduction to Operational Amplifier (Op-Amp)

The op-amp is a direct-coupled high-gain amplifier to which the feedback is externally connected to control its overall response characteristics. As it is mainly used to perform many linear operations as well as some non-linear functions, it is often called the *analog* or *linear integrated circuit.* The name *operational amplifier* coins from the application of this type of amplifier for specific electronic circuit functions or operations such as summation, scaling, differentiation and integration, and in analog computers.

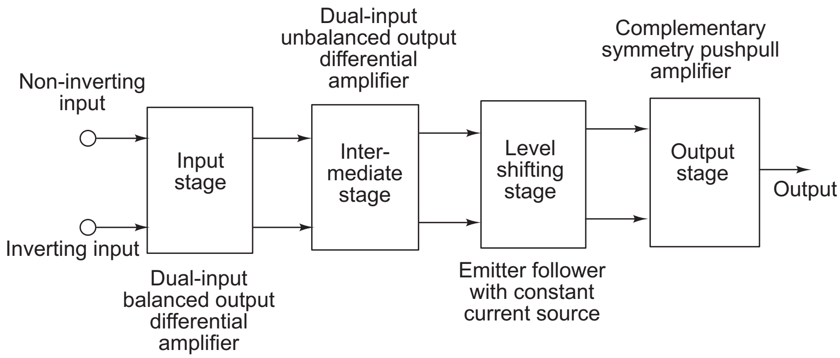


Figure 5.46 Basic block diagram of operational amplifier

Figure 5.46 shows the general block diagram of a typical op-amp. The major blocks are differential amplifiers, intermediate stages of differential, or dual input and single output amplifiers, DC level shift circuits and output stages. The operational amplifier is basically a differential amplifier and its function is to amplify the difference between the two input signals. The DC current source is a fundamental circuit component used to provide bias to the BJT and MOS op-amp circuits to improve their performance by providing a high voltage gain at low supply voltages. The circuits that can yield a precise voltage or current, which is independent of external influences, such as power supply and temperature variations are called *voltage references* and *current references.* The dual circuit of the current source, namely, the voltage source, voltage references and bandgap references are used to reduce supply sensitivity.

5.9.1 Circuit Symbol of an Operational Amplifier

The circuit schematic of an op-amp is in the form of a triangle as shown in Figure 5.47. The device has two input terminals and one output terminal. The terminal marked (–) is called the *inverting input* terminal and the terminal marked (+) is identified as the *non-inverting input* terminal. Figure 5.48 shows the op-amp with positive voltage supply V^+ and a negative voltage supply V^- terminals.

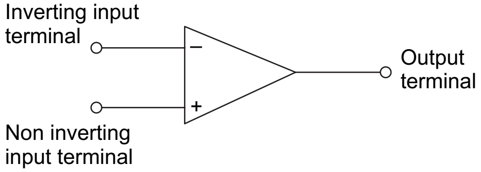


Figure 5.47 Op-amp circuit symbol

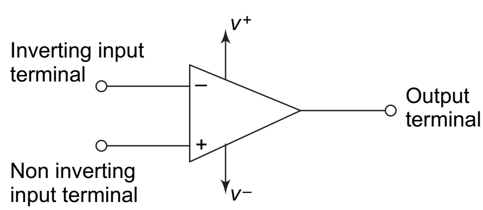


Figure 5.48 Op-amp circuit symbol with positive and negative voltage supply terminals

5.9.2 Ideal Operational Amplifier **[AU Nov/Dec, 2013]**

The *ideal* operational amplifier is a differential input and single-ended output device. The equivalent circuit and the transfer characteristics of an ideal op-amp are shown in Figure 5.49(a) and (b). The input impedance of the ideal op-amp is infinite and it means that the input current is zero. The output terminal of the op-amp acts as the output of an ideal voltage source which means that its small signal output impedance is zero. As shown in Figure 5.49(a), G represents the differential gain of the op-amp and the output voltage v_o is given by

$$v_o = G(v_1 - v_2) \tag{5.21}$$

where v_1 is the input at the non-inverting input terminal and v_2 is the input at the inverting input terminal.

Equation (5.21) identifies that the output is in phase with v_1 and out of phase with v_2. The transfer characteristics of an op-amp are shown in Figure 5.49(b).

The ideal op-amp produces an output in response only to the difference between the input signals v_1 and v_2. Thus, it maintains an ideal zero output signal for $v_1 = v_2$. When $(v_1 - v_2) \neq 0$, there exists a *common-mode input signal,* in response to which the ideal op-amp produces a zero output signal. This characteristic is called the *common-mode rejection*.

The op-amp consists of transistors biased in the active region with the voltages V^+ (or $+V_{CC}$) and V^- (or $-V_{EE}$). Therefore, the output voltage is limited by the output voltage levels. When the output voltage v_o reaches V^+, it saturates and is nearly equal to V^+. Similarly when the output voltage v_o approaches V^-, it saturates at a voltage nearly equal to V^-. The difference between the positive saturation level and positive supply voltage and the difference between the negative saturation level and the negative supply voltage is approximately 1 to 2V as shown in Figure 5.49(b). This concept is discussed in detail in Section. 1.11.

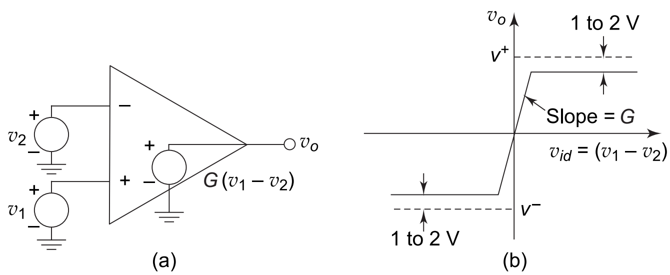


Figure 5.49 Ideal op-amp: (a) Equivalent circuit (b) Transfer characteristics

The ideal op-amp identifies the difference between two input signals that are applied at the inverting and non-inverting input terminals and amplifies the difference so obtained to produce an output signal. The output voltage is the voltage at the output terminal measured with respect to ground.

The characteristics of an ideal op-amp are as follows:

(i) Infinite input resistance, $R_i = \infty$
(ii) Zero output resistance, $R_o = 0$
(iii) Infinite voltage gain, $G = \infty$
(iv) Infinite bandwidth, $BW = \infty$
(v) Infinite common-mode rejection ratio, $CMRR = \infty$
(vi) Infinite slew rate, $SR = \infty$
(vii) Zero offset, i.e., when $v_1 = v_2$, $v_o = 0$
(viii) Characteristics do not drift with temperature

It can be observed that

(i) An ideal op-amp allows zero current to enter into its input terminals, i.e., $i_1 = i_2 = 0$. Due to infinite input impedance, any signal with source impedance can drive the op-amp without getting inflicted with any loading effect.

(ii) The gain of the ideal op-amp is infinite. Hence, the voltage between the inverting and non-inverting terminals is essentially zero for a finite output voltage.

(iii) The output voltage v_o is independent of the output current drawn from the op-amp, since $R_o = 0$. This means that the output can drive an infinite number of output devices of any impedance value.

If the input impedance is high and output impedance is low with respect to the feedback impedances connected externally, and if the voltage gain is sufficiently high, then the resulting amplifier performance becomes solely determined by the external feedback components.

5.9.3 Practical Operational Amplifier

A physical op-amp is not ideal. The equivalent circuit of a practical op-amp is shown in Figure 5.50. The equivalent circuit is useful in analyzing the operating principles of op-amps and in observing the effects of feedback.

For the practical op-amp, the open-loop gain $A_0 \neq \infty$, the input resistance $R_i \neq \infty$ and the output resistance $R_o \neq 0$. It can be observed from Figure 5.50 that the output is a voltage controlled voltage source with $A_0 v_{id}$ of equivalent Thevenin's voltage source and R_o of Thevenin's equivalent resistance looking back into the output terminal of the op-amp. The output voltage is given by

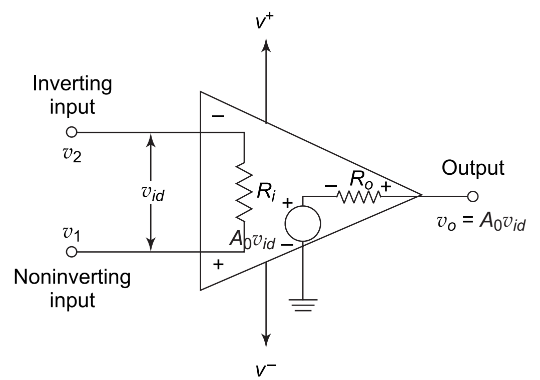


Figure 5.50 Equivalent circuit of a practical op-amp

$$v_o = A_0 v_{id} = A_0(v_1 - v_2) \tag{5.22}$$

where A_0 is the large-signal voltage gain and v_{id} is the difference between the input voltages v_1 and v_2.

From the equation above, it is evident that the op-amp amplifies the difference between the two input voltages and it does not amplify the input voltages themselves. The output voltage is directly proportional to the algebraic difference between the two input voltages. The polarity of the output voltage depends on the polarity of the difference between the input voltages.

The use of an op-amp is greatly enhanced by the negative feedback. The feedback helps in avoiding saturation of the output and the circuit operates in a linear manner.

Output Saturation

The dual supply voltages V^+ and V^- set the upper and lower bounds on the output characteristic of the op-amp. The input-output voltage transfer characteristics of Figure 5.49(b) show the regions of operation and approximate model of op-amp in the respective regions.

(i) In the linear region, the curve is approximately a straight line and its slope indicates the open-loop gain A_0. When A_0 is very large, the curve becomes steeper, and nearly aligns itself with the vertical axis. For a voltage of $v_{id} = 1\,\mu$V and $A_0 = 2 \times 10^5$, $v_o = A_0 \times v_{id} = 2 \times 10^5 \times 1 \times 10^{-6} = 0.2$ V. Hence, the op-amp is modelled with a dependent source of value $A_0 v_{id}$ as shown in Figure 5.50.

(ii) As v_{id} increases, v_o rises proportionally, until a point is reached when the internal transistors saturate that causes the characteristics to flatten out. This region is called the *positive saturation region* and in this region, v_o no longer depends on v_{id}. Then, the op-amp behaves as an independent source of value $+V_{sat}$. Similarly, the *negative saturation* occurs when v_{id} rises in the negative direction resulting in the op-amp acting as an independent source of value $-V_{sat}$.

For bipolar op-amps, such as IC 741, $+V_{sat}$ and $-V_{sat}$ are typically ±13 V which is about 2V below V_{CC} and V_{EE}, of value ±15V. When the op-amp is used with negative feedback, the operation must be confined within the linear region of operation.

5.10 Inverting Amplifier

The inverting amplifier is shown in Figure 5.51(a), and its alternate circuit arrangement is shown in Figure 5.51(b), with the circuit redrawn in a different way to illustrate how the voltage shunt feedback is achieved. The input signal drives the inverting input of the op-amp through resistor R_1.

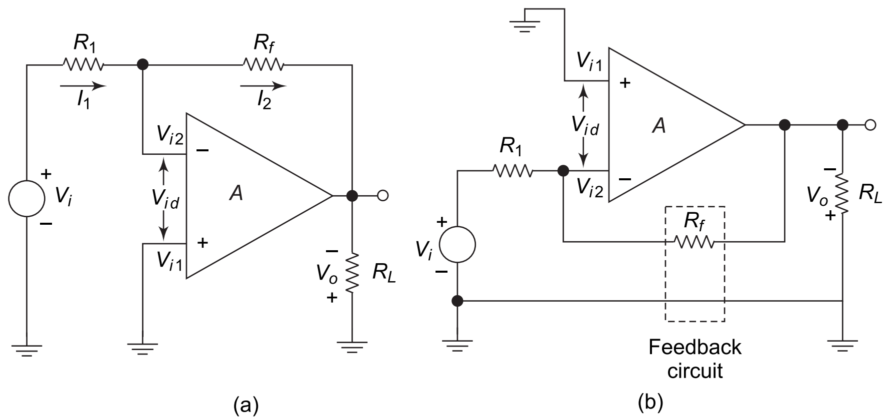


Figure 5.51 Closed-loop inverting amplifier

The op-amp has an open-loop gain of A, so that the output signal is much larger than the error voltage. Because of the phase inversion, the output signal is 180° out-of-phase with the input signal. This means that the feedback signal opposes the input signal and the feedback is *negative* or *degenerative*.

Virtual Ground

A *virtual ground* is a ground which acts *like* a ground. It may not have physical connection to ground. This property of an ideal op-amp indicates that the inverting and non-inverting terminals of the op-amp are at the same potential. The non-inverting input is grounded for the inverting amplifier circuit. This means that the inverting input of the op-amp is also at ground potential. Therefore, a virtual ground is a point that is at the fixed ground potential (0 V), though it is not practically connected to the actual ground or common terminal of the circuit.

The open-loop gain of an op-amp is extremely high, typically 200,000 for a 741. For example, when the output voltage is 10 V, the input differential voltage V_{id} is given by

$$V_{id} = \frac{V_o}{A} = \frac{10}{200{,}000} = 0.05 \text{ mV}$$

Furthermore, the open-loop input impedance of a 741 is around 2 MΩ. Therefore, for an input differential voltage of 0.05 mV, the input current is only

$$Ii = \frac{V_{id}}{R_i} = \frac{0.05\text{ mV}}{2\text{ M}\Omega} = 0.25\text{ nA}$$

Since the input current is so small compared to all other signal currents, it can be approximated as zero. For any input voltage applied at the inverting input, the input differential voltage V_{id} is negligibly small and the input current is ideally zero. Hence, the inverting input of Figure 5.51(a) acts as a virtual ground. The term *virtual ground* signifies a point whose voltage with respect to ground is zero, and yet no current can flow into it.

The expression for the closed-loop voltage gain of an inverting amplifier can be obtained from Figure 5.51(a). Since the inverting input is at virtual ground, the input impedance is the resistance between the inverting input terminal and the ground. That is, $Z_i = R_1$. Therefore, all of the input voltage appears across R_1 and it sets up a current through R_1 that equals

$$I_1 = \frac{V_i}{R_1} \tag{5.23}$$

This current must flow through R_f, because the virtual ground accepts negligible current. The left end of R_f is ideally grounded, and hence the output voltage appears wholly across it. Therefore,

$$V_o = -I_2R_f = -\frac{R_f}{R_1}V_i \tag{5.24}$$

The closed-loop voltage gain A_v is given by

$$A_v = \frac{V_o}{V_i} = \frac{-R_f}{R_1} \tag{5.25}$$

The input impedance can be set by selecting the input resistor R_1. Moreover, the above equation shows that the gain of the inverting amplifier is set by selecting a ratio of feedback resistor R_f to the input resistor R_1. The ratio R_f/R_1 can be set to any value less than or greater than unity. This feature of the gain equation makes the inverting amplifier with feedback very popular and it lends this configuration to a majority of applications.

Example 5.7

For the closed-loop inverting amplifier, $R_f = 10$ kΩ *and* $R_1 = 1$ kΩ. Determine the closed-loop voltage gain A_v.

Solution

The closed-loop voltage gain is

$$A_v = -\frac{R_f}{R_1} = -\frac{10}{1} = -10$$

The gain is 10 and the negative sign indicates the inverting mode or 180° phase-shift obtained at the output with respect to the input.

Practical considerations

(i) Setting the input impedance R_1 to be too high will pose problems for the bias current, and it is usually restricted to 10 kΩ.
(ii) The gain cannot be set very high due to the upper limit set by the gain-bandwidth ($GBW = A_v \times f$) product. The A_v is normally below 100.
(iii) The peak output of the op-amp is limited by the power supply voltages, and it is about 2 V less than supply, beyond which, the op-amp enters into saturation.
(iv) The output current may not be short-circuit limited, and heavy loads may damage the op-amp. When short-circuit protection is provided, a heavy load may drastically distort the output voltage.

5.11 Non-inverting Amplifier

The non-inverting amplifier with input signal applied to the non-inverting input and the output voltage feedback to the inverting input, i.e., in voltage-series mode is shown in Figure 5.52(a). Figure 5.52(b) represents the simplified circuit arrangement as usually followed. The op-amp provides an internal gain A. The external resistors R_1 and R_f form the feedback voltage divider circuit with an attenuation factor of β. Since the feedback voltage is at the inverting input, it opposes the input voltage at the non-inverting input terminal, and hence, the feedback is negative or degenerative.

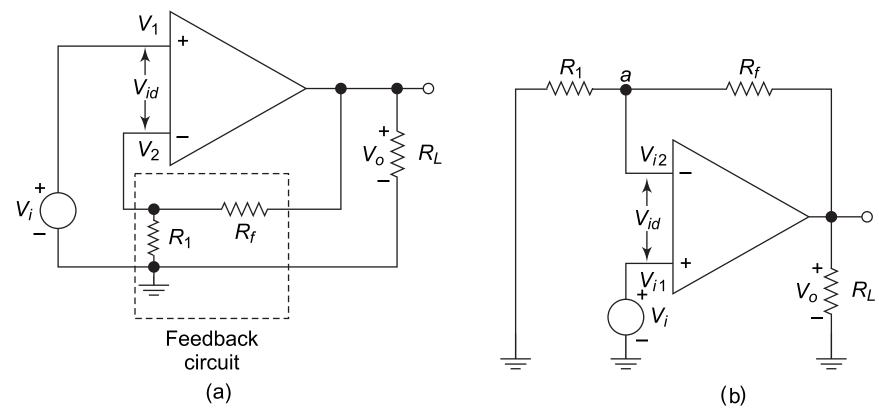


Figure 5.52 (a) Its alternate circuit arrangement (b) Closed-loop non-inverting amplifier

The differential voltage V_{id} at the input of the op-amp is zero, because node a is at the same voltage as that of the non-inverting input terminal. As shown in Figure 5.52(a), R_f and R_1 form a potential divider. Therefore,

$$Vi = \frac{R_1}{R_1 + R_f} \times V_o \tag{5.26}$$

since no current flows into the op-amp.

Equation (5.26) can be written as $\dfrac{V_o}{V_i} = \dfrac{R_1 + R_f}{R_1} = 1 + \dfrac{R_f}{R_1}$.

Hence, the voltage gain for the non-inverting amplifier is given by

$$A_v = \frac{V_o}{V_i} = 1 + \frac{R_f}{R_1}$$

Using the alternate circuit arrangement shown in Figure 5.52(b), the feedback factor of the feedback voltage divider network is

$$\beta = \frac{R_1}{R_1 + R_f} \tag{5.27}$$

Therefore, the closed-loop gain is

$$A_v = \frac{1}{\beta} = \frac{R_1 + R_f}{R_1} \tag{5.28}$$

$$= 1 + \frac{R_f}{R_1} \tag{5.29}$$

From the above equation, it can be observed that the closed-loop gain is always greater than one and it depends on the ratio of the feedback resistors. If precision resistors are used in the feedback network, a precise value of closed-loop gain can be achieved. The closed-loop gain does not drift with temperature changes or op-amp replacements.

The input resistance of the op-amp is extremely large (approximately infinity), since the op-amp draws negligible current from the input signal.

Example 5.8

The variable resistance varies from zero to 100 kΩ. Find out the maximum and the minimum closed-loop voltage gain.

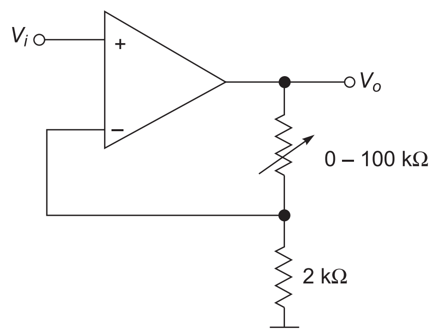


Figure E5.8

Solution

Given circuit is a non-inverting amplifier (Figure E5.8).

Therefore, $A_{Vf} = 1 + \dfrac{R_f}{R_1}$

Here $\quad R_f = 0 - 100\ \text{k}\Omega \quad \text{and} \quad R_1 = 2\ \text{k}\Omega$

When $\quad R_f = 0,\ A_{Vf} = 1 + \dfrac{0}{2 \times 10^3} = 1$

When $\quad R_f = 100\ \text{k}\Omega,\ A_{Vf} = 1 + \dfrac{100}{2 \times 10^3} = 51$

Example 5.9

Design a non-inverting amplifier circuit which is capable of providing a voltage gain of 15. Assume ideal op-amp and resistances used should not exceed 30 kΩ.

Solution Given $A_{CL} = 15$

$$A_{CL} = 1 + \frac{R_f}{R_1}$$

i.e.,

$$15 = 1 + \frac{R_f}{R_1}$$

$$\frac{R_f}{R_1} = 14$$

i.e.,

$$R_f = 14R_1$$

Select $R_1 = 1\ \text{k}\Omega$, i.e., $R_f = 14\ \text{k}\Omega$

The designed circuit is shown in Figure E5.9.

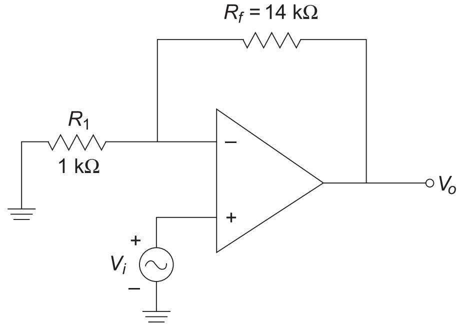


Figure E5.9

5.12 OSCILLATORS

The sine-wave is one of the most fundamental waveforms. The generation of sine-wave is a challenging task if ideal waveform characteristics are expected. In the sine-wave oscillators using op-amps, the required phase-shift of 180° in the feedback loop from output to input is obtained by using either L and C or R and C components.

5.12.1 Conditions for Oscillations

An oscillator is basically a feedback amplifier, in which a fraction of the output is fed back to the input with the use of a feedback circuit. The block diagram of an oscillator is shown in Figure 5.53.

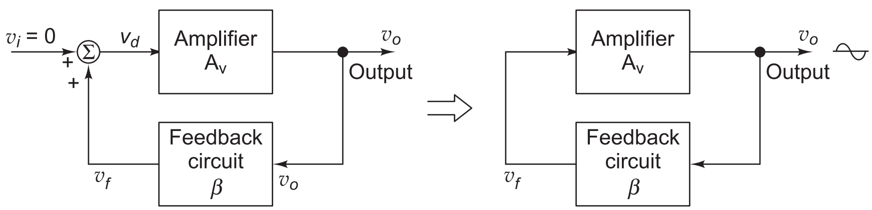


Figure 5.53 Block diagram of an oscillator

As illustrated in Figure 5.53,

$$v_d = v_f + v_i$$

$$v_o = A_v\, v_d = A_v(v_f + v_i)$$

$$v_f = \beta v_o = \beta A_v(v_f + v_i)$$

or

$$v_o = A_v\,(\beta v_o + v_i)$$

Therefore,

$$\frac{v_o}{v_i} = \frac{A_v}{1 - A_v\beta}$$

For an oscillator, $v_i = 0$, and $v_o \neq 0$. The Barkhausen Criteria for sustained oscillations are

$$|A_v \beta| = 1 \tag{5.30}$$

and

$$\angle A_v \beta = 0° \text{ or } 360° \tag{5.31}$$

That is,

(i) the magnitude of the loop gain, $A_v \beta$, must be unity and

(ii) the total phase-shift of the loop gain, $A_v \beta$, must be equal to 0° or 360°

Many different types of oscillators are available which are characterised by the types of components used in the feedback network, e.g. RC oscillator, LC oscillator and crystal oscillator.

5.12.2 *LC* Oscillators

A general form of *LC* oscillator is shown in Figure 5.54(a). Herewith the amplifier using op-amp is considered ideal and it has a non-zero output resistance R_o. Referring to its equivalent shown in Figure 5.54(b), we get $\beta = -\dfrac{V_f}{AV_f'}$.

Applying the voltage divider equation to Figure 5.54(b), we get

$$V_f = \frac{Z_1}{Z_1 + Z_3} V_o$$

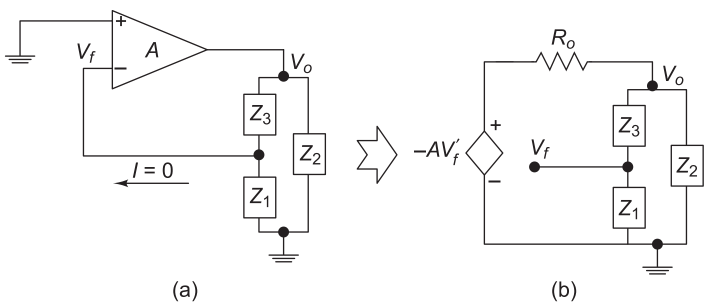


Figure 5.54 (a) LC Oscillator circuit using an ideal op-amp with output impedance R_o (b) Its equivalent circuit

$$V_o = -\frac{Z}{Z + R_o} AV_f'$$

where $Z = Z_2 \| (Z_1 + Z_3)$.

That is,

$$\frac{1}{AV_f'} = -\frac{1}{V_o}\frac{Z}{Z + R_o} = -\frac{1}{V_o}\frac{Z_2(Z_1 + Z_3)}{Z_2(Z_1 + Z_3) + R_o(Z_1 + Z_2 + Z_3)}$$

Solving for β, we get

$$\beta = -\frac{V_f}{AV_f'} = -\frac{Z_1 Z_2}{R_o(Z_1 + Z_2 + Z_3) + Z_2(Z_1 + Z_3)} \tag{5.32}$$

We know that for the LC tunable oscillators, the three impedances ($Z_1 + Z_2 + Z_3$) are purely reactive, i.e., the real part is equal to zero.

$$\beta = \frac{X_1 X_2}{jR_o(X_1 + X_2 + X_3) + X_2(X_1 + X_3)}$$

For β to be real,

$$X_1 + X_2 + X_3 = 0, \tag{5.33}$$

and

$$\beta(\omega_o) = -\frac{X_1}{X_1 + X_3}$$

where ω_o is the oscillation frequency. Using the above two equations, we get

$$\beta(\omega_o) = \frac{X_1}{X_2}.$$

Since $\beta(\omega_o)$ must be positive, X_1 and X_2 must have the same sign. This means that they are of the same kind of reactance, namely, two capacitors or two inductors. From the condition that the imaginary part equals zero, we find that if X_1 and X_2 are capacitors, X_3 must be an inductor and vice versa. Therefore, based on the choice of reactive elements, the LC oscillators are identified as Colpitts and Hartley oscillators.

Hartley Oscillator

The circuit diagram of Hartley oscillator is shown in Figure 5.55. The op-amp is connected to operate in inverting mode. An LC network consisting of inductive reactances X_1 and X_2 and a capacitive reactance X_3 forms the feedback network, and the phase-shift through the feedback network, is 180°. The reactances are:

$$X_1 = j\omega L_1$$

$$X_2 = j\omega L_2$$

and

$$X_3 = -\frac{j}{\omega C}$$

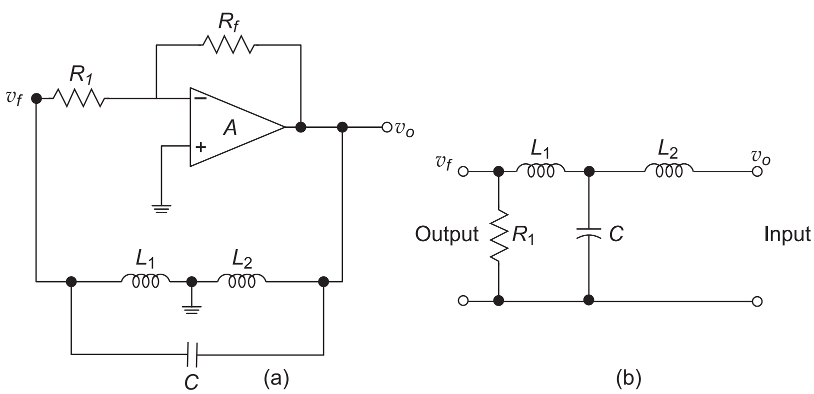


Figure 5.55 Hartley oscillator (a) Circuit diagram and (b) Feedback network

Using Eqn. (5.33), we get

$$\omega_o = \sqrt{\frac{1}{C(L_1 + L_2)}} \quad \text{and} \quad \beta(\omega_o) = \frac{L_1}{L_2}$$

The frequency of oscillation is $$f_o = \frac{1}{2\pi\sqrt{CL_T}} \tag{5.34}$$

where $L_T = L_1 + L_2$ or $L_T = L_1 + L_2 + 2M$

and M is the mutual inductance of coils L_1 and L_2.

Colpitts Oscillator

The circuit diagram of Colpitts oscillator is shown in Figure 5.56. The feedback signal is connected to (–) input terminal, such that the op-amp is working as an inverting amplifier. The feedback circuit consisting of two capacitive reactances X_1 and X_2, and the inductive element X_3 provides 180° phase-shift. The oscillation occurs when the feedback β is real. The reactances are:

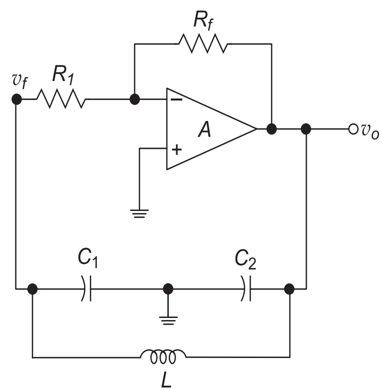


Figure 5.56 Colpitts oscillator

$$X_1 = \frac{1}{j\omega C_1} = -\frac{j}{\omega C_1}$$

$$X_2 = \frac{1}{j\omega C_2} = -\frac{j}{\omega C_2}$$

and $$X_3 = j\omega L$$

Using Eqn. (5.33), we get

$$\omega_o = \sqrt{\frac{1}{L\left(\frac{C_1 C_2}{C_1 + C_2}\right)}}$$

and $$\beta(\omega_o) = \frac{C_2}{C_1}$$

Thus, the frequency of oscillation for the Colpitts oscillator is

$$f_o = \frac{1}{2\pi\sqrt{LC_T}} \tag{5.35}$$

where $$C_T = \frac{C_1 C_2}{C_1 + C_2}.$$

The feedback factor β at the oscillation frequency is –1.

5.12.3 *RC* Oscillators

All the oscillators using tuned *LC* circuits operate well at high frequencies. At low frequencies, as the inductors and capacitors required for the timing circuit would be very bulky, *RC* oscillators are found to be more suitable. Two important *RC* oscillators are (i) *RC* phase shift oscillator and (ii) Wien Bridge oscillator.

RC Phase-shift Oscillator The *RC* phase-shift oscillator consisting of an op-amp serving as the amplifier stage and the cascaded RC network acting as the feedback circuit is shown in Figure 5.57. The op-amp is used in inverting configuration and it produces 180° phase-shift at the output. The cascaded RC networks connected in the feedback network path provide an additional phase-shift of 180°. Therefore, the total phase-shift around the loop is 360° (or 0°). When the gain of the amplifier is sufficiently large and the phase-shift of the cascaded RC network is exactly 180°, the circuit will oscillate at the frequency determined by the RC feedback network.

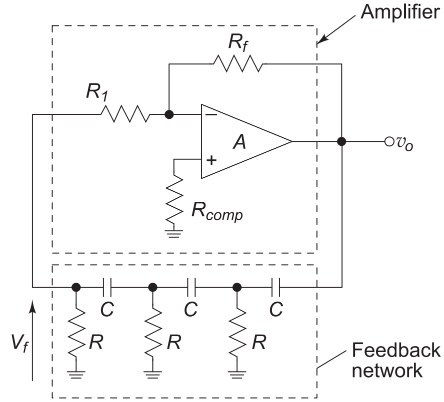


Figure 5.57 RC phase-shift oscillator

The frequency of oscillation for the RC phase shift oscillator is

$$f_0 = \frac{1}{2\pi\sqrt{6}RC} \tag{5.36}$$

Also we have

$$\left|\frac{R_f}{R_1}\right| = 29 \text{ or } R_f = 29\, R_1 \tag{5.37}$$

Therefore, the gain of the amplifier should be at least 29, and the total phase-shift around the loop should be exactly 360°.

For achieving the desired frequency of oscillation f_o, an available value of capacitor C is chosen and then the value of R is calculated using Eqn. (5.36). The desired amplitude of oscillations can be obtained by using Zener diodes connected *back-to-back* at the output of op-amp A.

Wien Bridge oscillator

Wien Bridge oscillator is the most commonly used audio frequency oscillator due to its inherent simplicity and stability. Figure 5.58(a) shows the Wien Bridge oscillator using op-amp. Since the op-amp is connected to operate in non-inverting mode, it produces no phase-shift at the output. The Wien Bridge circuit is connected between the input and output terminals of the amplifier. The bridge consists of a series *RC* network (shown as $Z_s(s)$) forming one arm of the bridge circuit, a parallel *RC* network (shown as $Z_p(s)$) forming the second arm, input resistance R_1 and feedback resistance R_f forming the third and fourth arms of the bridge circuit respectively as shown in Figure 5.58(b). The feedback circuit of the Wien Bridge oscillator is shown in Figure 5.58(c).

It is known that the total phase-shift around the circuit must be 0° or 360° for oscillations to occur. It is achieved when the bridge is balanced, i.e., at resonance. Thus the frequency of oscillation is the resonant frequency of the balanced Wien Bridge.

The frequency of oscillation of Wien Bridge oscillator is $f_0 = \frac{1}{2\pi RC}$. Also, we have $A_v = 1 + \frac{R_f}{R_1} = 3$ and hence $R_f = 2R_1$.

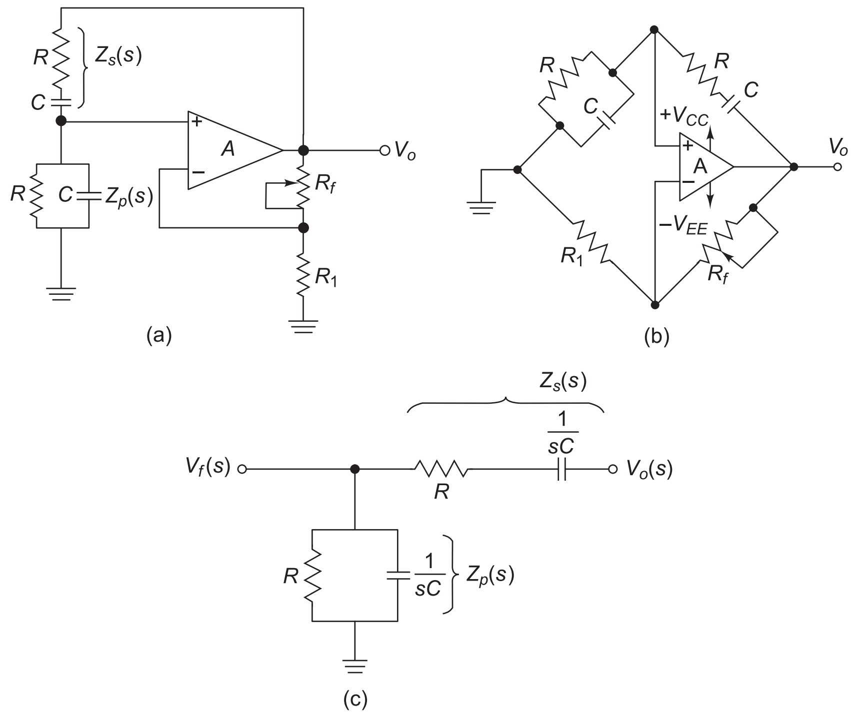


Figure 5.58 Wien Bridge oscillator: (a) Circuit diagram (b) Equivalent circuit (c) Feedback circuit of the Wien Bridge oscillator in s-domain

5.13 INTEGRATOR **[AU Nov/Dec 2013 & 2012]**

A circuit in which the output voltage waveform is the time integral of the input voltage waveform is called *integrator* or *integrating amplifier*. Integrator produces a summing action over a required time interval and the circuit is based on the general parallel-inverting voltage feedback model.

In order to achieve integration, the basic inverting amplifier configuration can be used with the feedback element Z_f replaced by a capacitor C_f as shown in Figure 5.59.

The expression for the output voltage $v_o(t)$ can be obtained as

$$v_o(t) = -\frac{1}{R_1 C_f}\int_0^t v_i(t)\,dt + v_o(0)$$

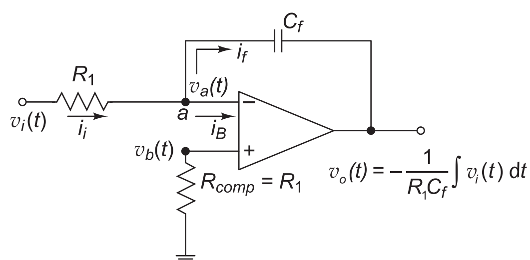


Figure 5.59 Integrator circuit

where $v_o(0)$ is the integration constant and is proportional to the value of the output voltage $v_o(t)$ at $t = 0$. The above equation indicates that the output voltage is directly proportional to the negative integral of the input voltage and inversely proportional to the time constant R_1C_f.

The input sinusoidal and square waveforms and the corresponding output waveforms of integrator circuit using op-amp are shown in Figure 5.60(a) and (b).

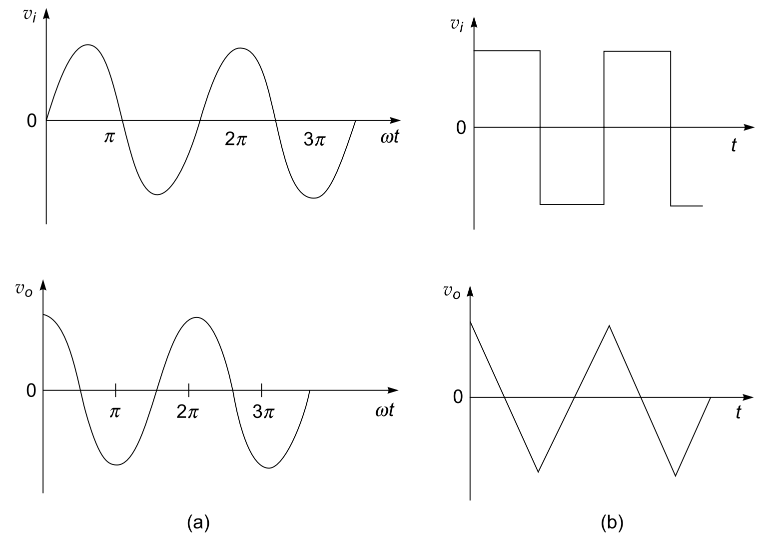


Figure 5.60 (a) Sine-wave input and its integrated cosine output and (b) Square-wave input and its triangular output

Applications of Integrator **[AU April/May, 2013]**

Integrators may be used in combination with summers and amplifiers to form analog computers which are used to model a variety of physical systems in real time. The integrator circuits are used as waveshaping circuits and used to convert square-waves into triangular waves. Further they are used for solving differential equations in analog to digital converters, and ramp generators.

5.14 Differentiator

The differentiator can perform the mathematical operation of differentiation, i.e., the output voltage is the differentiation of the input voltage. This operation is very useful to find the rate at which a signal varies with time.

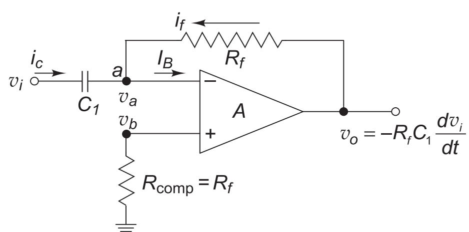


Figure 5.61 Differentiator

The ideal differentiator is obtained by interchanging the position of the resistor and capacitor in the ideal integrator circut, or it may be constructed from a basic inverting amplifier, if the input resistor R_1 is replaced by a capacitor C_1. The ideal differentiator circuit is shown in Figure 5.61.

The expression for the output voltage is

$$v_o = -R_f C_1 \frac{dv_i}{dt} \tag{5.38}$$

Thus, the output v_o is equal to the $R_f C_1$ times the negative instantaneous rate of change of the input voltage v_i with time. A differentiator performs the reverse of the integrator's function. The upper cut-off frequency is given by

$$f_a = \frac{1}{2\pi R_f C_1}$$

The input sinusoidal and square waveforms and the corresponding output waveforms of differentiator circuit using op-amp are shown in Figures 5.62(a) and (b).

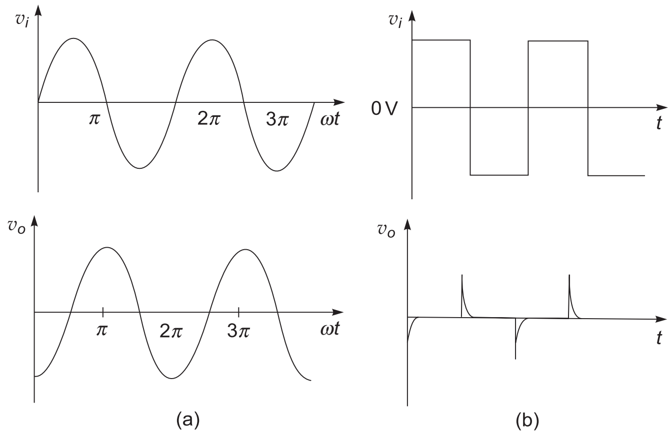


Figure 5.62 (a) Sine-wave input and its differentiated cosine output, (b) square-wave and its differentiated spike output

Applications of Differentiator

The differentiators can be used as waveshaping circuits. They can be used to convert triangular waves into square-waves. In fact, since differentiators tend to seek rapid changes in the input signal, they are quite useful as edge detectors in the FM demodulators. The integrators and differentiators may be used in combination with adders and amplifiers to form analog computers.

5.14.1 Comparison between an Integrator and a Differentiator

Since the process of integration involves the accumulation of signal over time, sudden changes in the signal are suppressed. Therefore, an effective smoothing of the signal is achieved and integration can be viewed as *low-pass filtering.*

Since the process of differentiation involves the identification of sudden changes in the input signal, constant and slowly changing signals are suppressed. Therefore, the differentiator can be viewed as a form of *high-pass filtering.*

5.15 Precision Rectifier **[AU April/May 2015 & 2014]**

The signal processing applications with very low voltage, current and power levels require rectifier circuits. The ordinary diodes cannot rectify voltages below the *cut-in* voltage of the diode. A circuit which can act as an *ideal diode* or *precision signal-processing rectifier circuit* for rectifying voltages which are below the level of *cut-in voltage* of the diode can be designed by placing the diode in the feedback loop of an op-amp.

5.15.1 Precision Diodes

Figure 2.60(a) shows the arrangement of a precision diode. It is a single diode arrangement and functions as a non-inverting precision half-wave rectifier circuit. It is called super diode, since it provides a nearly perfect rectifier. If v_i in the circuit of Figure 5.63(a) is positive, the op-amp output V_{OA} also becomes positive. Then the closed loop condition is achieved for the op-amp and the output voltage $v_o = v_i$. When $v_i < 0$, the voltage V_{OA} becomes negative and the diode is reverse biased. The loop is then broken and the output $v_o = 0$.

Consider the open loop gain A_{OL} of the op-amp is approximately 10^4 and the cut-in voltage V_γ for silicon diode is ≈ 0.7 V. When the input voltage $v_i > \frac{V_\gamma}{A_{OL}}$, the output of the op-amp V_{OA} exceeds V_γ and the diode D conducts. Then the circuit acts like a voltage follower for input voltage level $v_i > \frac{V_\gamma}{A_{OL}}$ $\left(\text{i.e., when } v_i > \frac{0.7}{10^4} = 70\mu\text{V}\right)$ and the output voltage v_o follows the input voltage during the positive half cycle for input voltages higher than 70 μV as shown in Figure 5.63(c). When v_i is negative or less than $\frac{V_\gamma}{A_{OL}}$, the output of op-amp V_{OA} becomes negative and the diode becomes reverse biased. The loop is then broken and the op-amp swings down to negative saturation. However, the output terminal is now isolated from both the input signal and the output of the op-amp and thus $v_o = 0$. No current is then delivered to the load R_L except for the small bias current of the op-amp and the reverse saturation current of the diode.

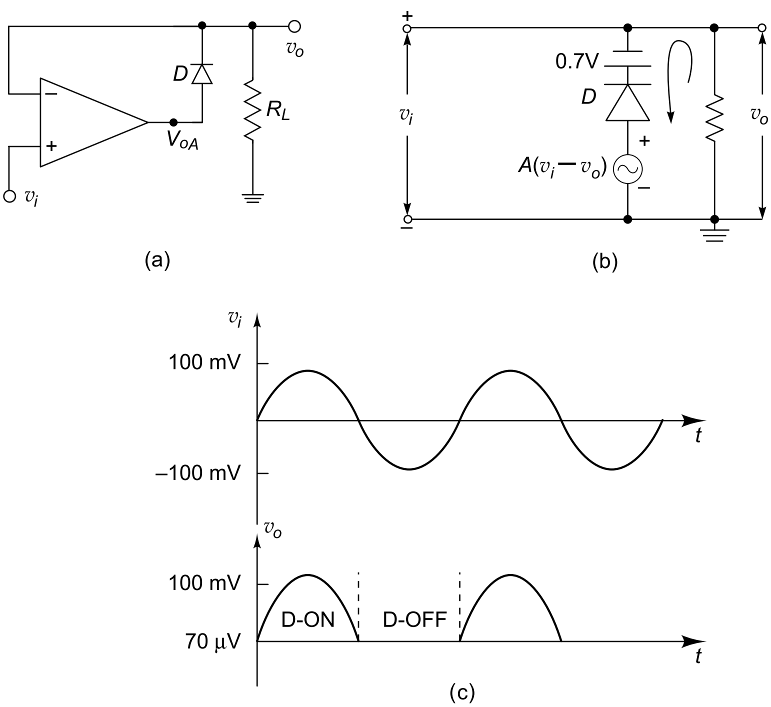


Figure 5.63(a) Precision diode (b) Equivalent circuit (c) Input and output waveforms

From the equivalent circuit shown in Figure 5.63(b), we have

$$v_o + 0.7 - A(v_i - v_o) = 0$$

$$v_o + Av_o = Av_i - 0.7$$

Therefore, $$v_o = \frac{A}{1+A}v_i - \frac{0.7}{1+A}$$

As A is large, $v_o \approx v_i$.

This circuit is an example of a non-linear circuit, in which linear operation is achieved over the region ($v_i > 0$) and non-linear operation is achieved over the remaining region ($v_i < 0$). Since the output swings to negative saturation level when $v_i < 0$, the circuit is basically of saturating form. Thus the frequency response is also limited. The precision diodes are used in Half-wave rectifier, Full-wave rectifier, Peak value detector, Clipper and Clamper circuits.

It can be observed that the precision diode shown in Figure 5.63(a) operates in the first quadrant with $v_i > 0$ and $v_o > 0$. The operation in third quadrant can be achieved by connecting the diode in reverse direction.

5.15.2 Half-wave Rectifier

A non-saturating half-wave precision rectifier circuit is shown in Figure 5.64(a). When $v_i > 0$, the voltage at the inverting input becomes positive, forcing the output V_{OA} to go negative. This results in forward biasing the diode D_1 and the op-amp output drops only by ≈ 0.7 V below the inverting input voltage. Diode D_2 becomes reverse-biased. The output voltage v_o is zero since no current flows in the feedback circuit through R_f. Hence, the output v_o is zero when the input is positive. When $v_i < 0$, the op-amp output V_{OA} becomes positive, forward biasing the diode D_2 and reverse biasing the diode D_1. The circuit then acts like an inverting amplifier circuit with a non-linear diode in the forward path. The gain of the circuit is unity when $R_f = R_i$.

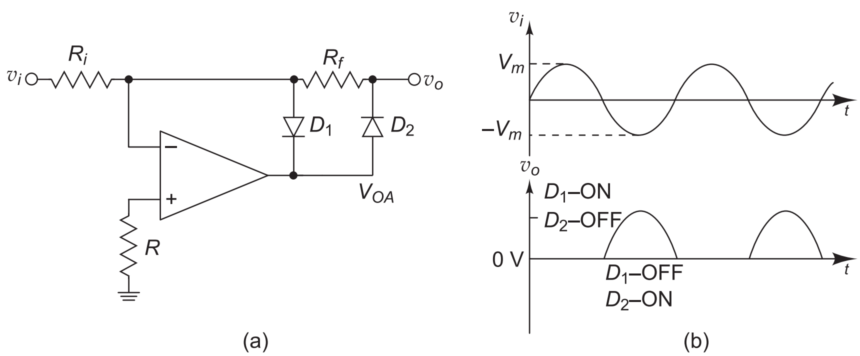


Figure 5.64(a) Non-saturating half-wave precision rectifier circuit (b) Input and output waveforms

The circuit operation can mathematically be expressed as

$$v_o = 0 \text{ when } v_i > 0$$

and $$v_o = -\frac{R_f}{R_i}v_i \text{ for } v_i < 0$$

The voltage V_{OA} at the op-amp output is

$$V_{OA} \approx -\ 0.7\text{ V for } v_i > 0$$

and $$V_{OA} \approx \frac{R_f}{R_i} v_i + 0.7\text{V for } v_i < 0.$$

The input and output waveforms are shown in Figure 5.64(b). The op-amp shown in the circuit must be a high-speed op-amp. This accommodates the abrupt changes in the value of V_{OA} when v_i changes sign and improves the frequency response characteristics of the circuit.

The advantages of half-wave rectifier are (i) it is a precision half-wave rectifier, and (ii) it is a non-saturating one.

The inverting characteristics of the output v_o can be circumvented by the use of an additional inversion for achieving a positive output.

5.15.3 Full-wave Rectifier [AU Nov/Dec, 2013; Nov/Dec, 2014]

The full-wave rectifier circuit commonly called an *absolute value circuit* is shown in Figure 5.65(a). The first part of the total circuit is a half-wave rectifier circuit considered earlier in Figure 5.65(a). The second part of the circuit is an inverting summing circuit.

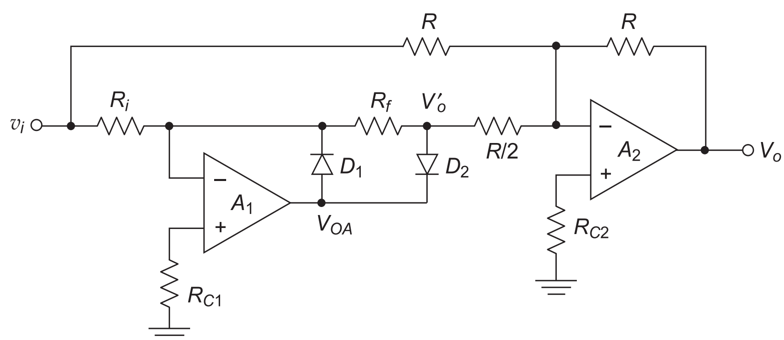


Figure 5.65(a) Active non-saturation full-wave rectifier circuit

For positive input voltage $v_i > 0$ V and assuming that $R_f = R_i = R$, the output voltage $V_o'' = -v_i$. The voltage V_o'' appears as (–) input to the summing op-amp circuit formed by A_2. The gain for the input V_o'', i.e., $\dfrac{-R}{(R/2)}$ is shown in Figure 5.65(a). The input v_i also appears as an input to the summing amplifier. Then, the net output is

$$\begin{aligned} V_o &= -v_i - 2V_o' \\ &= -v_i - 2(-v_i) = v_i \end{aligned}$$

Since $v_i > 0$V, V_o will be positive, with its input-output characteristics in first quadrant. For negative input $v_i < 0$V, the output V'_o of the first part of rectifier circuit is zero. Thus, one input to the summing circuit has a value of zero. However, v_i is also applied as an input to the summer circuit formed by the op-amp A_2. The gain for this input is $\left(-\frac{R}{R}\right) = -1$, and hence the output is $V_o = -v_i$. Since v_i is negative, V_o will be inverted and will thus be positive. This corresponds to the second quadrant operation of the circuit.

To summarise the operation of the circuit,

$$V_o = v_i \text{ when } v_i < 0 \text{ V and}$$

$$V_o = v_i \text{ for } v_i > 0 \text{ V, and hence } V_o = |v_i|$$

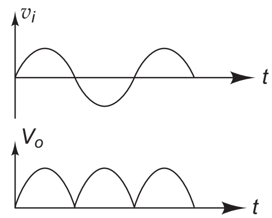


Figure 5.65(b) Input and output waveforms

It can be observed that this circuit is of non-saturating form. The input and output waveforms are shown in Figure 5.65(b).

5.16 Analog and Digital Data Conversions

In the application of signal processing, the measurement and analysis of signals are very important to identify their characteristics. If the signal is unknown, the process of analysis begins with the acquisition of the signal. The most common technique of acquiring signals is by *sampling*. Sampling a signal is the process of acquiring its values only at discrete points in time.

The definitions related to the process of sampling and the subsequent analog and digital conversion processes are:

(i) an *analog signal* is a signal that is defined over a continuous period of time in which the amplitude may assume a continuous range of values.
(ii) the term *quantisation* refers to the process of representing a variable by a finite set of discrete values.
(iii) a *quantised variable* is the signal variable that can assume only finite distinct values.
(iv) a *discrete time signal* is the one that is defined at particular points of time only. Therefore, the independent time variable is quantised. When the amplitude of a discrete-time signal is allowed to assume a continuous range of values, the function is called a *sampled-data* signal. A sampled data signal could result from sampling an analog signal at discrete points of time.
(v) a *digital signal* is a function, in which the time and amplitude are quantised. A digital signal is always represented by a sequence of *words,* where each word can contain a finite number of *bits* (binary digits).

A Digital to Analog Converter (DAC) converts digital data into its equivalent analog data. The analog data is required to drive motors and other analog devices. An Analog to Digital Converter (ADC) converts analog data into its equivalent digital data, i.e., binary data. The A/D converter and D/A converter are also called *data converters* and they are also available as monolithic integrated circuits.

Figure 5.66 shows a typical application in which A/D and D/A conversions are employed. An analog input signal from a transducer is band-limited by *anti-aliasing* filter. The signal is then sampled at a frequency rate higher than twice the maximum frequency of the band-limited input signal. That is, when the A/D converter is operated at a rate of f_s samples/second, the highest frequency component of the input signal can be less than $f_s/2$. The A/D converters normally require the input to be held constant during the conversion process. Hence, a *Sample-and-Hold Amplifier* (SHA) is introduced in the loop as shown in Figure 5.66 before A/D converter. The SHA freezes the band-limited signal just before the start of each conversion. The digital signal from A/D converter may be processed, transmitted and recorded in digital form by the digital signal processor (DSP) block. Then, the digital signal is converted into analog signal by D/A converter for use in analog form. The D/A converter is usually operated at the same frequency f_s as that of A/D converter. The output of D/A converter is commonly a staircase signal, which is passed through a smoothing filter to eliminate the quantisation noise effects. A *deglitcher* may be introduced in the loop before the smoothing process, to remove any output glitches generated during input code variations.

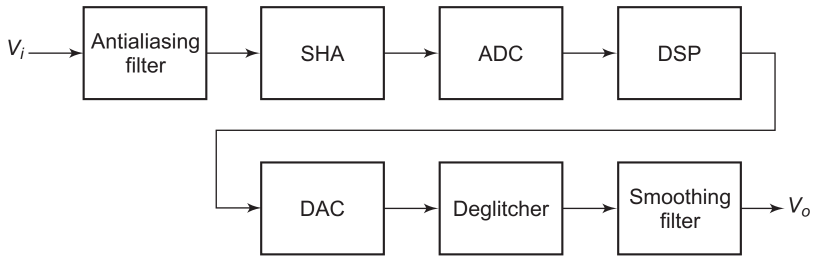


Figure 5.66 Sampled data system using A/D and D/A converters

The schematic structure shown in Figure 5.66 is prevalent either in full or in part in numerous applications such as digital signal processing, direct signal control, digital audio mixing, music and video synthesis, pulse-code modulation (PCM) communication, data acquisition and digital microprocessor based instrumentation.

5.17 D/A Converters (DAC)

The D/A converter converts digital or binary data into its equivalent analog value. The input digital data for a D/A converter is an n-bit binary word D. The bit b_1 is called the most significant bit (MSB) and bit b_n the least significant bit (LSB). Then, the quantity D can be represented by $D = b_1 2^{-1} + b_2 2^{-2} + b_3 2^{-3} + ... + b_n 2^{-n}$. The D/A converter accepts the binary input D and produces an analog output, which is proportional to D using a reference voltage V_R. The converted analog value is either in voltage or current form. For a voltage output D/A converter, the conversion characteristic may be expressed as

$$V_o = KV_{FS}\left(b_1 2^{-1} + b_2 2^{-2} + b_3 2^{-3} + \cdots + b_n 2^{-n}\right) \tag{5.39}$$

where V_o = output voltage

V_{FS} = full-scale range of voltage

K = scaling factor, usually unity

$b_1 \ldots b_n$ = n-bit binary fractional word with binary point located at the left

b_1 = most significant bit (MSB) of weight $V_{FS}/2$

b_n = least significant bit (LSB) of weight $V_{FS}/2^n$

The symbolic representation of an n-bit D/A converter is shown in Figure 5.67.

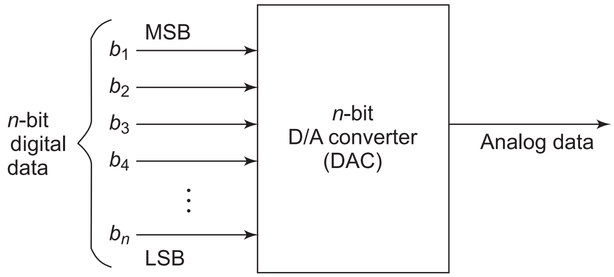


Figure 5.67 *n*-bit D/A converter

5.17.1 Weighted Resistor Type D/A Converter

[AU Nov/Dec, 2013; Nov/Dec, 2014; Nov/Dec, 2012; April/May, 2015; April/May, 2014]

In the weighted resistor type D/A converter, each digital level is converted into an equivalent analog voltage or current. In a 4-bit D/A converter which accepts data from 0000 to 1111, there are 15 discrete levels of input above the zero level, and hence it is convenient to divide the output analog signal into 15 levels above zero. The LSB of the digital data causes a change in the analog output that is equal to $1/15^{th}$ of the full-scale analog output voltage (V_R). Therefore, the weighted resistor network is designed in such a way that a 1 in LSB (2^0) position results in $V_R \times 1/15$ at the output. A 1 in the 2^1 bit position must cause a change in the analog output voltage that is equal to $2/15^{th}$ of V_R (i.e., twice the size of the LSB). Similarly, a 1 in 2^2 and 2^3 bit positions must cause a change of $V_R \times 4/15$V and $V_R \times 8/15$V respectively as the analog output. It is important to note that the sum of the weights assigned to various bit positions of a 4-bit D/A converter must be equal to 1, i.e., $(1/15 + 2/15 + 4/15 + 8/15 = 15/15)\ V_R = V_{FS}$. In general, the weight assigned to the LSB is $1/(2^n - 1)$, where n is the number of bits in the digital input.

Thus, the 4-bit weighted resistor network shown in Figure 5.68(a) performs the following D/A conversion:

1. The 2^0 bit is changed to $1/15^{th}$ of V_R, 2^1 bit to $2/15^{th}$ of V_R, 2^2 bit to $4/15^{th}$ of V_R and 2^3 bit to $8/15^{th}$ of V_R.
2. These four voltages are added together to form the analog output voltage using an op-amp summer circuit.

The resistor R_0, R_1, R_2 and R_3 form the voltage divider network connected with the op-amp and R_L is the load resistor which should be large enough so as not to load the divider network. The LSB should be connected with the highest input resistance R_0 while the 2^1 bit is connected with a resistance of half the value of LSB resistor, i.e., $R_0/2$. Therefore, its current contribution at the summing junction of op-amp will be twice that of LSB. The 2^2 bit is connected with a $1/4^{th}$ of LSB resistance, i.e., $R_0/4$. Similarly, the MSB is connected with $1/8^{th}$ of the LSB resistance, i.e., $R_0/8$. The output is the sum of these four attenuated voltages.

The operating principle of the circuit is explained with the following illustration.

Illustration

The equivalent circuit of Figure 5.68(a), when applied with the digital data of 0001, is shown in Figure 5.68(b). The analog output voltage V_o can be calculated using Millman's theorem, which states that the voltage at any node in a resistive network is equal to the sum of the currents entering the node divided by the sum of the conductance connected at the node. Then the output voltage is expressed as

$$V_o = \frac{\dfrac{V_R}{R_0} + \dfrac{V_R}{R_1} + \dfrac{V_R}{R_2} + \dfrac{V_R}{R_3}}{\dfrac{1}{R_0} + \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} \tag{5.40}$$

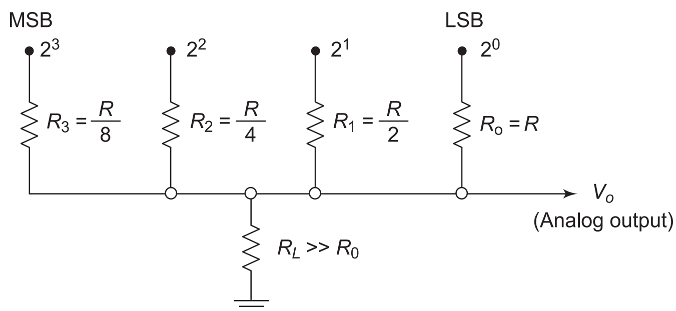


Figure 5.68(a) Four-bit weighted resistor D/A converter

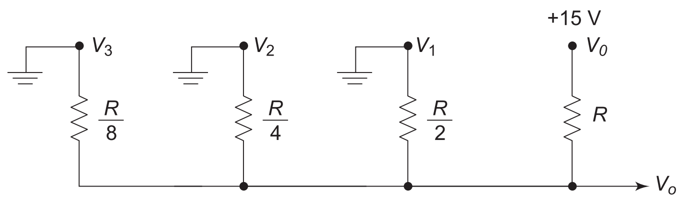


Figure 5.68(b) Equivalent circuit of a 4-bit weighted resistor D/A converter for input 0001

For the weighted resistor network, assuming $R_1 = R$, $R_2 = R/2$, $R_3 = R/4$ and $R_4 = R/8$, and applying the Millman's theorem to the circuit of Figure 5.68(b), we get

$$V_o = \frac{\dfrac{V_R}{R} + \dfrac{V_R}{R/2} + \dfrac{V_R}{R/4} + \dfrac{V_R}{R/8}}{\dfrac{1}{R} + \dfrac{1}{R/2} + \dfrac{1}{R/4} + \dfrac{1}{R/8}} \tag{5.41}$$

Figure 5.69(a) shows the circuit of an n-bit D/A converter using op-amp as a summing amplifier. It employs a binary-weighted resistor network to generate the terms $b_i 2^{-i}$ where $i = 1, 2, \ldots n$. The circuit also uses n-electronic switches controlled by the binary input word $b_1, b_2, \ldots b_n$ and a reference voltage $-V_R$. The switches are of single pole double throw (SPDT) type. If the binary input to a switch is 1, then the switch connects the resistance to the reference voltage $-V_R$. When the input bit to the switch is 0, it connects the resistor to ground.

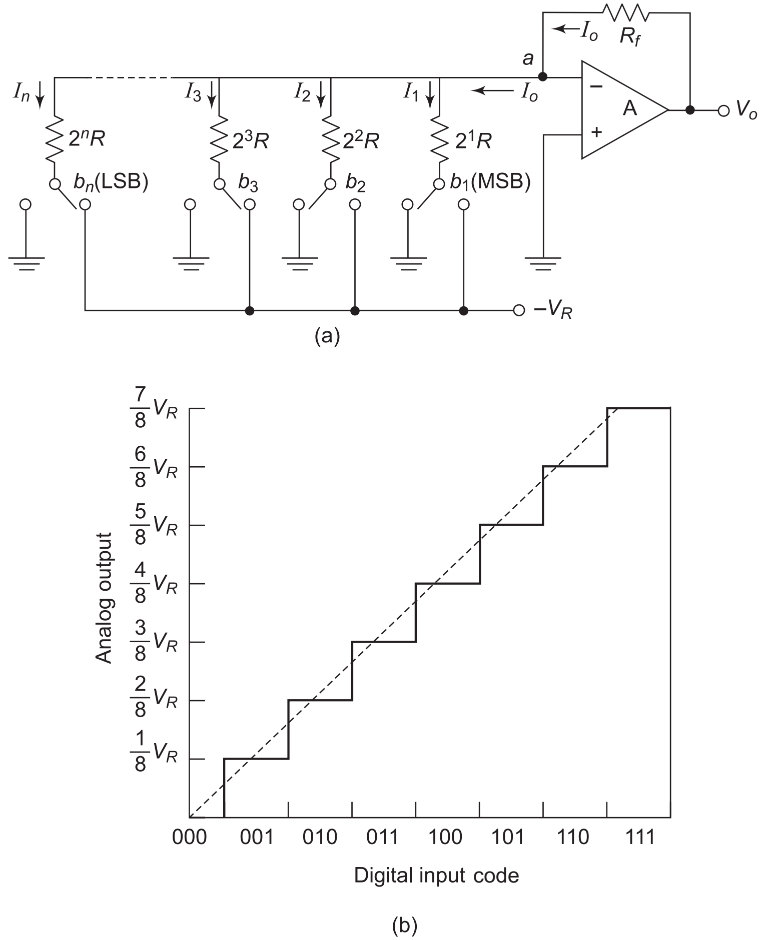


Figure 5.69 An n-bit weighted resistor D/A converter: (a) Circuit diagram (b) Transfer characteristic

Considering an ideal op-amp A, the output current I_o is given by

$$I_o = I_1 + I_2 + \cdots + I_n$$

$$= \frac{V_R}{2^1 R} b_1 + \frac{V_R}{2^2 R} b_2 + \cdots + \frac{V_R}{2^n R} b_n$$

$$= \frac{V_R}{R}[b_1 2^{-1} + b_2 2^{-2} + \cdots + b_n 2^{-n}]$$

Then the output voltage $$V_o = I_o R_f = \frac{V_R}{R}[b_1 2^{-1} + b_2 2^{-2} + \cdots + b_n 2^{-n}] \qquad (5.42)$$

Using Eqs. (5.39) and (5.42), it can be seen that if $R_f = R$, then $K = 1$ and $V_{FS} = V_R$.

The n-bit D/A converter circuit shown in Figure 5.69(a) uses a negative reference voltage, thus producing a positive staircase voltage. The analog output voltage waveform for 3-bit weighted resistor D/A converter is shown in the transfer characteristics of Figure 5.69(b) for an input binary word 000, 001, … 111.

It can be noted that

(i) The D/A converter output is the result of multiplying the analog signal V_R by the signal data. Therefore, if V_R is made variable, then the D/A converter is called a *multiplying D/A converter* [MDAC].
(ii) Higher the value of n, finer is the resolution of conversion, and closer is the staircase to a continuous ramp waveform. DACs are available for word lengths ranging from 6 bits to 20 bits or more with 6, 8, 10, 12 and 14 bits being common.
(iii) The op-amp in Figure 5.69(a) can be connected in non-inverting mode also.
(iv) The op-amp is operated as a *current-to-voltage* converter.
(v) Polarity of V_R is chosen depending on the switches to be operated.
(vi) The accuracy and stability of D/A converter are based on the accuracy of resistors and their temperature dependence and the resistors have to handle varying currents based on bit values.
(vii) The switches are in series with resistors, and therefore the finite ON resistance of the switch must be very low. The bipolar transistor does not perform well as voltage switches due to the voltage offset when it is in saturation. Therefore MOSFET devices are preferred as efficient electronic switches which are discussed in Section 5.8.

The main disadvantage of binary weighted D/A converter is the requirement of wide range of resistor values. As the length of the binary word is increased, the range of resistor values needed also increases. For an 8-bit D/A converter, the resistor values to be connected with the bits are $2^0R + 2^1R + \cdots + 2^7R$. Therefore, the largest resistor corresponding to bit b_8 is 128 times the value of the smallest resistor corresponding to bit b_1. The fabrication of such large value of resistors of the order of $M\Omega$ is not practically possible in monolithic circuit fabrication. In addition, the voltage drop variations across such high value resistors due to the bias currents affect the accuracy. Therefore, the limitations in achieving and maintaining resistor ratios restrict the use of weighted resistor D/A converters to below 8-bits of word length.

The R-$2R$ ladder type D/A converter is a better choice for practical applications and it overcomes such drawbacks.

5.17.2 *R*-2*R* Ladder D/A Converter

[AU Nov/Dec, 2014; Nov/Dec, 2012; April/May, 2015]

A wide range of resistor values is required in the design of binary weighted resistor D/A converter. In R-$2R$ ladder D/A converter, resistors of only two values, i.e., R and $2R$ are used. Hence, it is suitable for integrated circuit fabrication. The typical values of R used vary from 2.5 kΩ to 10 kΩ. The principle of operation of a ladder type network for 4-bit D/A conversion is shown in Figure 5.70(a), with 4-bit binary input, $b_1\, b_2\, b_3\, b_4$, analog output V_o and one terminating resistor $2R$.

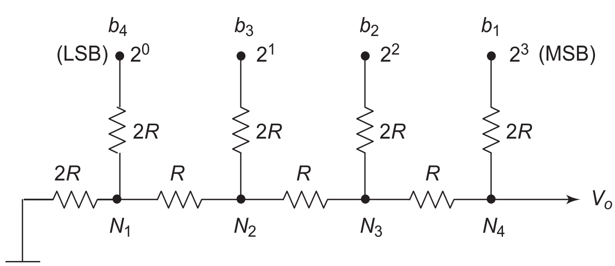


Figure 5.70(a) Four-bit R-2R ladder type D/A converter

In this ladder circuit, the output voltage is a weighted sum of digital inputs. For example, if the 4-bit binary input, $b_1\, b_2\, b_3\, b_4$ is 1000, i.e., if MSB is 1, while the other three inputs are 0, the circuit shown in Figure 5.70(a) can be modified as shown in Figure 5.70(b).

Here, the terminating resistor ($2R$) and the resistor connected to b_4 input ($2R$) are combined at node N_1 to form an equivalent resistor (R) as shown in the equivalent circuit of 1st stage in Figure 5.70(c).

Then, at node N_2, the resistor connected with b_3 input ($2R$) can be combined with the resistor ($R + R = 2R$) to form the 2nd stage of equivalent circuit as shown in Figure 5.70(d).

Similarly, at Node N_3, the equivalent resistor is R as shown in the equivalent circuit of stage 3 in Figure 5.70(e). Then, the analog output voltage V_o is given by

$$V_o = \frac{V_R \times 2R}{R + R + 2R} = \frac{V_R}{2} \tag{5.43}$$

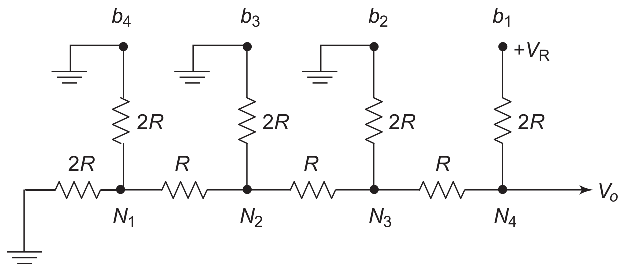


Figure 5.70(b) Equivalent circuits for binary input $b_1\, b_2\, b_3\, b_4 = 1000$

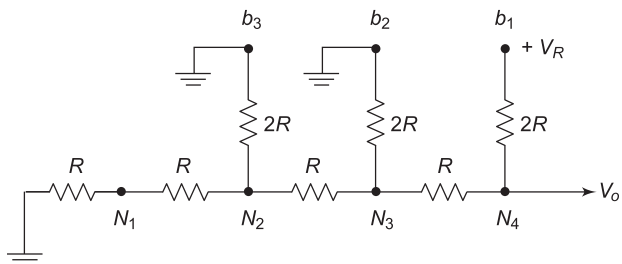


Figure 5.70(c) Equivalent circuit of 1st stage

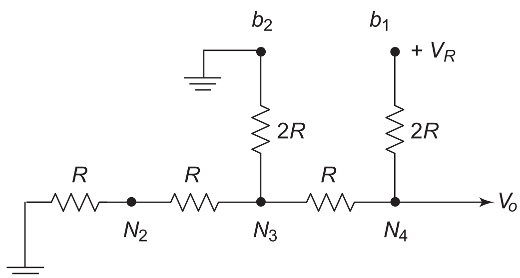


Figure 5.70(d) Equivalent circuit of 2nd stage

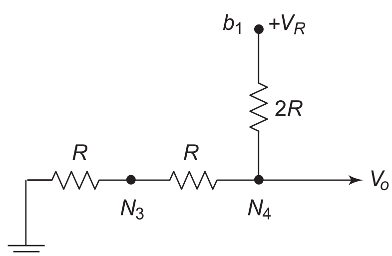


Figure 5.70(e) Equivalent circuit of 3rd stage

Thus, for digital input $b_1\, b_2\, b_3\, b_4 = 1000$, i.e., when MSB = 1, the output is $V_R/2$. Similarly, it can be found that for digital input $b_1\, b_2\, b_3\, b_4 = 0100$, i.e., when second MSB = 1, the output is $V_R/4$; for $b_1\, b_2\, b_3\, b_4 = 0010$, the output is $V_R/8$ and for $b_1\, b_2\, b_3\, b_4 = 0001$, i.e., when LSB = 1, the output becomes $V_R/16$.

Since the resistive ladder is a linear network, the principle of superposition can be used to find the total analog output voltage for a particular digital input by adding the output voltages caused by the individual digital inputs. This can be represented for an n-bit D/A converter as follows:

$$V_o = \frac{V_R}{2^1} + \frac{V_R}{2^2} + \frac{V_R}{2^3} + \cdots + \frac{V_R}{2^n} \tag{5.44}$$

where n is the total number of bits at the input.

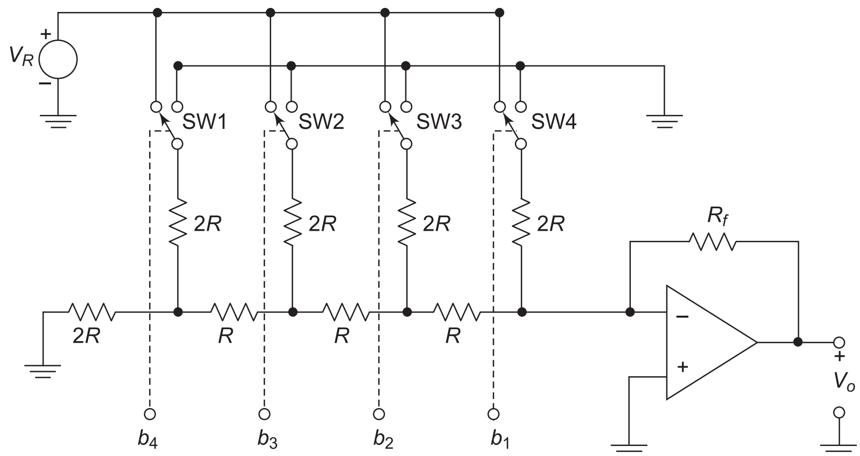


Figure 5.71 Four-bit R-2R ladder D/A converter

Figure 5.71 shows a practical circuit arrangement of a 4-bit D/A converter using an op-amp. The inverting input terminal of the op-amp acts as summing junction for the ladder inputs. Using Eqn. (5.44) the output voltage V_o is expressed by

$$V_o = -V_R \frac{R_f}{R}\left(\frac{b_1}{2^1} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \frac{b_4}{2^4}\right) \tag{5.45}$$

$$= -V_R \frac{R_f}{R \times 2^4}\,(b_1 2^3 + b_2 2^2 + b_3 2^1 + b_4 2^0)$$

Or, more generally for an n-bit input signal, assuming $R_f = R$

$$V_o = -\frac{V_R}{2^n}\,(b_1 2^{n-1} + b_2 2^{n-2} + \cdots + b_n 2^0)$$

The resolution of the R-$2R$ ladder type D/A converter with current output is given by

$$\text{Resolution } I = \frac{1}{2^n} \times \frac{V_R}{R} \tag{5.46}$$

The resolution of the R-$2R$ ladder type D/A converter with voltage output is given by

$$\text{Resolution } V = \frac{1}{2^n} \times \frac{V_R}{R} \times R_f \tag{5.47}$$

where R_f is the feedback resistance of the op-amp.

5.18 A/D Converters (ADC)

An A/D converter does the inverse function of a D/A converter. It converts an analog signal into its equivalent n-bit binary coded digital output signal. The analog input is sampled at a frequency much higher than the maximum frequency component of the input signal. The digital output from an A/D converter can be in serial or parallel form.

The A/D converter accepts an analog input v_i and produces an output binary word $b_1, b_2 \ldots b_n$ of fractional value D such that

$$D = b_1 2^{-1} + b_2 2^{-2} + \cdots + b_n 2^{-n} \tag{5.48}$$

where b_1 is the MSB and b_n is the LSB. The symbolic representation of an n-bit A/D converter is shown in Figure 5.72(a). Two additional control pins START input and End of Conversion (EOC) output are provided with A/D converters. The START input initiates the conversion and the EOC announces when the conversion is complete. The output can be of parallel or serial form. Usually latches, control logic and buffers are provided to enable interfacing of the A/D converter to microprocessors or LCD/LED displays directly.

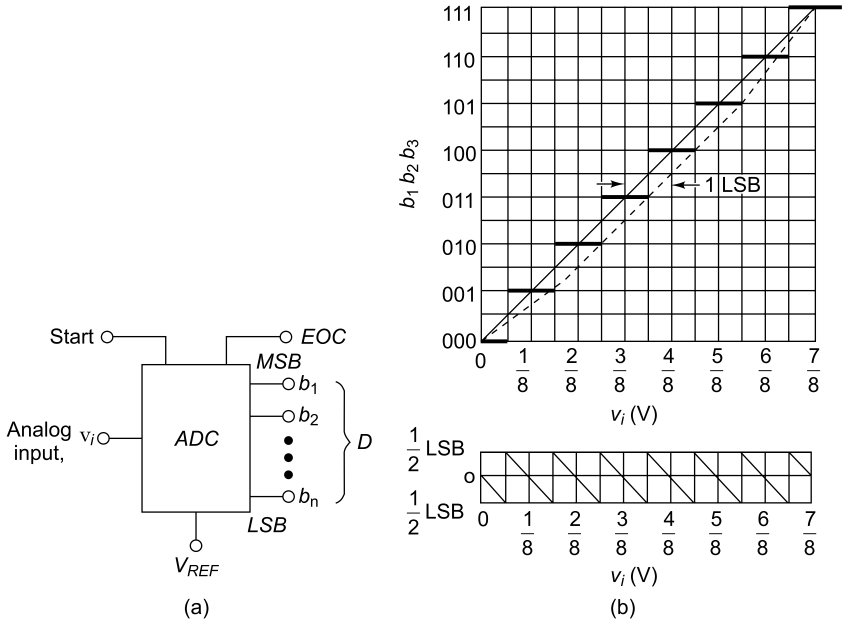


Figure 5.72 A/D converter (a) Symbolic representation (b) Ideal transfer characteristics and quantisation noise for a 3-bit A/D converter

Figure 5.72(b) shows the ideal characteristics of a 3-bit A/D converter with V_{FS} = 1.0 V where V_{FS} is the full-scale analog voltage. The A/D conversion process divides the analog input into 2^n intervals. These intervals are called *code ranges* and all the values of v_i falling within a code range are represented by the particular code. For instance, the code 110 corresponding to $v_i = \frac{6}{8}$ V represents all inputs of value $\frac{6}{8} \pm \frac{1}{16}$ V. Hence, the output can err by $\pm\frac{1}{2}$ LSB.

The general block diagram of an A/D converter is shown in Figure 5.73. It consists of an antialiasing filter or prefilter, Sample-and-Hold amplifier, a quantiser and an encoder. The prefilter avoids the aliasing of high frequency signals. The Sample-and-Hold circuit holds the input analog signal into the A/D converter at a constant value during the conversion time. The quantiser segments the reference voltage signal into subranges. Typically, for an n-bit digital output code, there are 2^n subranges. The digital processor forms the encoder circuit which encodes the subrange into the corresponding digital bits. Therefore, the analog input signal is converted into an equivalent digital output code within the *conversion time.*

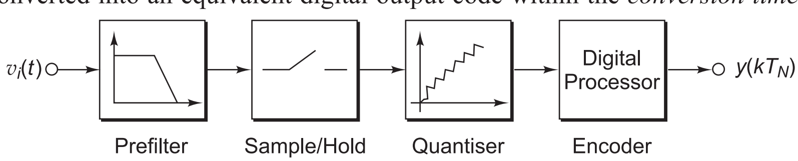


Figure 5.73 General block diagram of an A/D converter

5.18.1 Simultaneous Type (Flash Type) A/D Converter

[AU Nov/Dec, 2014; April/May, 2014; April/May, 2015]

The simultaneous type A/D converter is based on comparing an unknown analog input voltage with a set of reference voltages. To convert an analog signal into a digital signal of n output bits ($2^n - 1$) number of comparators are required. For example, a 2-bit A/D converter requires 3 or ($2^2 - 1$) comparators, while a 3-bit converter needs 7 or ($2^3 - 1$) comparators. The block diagram of a 2-bit simultaneous type A/D converter is shown in Figure 5.74.

As shown in Figure 5.74, the three op-amps are used as comparators. The non-inverting inputs of all the three comparators are connected to the analog input voltage. The inverting input terminal of the op-amps are connected to a set of reference voltages $V/4$, $2V/4$ and $3V/4$ respectively, which are obtained using a resistive divider network and power supply +V.

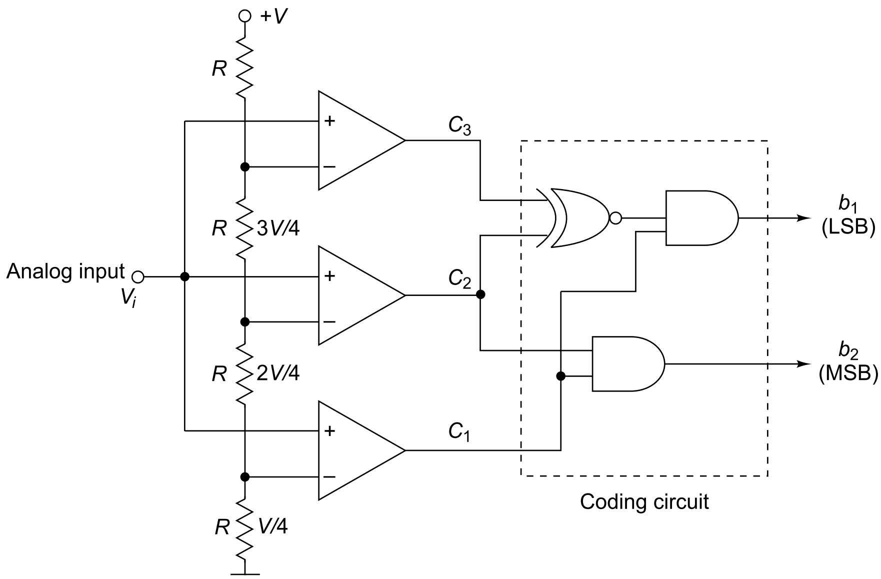


Figure 5.74 Block diagram of 2-bit simultaneous type A/D converter

The output of a comparator is in *positive* saturation state when the voltage at the non-inverting input terminal is more than the voltage at the inverting terminal and it is in *negative* saturation state otherwise. When the analog input voltage is less than $V/4$, the voltage at the non-inverting terminals of the three comparators is less than their respective inverting input voltages, and hence, the comparator outputs are $C_1\ C_2\ C_3$ = 000. When the analog input is between $V/4$ and $V/2$, the comparator outputs are $C_1\ C_2\ C_3$ = 100. Table 5.2 shows the comparator outputs for different ranges of analog voltage and their corresponding digital outputs.

Since there are four ranges of analog input voltages, this can be coded using a 2 bit digital output (b_2, b_1) as shown in Table 5.2. The coding circuit for encoding the three comparator outputs into two digital outputs is shown inside the dotted square of Figure 5.74 using the simplified expressions for b_1 and b_2 as discussed below.

From Table 5.2, logic expressions for b_2 and b_1 can be written as

$$b_2 = C_1C_2\overline{C_3} + C_1C_2C_3 = C_1C_2\left(\overline{C_3} + C_3\right) = C_1C_2 \tag{5.49}$$

$$b_1 = C_1\overline{C_2}\,\overline{C_3} + C_1C_2C_3 = C_1\left(\overline{C_2 \oplus C_3}\right) \tag{5.50}$$

Table 5.2 *Comparator and digital outputs for a 2-bit simultaneous type A/D converter*

Analog Input Voltage (V_i)	Comparator Outputs			Digital Outputs	
	C_1	C_2	C_3	b_2	b_1
$0 \le V_i \le V/4$	0	0	0	0	0
$V/4 \le V_i \le V/2$	1	0	0	0	1
$V/2 \le V_i \le 3V/4$	1	1	0	1	0
$3V/4 \le V_i \le V$	1	1	1	1	1

Similarly, a 3-bit A/D converter can be constructed using seven ($2^3 - 1$) comparators as shown in Figure 5.75. The comparator and digital outputs for eight different ranges of analog input voltage are given in Table 5.3.

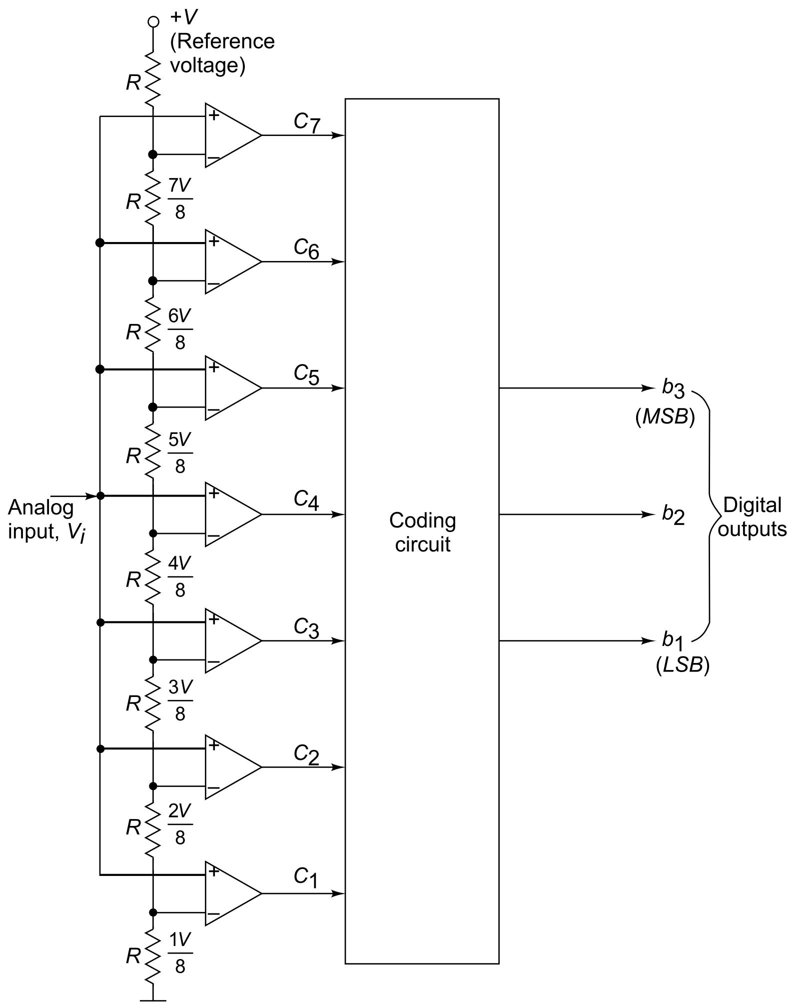


Figure 5.75 Block diagram of 3-bit simultaneous type A/D converter

From Table 5.3, it is clear that the logic expressions for (b_3, b_2 and b_1) are complex due to their dependence on seven input variables (C_1, C_2, ... C_7). Hence, the coding circuit is implemented using a priority encoder. The IC 74148 is an 8 to 3 priority encoder with active LOW inputs and outputs. Since the comparator outputs are active HIGH, they are connected to the inputs of encoder through inverters and the outputs of encoder are inverted once again to get active HIGH digital outputs b_3, b_2 and b_1 as shown in Figure 5.76.

Table 5.3 *Comparator and digital outputs for 3-bit simultaneous type A/D converter*

Analog input voltage (V)	Comparator Outputs							Digital Outputs		
	C_1	C_2	C_3	C_4	C_5	C_6	C_7	b_3	b_2	b_1
$0 \le V_i \le V/8$	0	0	0	0	0	0	0	0	0	0
$V/8 \le V_i \le 2V/8$	1	0	0	0	0	0	0	0	0	1
$V/8 \le V_i \le 3V/8$	1	1	0	0	0	0	0	0	1	0
$V/8 \le V_i \le 4V/8$	1	1	1	0	0	0	0	0	1	1
$V/8 \le V_i \le 5V/8$	1	1	1	1	0	0	0	1	0	0
$V/8 \le V_i \le 6V/8$	1	1	1	1	1	0	0	1	0	1
$V/8 \le V_i \le 7V/8$	1	1	1	1	1	1	0	1	1	0
$V/8 \le V_i \le V$	1	1	1	1	1	1	1	1	1	1

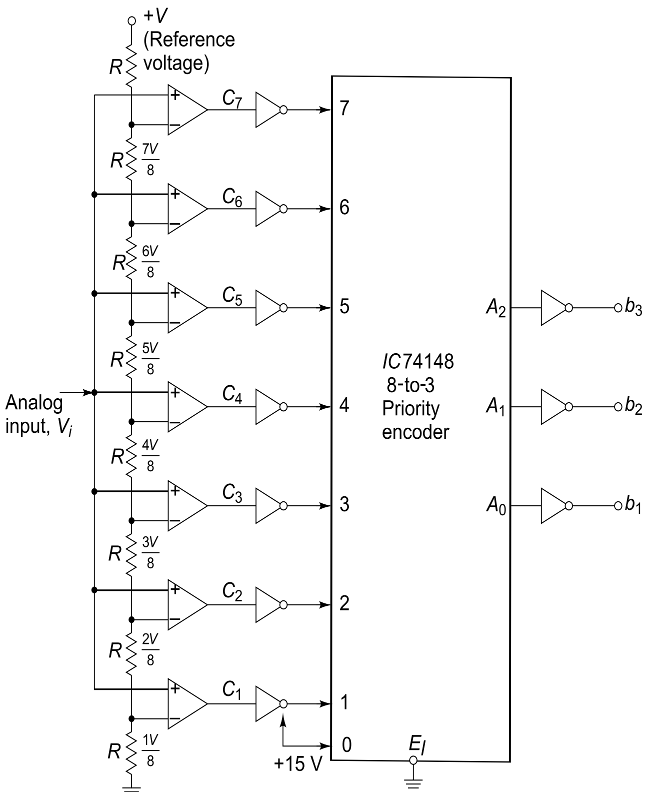


Figure 5.76 Logic diagram of 3-bit simultaneous type A/D converter

Advantages

(i) Simultaneous type A/D converter is the fastest because A/D conversion is performed simultaneously through a set of comparators. Hence, it is also called *flash type* A/D converter. Typical conversion time is 100 ns or less.

(ii) The construction is simple and easier to design.

Disadvantages

The simultaneous type A/D converter is not suitable for A/D conversion with more than 3 or 4 digital output bits. It is becouse of the fact that $(2^n - 1)$ comparators are required for an n-bit A/D converter and the number of comparators required doubles for each added bit.

5.18.2 Successive Approximation Type A/D Converter

The conversion time is maintained constant in successive approximation type A/D converter, and it is proportional to the number of bits in the digital output, unlike the counter and continuous type A/D converters.

The basic principle of this A/D converter is that the unknown analog input voltage is approximated against an n-bit digital value by trying one bit at a time, beginning with the MSB. The principle of successive approximation process for a 4-bit conversion is shown in Figure 5.77.

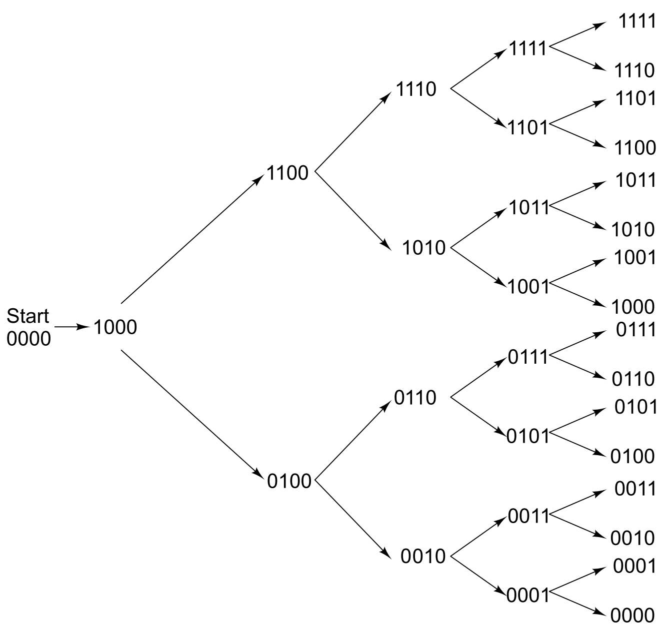


Figure 5.77 Successive approximation principle for 4-bit digital output

This type of A/D converter operates by successively dividing the voltage range by half, as explained in the following steps.

(i) The MSB is initially set to 1 with the remaining three bits set as 0. The digital equivalent is compared with the unknown analog input voltage.

(ii) If the analog input voltage is higher than the digital equivalent, the MSB is retained as 1 and the second MSB is set to 1. Otherwise, the MSB is reset to 0 and the second MSB is set to 1.

(iii) Comparison is made as given in step 1 to decide whether to retain or reset the second MSB. The third MSB is set to 1 and the operation is repeated down to LSB and by this time, the converted digital value is available in the SAR.

From Figure 5.77, it can be seen that the conversion time is constant (i.e., four cycles for 4-bit A/D converter) for various digital outputs. This method uses a very efficient search strategy to complete an *n*-bit conversion in just n-clock periods. Therefore, for an 8-bit successive approximation type A/D converter, the conversion requires only 8 cycles, irrespective of the amplitude of analog input voltage.

The functional block diagram of successive approximation type A/D converter is shown in Figure 5.78. The circuit employs a *successive approximation register* (SAR) which finds the required value of each successive bit by *trial and error* method. The output of the SAR is fed to an *n*-bit D/A converter. The analog output equivalent of the D/A converter is applied to the non-inverting input of the comparator, while the other input of the comparator is connected with an unknown analog input voltage V_i under conversion. The comparator output is used to activate the successive approximation logic of SAR.

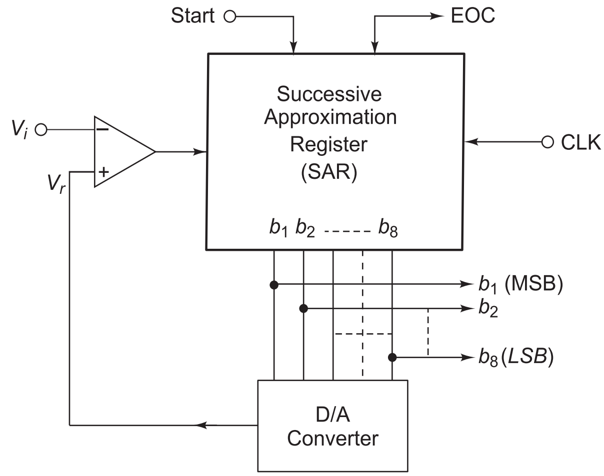


Figure 5.78 Functional block diagram of successive approximation type A/D converter

When the START command is applied, the SAR sets the MSB (b_1) of the digital signal, while the other bits are made zero, so that the trial code becomes 1 followed by zeros. For example, for an 8-bit A/D converter the trial code is 10000000. The output of the SAR is converted into analog equivalent V_r and gets compared with the input signal V_i. If V_i is greater than the D/A converter output, then the trial code 10000000 is less than the correct digital value. The MSB is retained as 1 and the next significant bit is made 1 and the testing is repeated. If the analog input V_i is now less than the D/A converter output, then the value 11000000 is greater than the exact digital equivalent. Therefore, the comparator resets the second MSB to 0 and proceeds to the next most significant bit. This process is repeated for all the remaining lower bits in sequence until all the bit positions are tested. The EOC signal is sent out when all the bits are scanned and the value of D/A converter output just crosses V_i.

Table 5.4 shows the flow of conversion sequence and Figure 5.79 shows the output response with the associated waveforms. It can be observed that the D/A converter output voltage gets successively closer to the analog input voltage V_i. For an 8-bit A/D converter, it requires 8 pulses to compute the output irrespective of the value of the analog input.

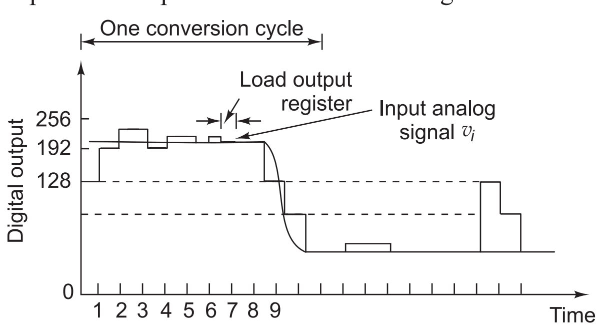


Figure 5.79 Output response for an analog input

Table 5.4 *Successive approximation conversion sequence*

Correct Digital Representation	Successive Approximation Register (SAR) Output V_i at Different Stages in the Conversion	Comparator Output
11010100	10000000	1 (initial output)
	11000000	1
	11100000	0
	11010000	1
	11011000	0
	11010100	1
	11010110	0
	11010101	0
	11010100	

Successive approximation ICs are available as monolithic circuits. The AD7582 from Analog Devices Corporation provides a 28-pin DIP CMOS package for 12-bit A/D conversion using successive approximation technique.

5.19 MULTIVIBRATOR USING 555 TIMER IC

The 555 integrated circuit timer was first introduced by Signetics Corporation as Type SE555/NE555. It is available in 8-pin circular style TO-99 Can, 8-pin mini-DIP and 14-pin DIP as shown in Figure 5.80. The 555 IC is widely popular and various manufacturers provide the IC. The IC 556 contains two 555 timers in a 14-pin DIP package and Exar's XR-2240 contains a 555 timer with a programmable binary counter in a single 16-pin package.

The 555 timer can be operated with a DC supply voltage ranging from +5V to +18V. This feature makes the IC compatible to TTL/CMOS logic circuits and op-amp based circuits. The IC 555 timer is very versatile and its applications include oscillator, pulse generator, square and ramp wave generator, *one-shot* multivibrator, safety alarm and timer circuits, traffic light controllers, etc. The 555 timer can provide time delay, ranging from microseconds to hours.

5.19.1 General Description of the IC 555

Figure 5.81 shows the functional block diagram of 555 IC timer. The positive DC power supply terminal is connected to pin 8(V_{CC}) and negative terminal is connected to pin 1(*Gnd*). The ground pin acts as a common ground for all voltage references while using the IC. The *output* (pin 3) can assume a HIGH level (typically 0.5V less than V_{CC}) or a LOW level (approximately 0.1V).

Two comparators, namely, upper comparator (UC) and lower comparator (LC) are used in the circuit. Three 5 kΩ internal resistors provide a potential divider arrangement. It provides a voltage of $(2/3)V_{CC}$ to the (–) terminal of the upper comparator and $(1/3)V_{CC}$ to the (+) input terminal of the lower comparator. A *control* voltage input terminal (pin 5) accepts a modulation control input voltage applied externally.

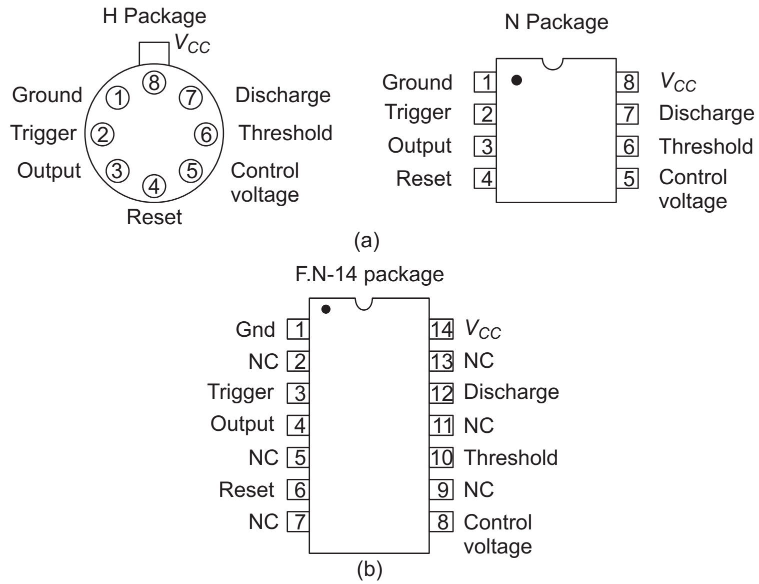


Figure 5.80 Pin configurations of IC 555 Timer

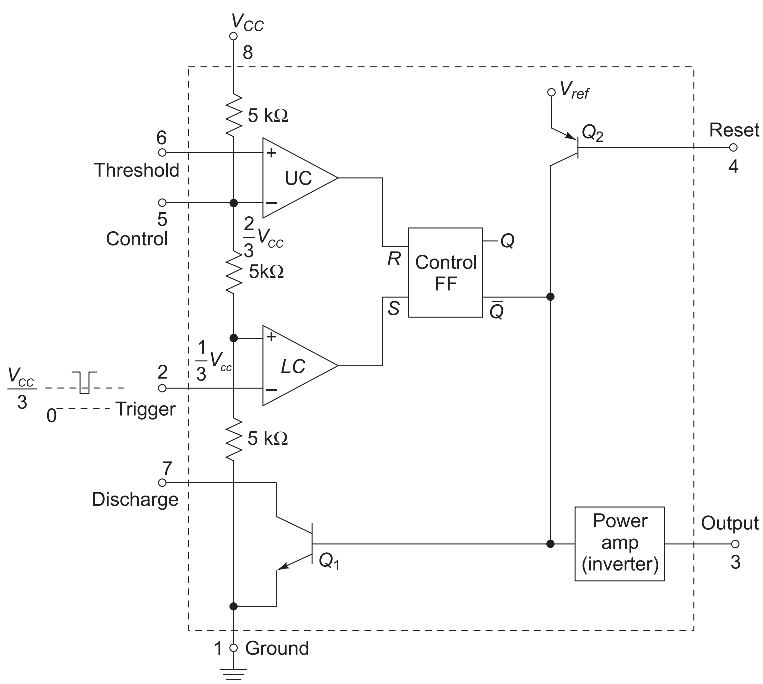


Figure 5.81 Functional block diagram of IC 555 Timer

Pin 5 is connected to ground through a bypassing capacitor of 0.1 μF. It bypasses the noise or ripple from the supply. The (+) input terminal of the UC is called the *threshold terminal* (pin 6) and the (–) input terminal of the LC is the *trigger terminal* (pin 2). The operation of the IC can be summarised as shown in Table 5.5.

Table 5.5 *States of operation of IC 555*

Sl. No.	Trigger (pin 2)	Threshold (pin 6)	Output State (pin 3)	Discharge State (pin 7)
1	Below $(1/3)V_{CC}$	Below $(2/3)V_{CC}$	High	Open
2	Below $(1/3)V_{CC}$	Above $(2/3)V_{CC}$	Last state remains	Last state remains
3	Above $(1/3)V_{CC}$	Below $(2/3)V_{CC}$	Last state remains	Last state remains
4	Above $(1/3)V_{CC}$	Above $(2/3)V_{CC}$	Low	Ground

The standby (stable) state makes the output $\bar{Q}$ of flip-flop (FF) HIGH. This makes the output of inverting power amplifier LOW. When a negative going trigger pulse is applied to pin 2, as the negative edge of the trigger passes through $\frac{1}{3}(V_{CC})$, the output of the lower comparator becomes HIGH and it sets the control FF making $Q = 1$ and $\bar{Q} = 0$. When the threshold voltage at pin 6 exceeds $\frac{2}{3}(V_{CC})$, the output of upper comparator goes HIGH. This action resets the control FF with $Q = 0$ and $\bar{Q} = 1$.

The *reset* terminal (pin 4) allows the resetting of the timer by grounding the pin 4 or reducing its voltage level below 0.4 V. This makes the *output* (pin 3) low overriding the operation of lower comparator. When not used, the *reset* terminal is connected to V_{CC}. Transistor Q_2 isolates the reset input from the *FF* and transistor Q_1. The reference voltage V_{ref} is made available internally from V_{CC}. Transistor Q_1 acts as a *discharge* transistor. When *output* (pin 3) is high, Q_1 is OFF making the *discharge* terminal (pin 7) *open*. When the output is low, Q_1 is forward-biased to ON condition. Then, the *Discharge* terminal appears as a short circuit to ground.

5.19.2 Astable Operation of the Timer IC 555 **[AU Nov/Dec, 2011]**

The functional diagram of the IC 555 connected for astable operation is shown in Figure 5.82. The device connection diagram with external components is shown in Figure 5.83. Resistors R_A and R_B form the timing resistors. The *discharge* (pin 7) terminal is connected to the junction of R_A and R_B. *Threshold* (pin 6) and *trigger* (pin 2) terminals are connected to the v_c terminal, and *control* (pin 5) terminal is by-passed to ground through a 0.01 μF capacitor.

When the power supply V_{CC} is connected to the circuit, the capacitor C charges towards V_{CC}. The charging rate is determined by the time constant $(R_A + R_B)C$. During this period, the *output* (pin 3) is high, since $R = 0$, $S = 1$ and thus $Q = 0$. When the capacitor voltage reaches and rises just above $(2/3)V_{CC}$, the upper comparator triggers, and resets the flip-flop (FF). This makes internal *discharge* transistor Q_1 ON, resulting in the capacitor C discharging towards ground through the resistance R_B and Q_1. The time constant for discharging is R_BC. Since current can also flow through R_A into Q_1, the resistance R_A and R_B are to be made large enough to limit this current. A maximum current of 0.2A can be allowed to flow through the ON transistor Q_1.

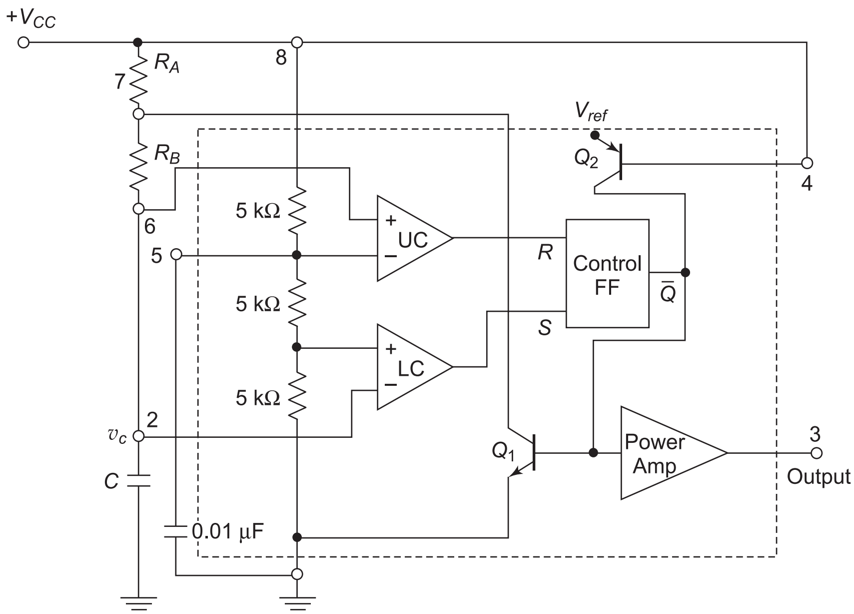


Figure 5.82 Functional diagram of astable multivibrator using IC 555 Timer

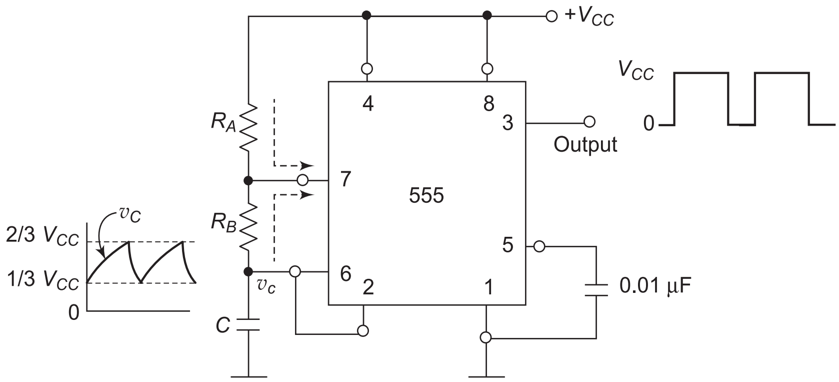


Figure 5.83 Connection diagram of astable multivibrator

When the timing capacitor C discharges, as it reaches and goes just less than $(1/3)V_{CC}$, the lower comparator gets triggered. This sets the flip-flop making $Q = 0$. This results in unclamping the external timing capacitor by switching Q_1 OFF. This cycle of charging to $(2/3)V_{CC}$, and discharging to $(1/3)V_{CC}$ repeats. Figure 5.84 shows the timing sequence and capacitor voltage waveform during the astable operation. The output (pin 3) is high during the internal charging of the capacitor from $(1/3)V_{CC}$ to $(2/3)V_{CC}$. The capacitor voltage v_c for a low pass RC circuit is given by

$$v_c = V_{CC}(1 - e^{t/RC})$$

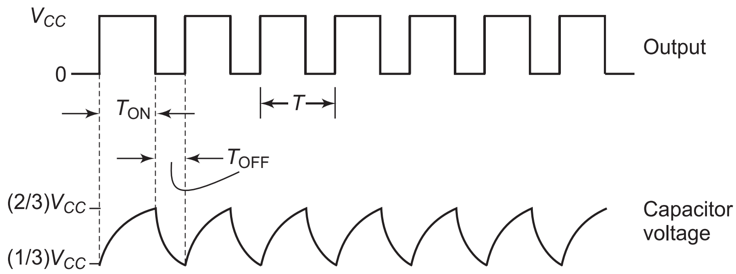


Figure 5.84 The timing waveform of astable multivibrator

Here we assume a step input of V_{CC} volt.

If t_1 is the time taken by the capacitor to charge from 0 to $(2/3)V_{CC}$, then

$$(2/3)V_{CC} = V_{CC}(1 - e^{-t_1/RC})$$

Therefore, $t_1 = 1.098\ RC$

If t_2 is the time taken by the capacitor to charge from 0 to $(1/3)V_{CC}$, then

$$(1/3)V_{CC} = V_{CC}(1 - e^{-t_2/RC})$$

Therefore, $t_2 = 0.405\ RC$

Then, the time taken by the capacitor to charge from $(1/3)V_{CC}$ to $(2/3)V_{CC}$ is given by

$$\begin{aligned} t_{ON} &= t_1 - t_2 \\ &= 1.098\text{RC} - 0.405\ RC \\ &\approx 0.693\ \text{RC} \end{aligned}$$

Thus, t_{ON} for the circuit is

$$t_{ON} = 0.693(R_A + R_B)C$$

where R_A and R_B form the charging path. The output is in LOW level during the period of discharging from $(2/3)V_{CC}$ to $(1/3)V_{CC}$ and the voltage across the capacitor in such a condition is expressed by

$$(1/3)V_{CC} = (2/3)V_{CC}e^{-t/\text{RC}}$$

Hence, $t = 0.693\ \text{RC}$

Therefore, for the astable circuit,

$$t_{OFF} = 0.693\,R_BC$$

where R_B forms the discharging path for the current.

Thus, total time, $T = T_{ON} + T_{OFF}$ or $T = 0.693(R_A + 2R_B)C$

and $$f = \frac{1}{T} = \frac{1.45}{(R_A + 2R_B)C}$$

The *duty cycle D* of any pulse generator circuit is

$$D = \frac{\textit{high (ON) interval}}{\textit{period}} \times 100\% = \frac{T_{ON}}{T} \times 100\%$$

$$D = \frac{R_A + R_B}{R_A + 2R_B} \times 100\%$$

5.19.3 Schmitt Trigger using Timer IC 555

The timer IC 555 can be used to function as a Schmitt trigger with variable threshold voltage levels.

The use of timer IC 555 connected as a Schmitt trigger circuit in astable mode of operation is shown in Figure 5.85(a). The two internal comparator inputs (pins 2 and 6) are connected together and externally biased with a voltage $V_{CC}/2$ through R_1 and R_2 potential divider network. Since the voltage at pins 6 and 2 will trigger the upper comparator (UC) at $(2/3)V_{CC}$ and the lower comparator (LC) at $(1/3)V_{CC}$, the bias provided by R_1 and R_2 is centered within these two threshold levels. The input and output waveforms are shown in Figure 5.85(b).

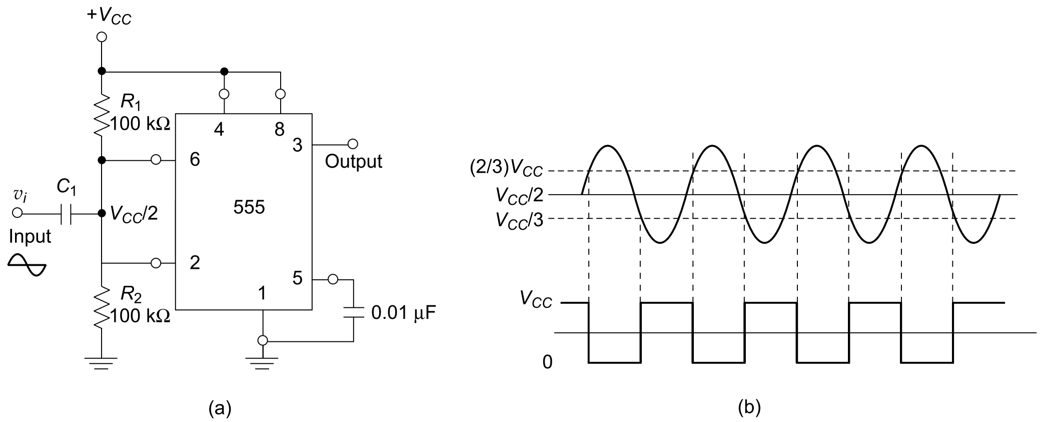


Figure 5.85 Schmitt trigger using IC 555 Timer: (a) Connection diagram, (b) Input and output waveforms

When a sine wave input of sufficient amplitude (v_i), where v_i is greater than $[(2/3)V_{CC} - (1/3)V_{CC}]$ is applied to the circuit, it causes the internal flip-flop to alternatively *Set* and *Reset* generating a square wave output.

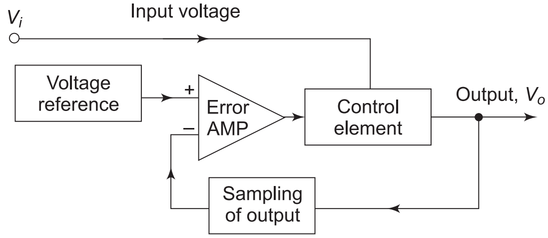
Unlike a conventional multivibrator type of square wave generator that divides the input frequency by 2, the main advantage of Schmitt trigger is that it simply converts the sine-wave signal into square-wave signal of the same frequency. Hence, this circuit can be used as a wave shaper.

5.20 Linear Voltage Regulators

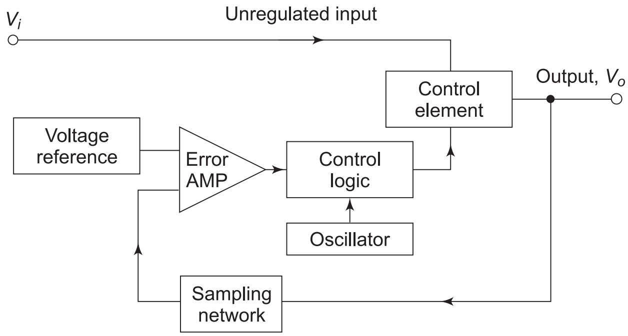
All electronic circuits need DC power supply either from battery or power pack units. It may not be economical and convenient to depend upon battery power supply. Hence, many electronic equipments contain circuits which convert the ac supply voltage into DC voltage at the required level. The DC voltages obtained from such circuits should be stable.

In an unregulated power supply, the output voltage changes whenever the input voltage or load changes. An ideal regulated power supply is an electronic circuit designed to provide a predetermined DC voltage which is independent of the load current and variations in the input voltage. The DC voltage regulated power supplies are employed to provide a stable DC voltage independent of the load current, temperature and AC line voltage variations and to attenuate the ripple.

The commonly used voltage regulators can be classified into (i) Linear voltage regulators and (ii) Switching regulators. The main difference between these two regulators is seen from the block diagrams given in Figures 5.86(a) and (b).



(a)



(b)

Figure 5.86 (a) Block diagram of a linear voltage regulator (b) Block diagram of a switching voltage regulator

The switching regulators make use of a power transistor, which acts as a high frequency switch. Therefore, the transistor does not pass current continuously and it results in improved efficiency in regulation. The switching regulators can generate output voltage of opposite polarity, multiple output voltages, or isolated outputs.

5.20.1 Basics of Voltage Regulator

Linear Mode Power Supply

The basic building blocks of a linear power supply are shown in Figure 5.87. A transformer supplies ac voltage at the required level. This bidirectional ac voltage is converted into a unidirectional and pulsating DC using a rectifier. The unwanted ripple contents of this pulsating DC are removed by a filter to get a pure DC voltage. The output of the filter is fed to a voltage regulator which gives a steady DC output, independent of load variations and input supply fluctuations.

The performance of a voltage regulator is usually defined in terms of the line regulation, load regulation and ripple rejection.

Line Regulation

Line regulation is defined as the change in output voltage for a change in line-supply voltage keeping the load current and temperature constant. Line regulation is given by

$$\text{Line regulation} = \frac{\text{change in output voltage}}{\text{change in input voltage}} = \frac{\Delta V_o}{\Delta V_i}$$

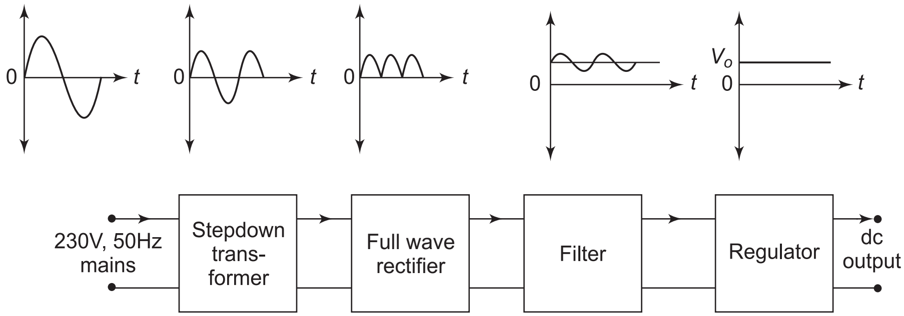


Figure 5.87 Basic building blocks of linear mode power supply

Load Regulation

Load regulation is expressed as

$$\text{Load regulation} = \frac{(V_{\text{no load}} - V_{\text{full load}})}{V_{\text{no load}}}$$

or
$$\text{Load regulation} = \frac{V_{\text{no load}} - V_{\text{full load}}}{V_{\text{full load}}}$$

where $V_{no\ load}$ is the output voltage at zero load current and $V_{full\ load}$ is the output voltage at rated full load current. This is usually denoted in percentage. The plot of the output voltage V_o versus the load current I_L for a full wave rectifier is shown in the load regulation characteristics of Figure 5.88. The drop in the characteristic curve is a measure of the internal resistance of the power supply.

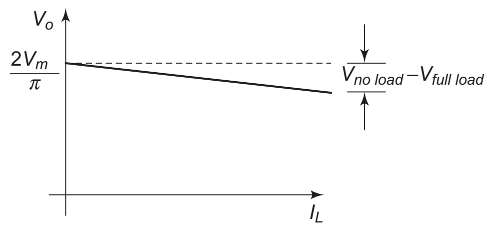


Figure 5.88 Load regulation characteristics

Ripple Rejection Ratio (RRR)

The ac performance of linear IC regulators is specified by a parameter called *ripple rejection ratio*. It is defined as the ratio of peak-to-peak input ripple voltage $\Delta V_{o(\text{unreg})}$ or V_r to the peak-to-peak output ripple voltage $\Delta V_{o(\text{reg})}$. This is typically 60 dB or more for the commonly used voltage regulators.

The Ripple Rejection Ratio (*RRR*) is expressed in decibels as $RRR = 20 \log \frac{\Delta V_{o(\text{unreg})}}{\Delta V_{o(\text{reg})}}$. The *RRR* specification is used particularly in connection with voltage regulators. This provides an indication of the amount of ripple, normally 100 Hz ripple fed to the output.

5.21 Adjustable Voltage Regulators using LM317 and LM 337

The fixed voltage regulators are designed and preset for a particular voltage of *positive/negative* polarities. There are applications which require

(i) regulated voltage sources which are precisely variable, and
(ii) some supply voltages which are not available from standard fixed voltage regulators.

The most popular variable voltage regulators for such applications are LM117 / LM317 and IC723. The LM117/LM317 and LM137/LM337 families are adjustable three terminal positive and negative voltage regulators respectively. There are several variations of these ICs with different current ratings. The operating specifications are identified by their different suffixes and numbers.

5.21.1 LM117/LM317 Adjustable Positive Voltage Regulators

The LM117/LM317 series regulators are adjustable three terminal positive voltage regulators and they are capable of supplying output current of 0.1 A to 1.5 A, over a range of 1.2 V to 37 V output voltage range.

Circuit Connection

The internal functional diagram and the basic circuit connection for the LM317 regulator are shown in Figure 5.89(a) and (b) respectively. The LM317 needs two resistors R_1 and R_2 for setting the output voltage. Normally capacitors are not required. However, when the LM317 is placed more than 15cm apart from the output filter capacitor C_2 of the power supply circuit, an input bypass capacitor, C_1 of 0.1 μF disc or 1 μF tantalum is needed to be connected as shown in Figure 5.89(b). An additional aluminium or tantalum elec-

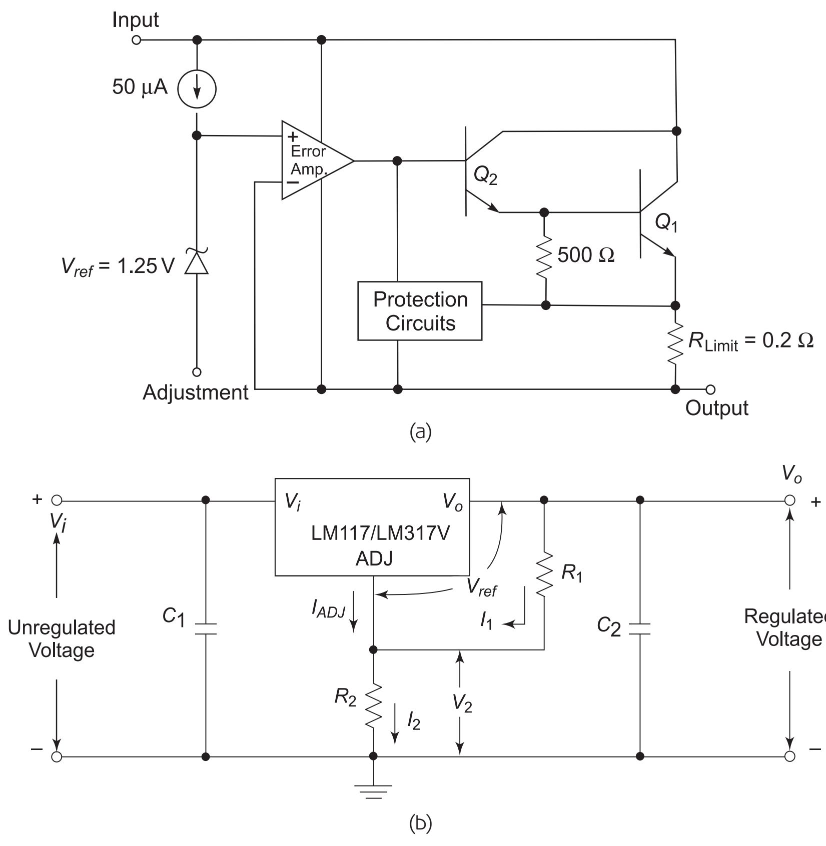


Figure 5.89 (a) LM317 voltage regulator functional diagram (b) Circuit connection for LM317 regulator

trolytic capacitor of range 1 to 1000 μF may be connected at the output to improve the transient response characteristics. The unregulated voltage is applied at the terminal V_i of LM317 with respect to ground and it must be higher than the required regulated voltage V_o, by a value of at least 2V.

When the circuit is connected as shown in Figure 5.89(b), the IC develops a typical value of *reference voltage* V_{ref} = 1.25V, between the output and adjustment terminals. This voltage appears across R_1 resulting in a current flow of $I_1 = \dfrac{V_{\text{ref}}}{R_1}$.

This current also flows through R_2 and an additional current of I_{ADJ} flows out of the adjustment terminal of the regulator through R_2. Thus the net current through R_2 is $I_2 = I_1 + I_{ADJ}$.

The voltage across R_2 is $V_2 = I_2 R_2 = (I_1 + I_{ADJ}) R_2$.

The net output voltage V_o is then given by

$$V_o = V_{\text{ref}} + V_2 = V_{ref} + R_2 (I_1 + I_{ADJ}) \tag{5.51}$$

Substituting $I_1 = \dfrac{V_{\text{ref}}}{R_1}$ in the above equation,

$$V_o = V_{\text{ref}} \left(1 + \frac{R_2}{R_1}\right) + I_{ADJ} R_2 \tag{5.52}$$

The last term $I_{ADJ} R_2$ indicates an error term of a typical value of 50 μA, and when R_2 is low, this term can be neglected.

Then,
$$V_o = 1.25 \left(1 + \frac{R_2}{R_1}\right) \tag{5.53}$$

Hence, the output voltage V_o is a function of R_2 for a specific value of R_1, which is normally 240 Ω. The resistor R_1 is to be connected directly to the regulator output.

Protection diodes can be connected to the regulator circuit as shown in Figure 5.90, when output capacitors of value greater than 25 μF are used. The diode D_1 prevents the capacitors from discharging into the regulator.

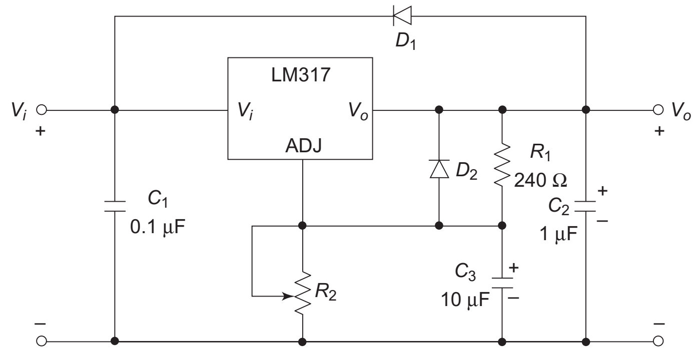


Figure 5.90 LM317 regulator with capacitors and protective diodes

Figure 5.91 shows the circuit arrangement of an adjustable positive voltage regulator using LM317.

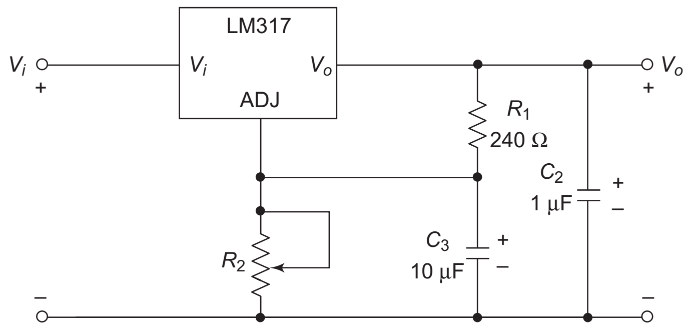


Figure 5.91 LM317 adjustable positive voltage regulator

5.22 General Purpose Voltage Regulator using IC723

[AU April/May, 2013; April/May, 2014; Nov/Dec, 2010; Nov/Dec, 2013]

The three-terminal regulators such as 7805, 7815, 7905, 7915, etc. are capable of producing only fixed positive, or negative output voltages. Moreover, such regulators do not have short circuit protection. Therefore, these three terminal regulators evolved into dual polarity variable voltage regulators. They have provision for regulating positive and negative voltage inputs. The evolution further led to the monolithic linear voltage regulators and monolithic switching regulators. The monolithic linear voltage regulator type IC 723 is discussed in this section.

The IC 723 general purpose regulator overcomes the limitations of three terminal fixed voltage regulators. The IC 723 is a low current device, and can be employed for providing a load current up to 10 A or more by the addition of external transistors.

Figure 5.92 shows the detailed internal circuit diagram of IC 723. It consists of a temperature compensated Zener diode D_1, biased with a constant current source. The reference voltage V_{ref} is available from a buffer amplifier realised by transistors Q_1 through Q_6 and Zener diodes, D_1 and D_2. The V_{ref} has a typical value of 7.15 V. The series-pass element is realised by Darlington-connected transistors Q_{14} and Q_{15}. They boost the output current of the regulator to a value of 150 mA. The terminals $+V$ and V_c are accessible externally. Current limit protection is provided by the transistor Q_{16}. *The sense voltage* is the voltage drop obtained across a suitable resistor connected between the terminals *current limit* (CL) and *current sense* (CS) of the IC 723. Transistors Q_7 through Q_{13} and resistors R_{10} and R_{11} form the error amplifier. The transistors Q_{11} and Q_{12} with their non-inverting and inverting input terminals produce a differential amplifier. The transistor Q_{13} acts as the current source for the differential amplifier.

The functional block diagram of IC 723 is shown in Figure 5.93(a). Figures 5.93(b) and (c) show the pin diagrams for a 14-pin DIP and 10-pin Metal Can packages for the device. The Zener diode, the constant current source and reference amplifier form one section of the IC. The constant current source helps in maintaining a fixed output voltage from Zener diode D_2. The error amplifier, series pass element Q_1 and current limit transistor Q_2 form the second section. The error amplifier compares the input voltages applied at non-inverting (NI) and inverting (INV) input terminals. The error signal obtainable at the output of error amplifier drives the series pass element Q_1.

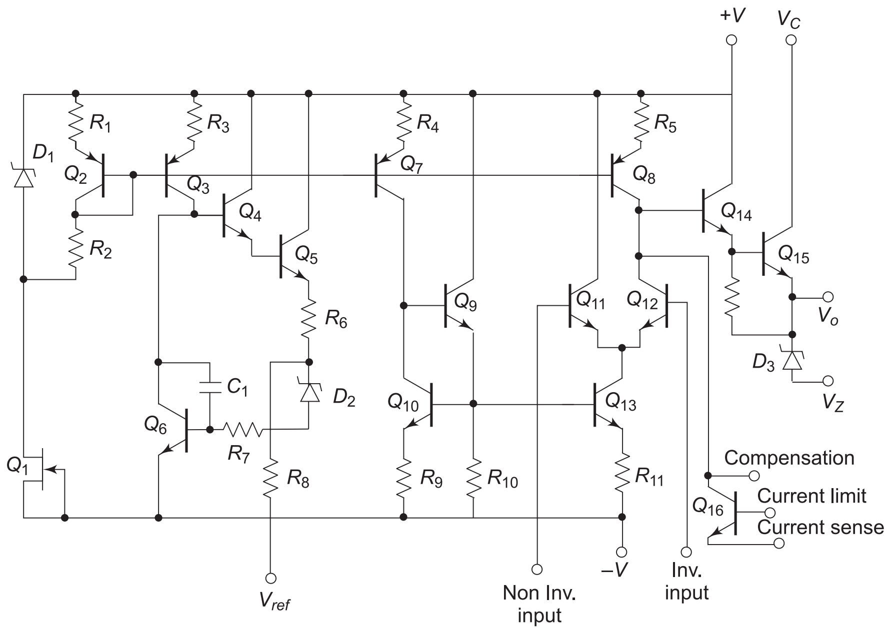


Figure 5.92 IC 723 Regulator internal circuit diagram

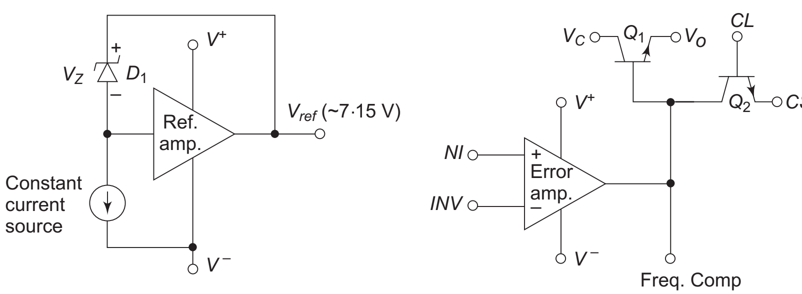


Figure 5.93(a) Functional block diagram of IC 723

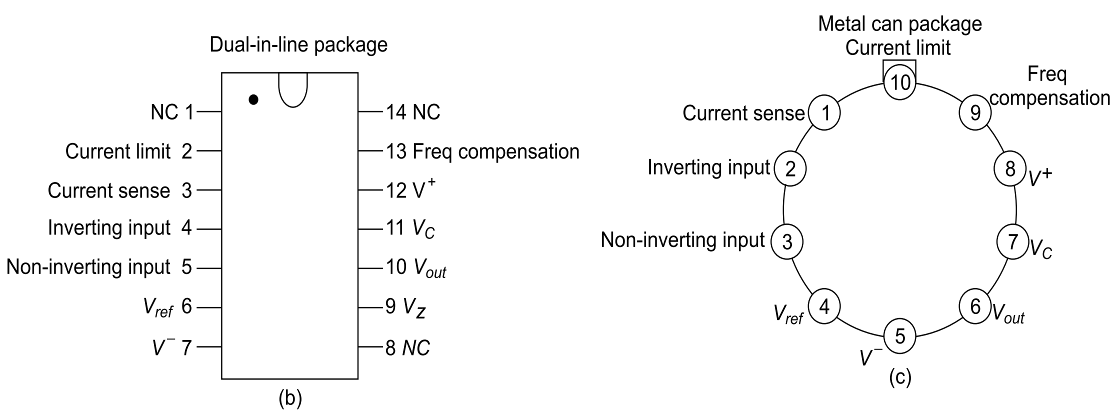


Figure 5.93(b) Pin diagram of 14-pin DIP (c) Pin diagram of 10-pin Metal Can packages

5.22.1 Low Voltage Regulator using IC 723

Figure 5.94 shows the functional block diagram for a low voltage regulator using IC 723. This circuit arrangement is used for regulating voltages ranging from 2V to 7V, and hence it is called a low voltage regulator. The output voltage is directly fed back to the INV input terminal. The non-inverting input (NI) is obtained across the potential divider formed by resistor R_1 and R_2. Hence, voltage at NI terminal is given by

$$V_{NI} = V_{\text{ref}} \times \frac{R_2}{R_1 + R_2}$$

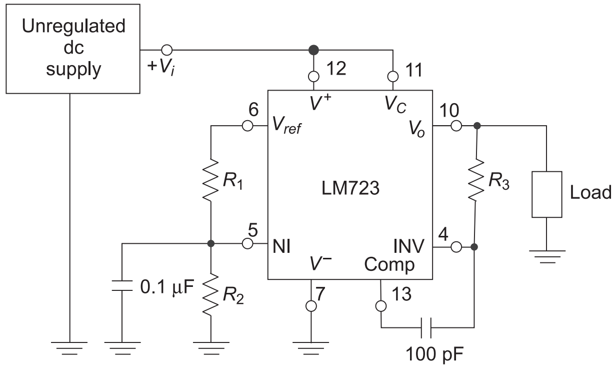


Figure 5.94 Functional block diagram of a low voltage regulator using IC 723

The error amplifier amplifies the difference and it drives the pass transistor Q_1. Depending on the error signal, the pass transistor Q_1, acting as control element, minimises the difference between the NI and INV inputs of error amplifier. Therefore, the output voltage V_o is given by

$$V_o = V_{\text{ref}} \times \frac{R_2}{R_1 + R_2} = 7.15 \times \frac{R_2}{R_1 + R_2}$$

Now, assuming that the output voltage is low, the INV terminal input goes down, making the output of error amplifier more positive. This drives the NPN pass transistor further into conduction. Hence, higher current is driven into the load, thereby causing the output voltage to increase. This compensates for the drop in output voltage. In a similar manner, any rise in load voltage gets regulated.

5.22.2 High Voltage Regulator Circuit using IC 723 **[AU May, 2015]**

The IC 723 can be used for designing a high voltage regulator for output voltages ranging from 7V to 37V. The circuit connection diagram is shown in Figure 5.95. The non-inverting input (NI) terminal is directly connected to V_{ref} through R_3. The inverting input (INV) terminal is connected to the junction of resistors R_1 and R_2 connected with the output V_o. The resistor R_3 is selected to be equal to $R_1 \parallel R_2$. Then the error amplifier acts as a noninverting amplifier with a voltage gain of $A_V = 1 + \frac{R_1}{R_2}$.

Therefore, the output voltage for the circuit is

$$V_o = V_{\text{ref}}\left(1 + \frac{R_1}{R_2}\right) = 7.15\left(1 + \frac{R_1}{R_2}\right)$$

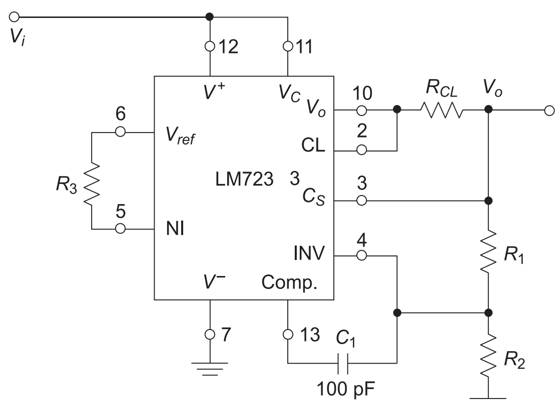


Figure 5.95 Functional block diagram of a high voltage regulator using IC 723

5.22.3 Current Limit Protection **[AU May, 2015]**

The limitation of the regulator IC 723 is that it has no built-in thermal protection and short circuit protection. Therefore, the current limit protection in regulator ICs is necessary for providing protection against short circuit condition across the load.

An active current limiting circuit for IC 723 is shown in Figure 5.96(a). This circuit prevents the load current from increasing beyond a safe value. The operation of the circuit can be explained as follows. The series pass element Q_1, which is part of the regulator circuit, is shown connected in series with a current limiting resistor R_{CL}. The voltage drop across the resistance R_{CL} can bias the transistor Q_2 and turn it ON. Assume that the circuit can supply a maximum current of $I_{L(max)}$. The output voltage remains constant for any value of I_{CL} up to the maximum current $I_{CL(\max)}$. In such normal load conditions, the voltage V_{CL} across the resistor R_{CL} (i.e., $V_{CL} = I_{CL} \times R_{CL}$) is insufficient to turn transistor Q_2 ON. Therefore, Q_1 supplies the current demanded by the load conditions at the fixed output voltage V_L. Now, consider that the load current I_L increases. This leads to more current through R_{CL}, and the voltage drop V_{CL} increases too. This turns the transistor Q_2 ON. Hence, any current, which is in excess of $I_{L(max)}$ is diverted away from the base of Q_1. This effectively reduces the emitter current of Q_1, and thus the load current reduces. Similarly, when the load current reduces, the drop across R_{CL} drops, turning Q_2 OFF and allowing Q_1 to pass I_L.

The curve shown in Figure 5.96(b) shows the output characteristics of series-pass voltage regulator using such a simple current limiting method. The transistor Q_2 supplies an additional small amount of current to the load when the current limiting takes place.

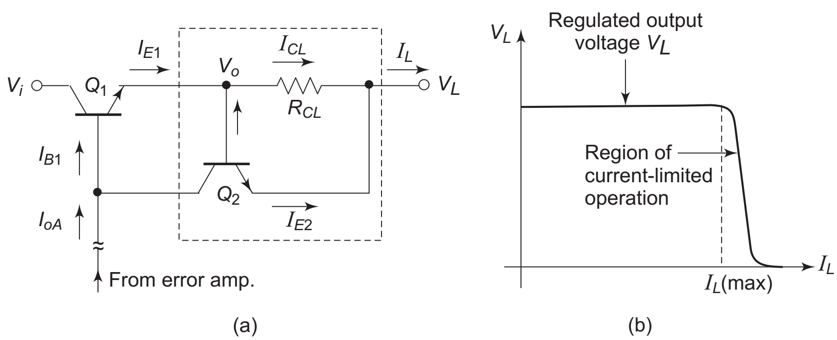


Figure 5.96 (a) Current limiting circuit (b) Its output characteristics

Two Mark Questions and Answers

1. **What is meant by doping in a semiconductor?** **[AU Nov/Dec, 2016]**

   The process of adding impurity to intrinsic semiconductor is known as *doping*. As a result of doping, an extrinsic semiconductor is formed.

2. What is barrier potential? **[AU Nov/Dec, 2016]**

With no applied voltage across the *PN* junction diode, a barrier is set-up across the junction which prevents movement of charge carriers i.e., electrons and holes. This leads to induced electric field across the depletion layer, and an electrostatic potential difference is established between the *P*- and *N*-regions. Such potential is called the barrier potential, junction barrier, diffusion potential, or contact potential, V_o.

3. Sketch the forward bias characteristics of the *PN* junction diode. **[AU April/May, 2015]**

Refer to Figure 5.6.

4. Differentiate between intrinsic and extrinsic semiconductors.
[AU April/May 2013 & 2011]

	Intrinsic Semiconductor	Extrinsic Semiconductor
1	It is pure semi-conducting material and no impurity atoms are added to it.	It is prepared by doping a small quantity of impurity atoms of the pure semi-conducting materials.
2	The number of free electrons in the conduction band and the number of holes in the valence band are exactly equal and very small indeed.	The number of free electrons and holes are never equal. There are excess of electrons in *N*-type semi-conductors and excess of holes in *P*-type semi-conductors.
3	Its electrical conductivity is a function of temperature alone.	Its electrical conductivity depends upon the temperature as well as on the quantity of impurity atoms doped in the structure.
4	Its electrical conductivity is low.	Its electrical conductivity is high
5	*Examples*: Crystalline forms of pure silicon and germanium.	*Examples*: Silicon "Si" and germanium "Ge" crystals with impurity atoms of As, Sb, P etc. or In B, Al, etc.

5. What is the principle of operation of a *PN* junction diode in reverse bias condition?
[AU May/June, 2014]

As shown in Figure 5.7, when reverse bias is applied to the *PN* junction, the holes in the *P*-side more towards the negative terminal of the battery and the electrons are attracted towards the positive terminal of the battery. This increases the depletion width and hence, a very small current of the order of few microamperes flows across the junction.

6. Write any two applications of Zener diode. **[AU May/June, 2014]**

Zener diode is used as

(a) Voltage regulator,

(b) Voltage clipper circuits and

(c) For controlling the output amplitude.

7. What is Zener breakdown? **[AU Nov/Dec, 2011]**

When the *P* and *N* regions are heavily doped, direct rupture of covalent bonds takes place because of the strong electric field at the junction of *PN* diode. The new electron-hole pair, so created, increases the reverse current in a reverse biased *PN* diode. As a result of heavy doping of *P* and *N* regions, the depletion width becomes very small and the field across the depletion region becomes very high and it is due to ruptures of the covalent bond. This breakdown is known as *Zener breakdown*.

8. If a transistor has an α of 0.97, find the value of β. **[AU April/May, 2017]**

Refer to Example 5.3.

9. The common-base DC current gain of a transistor is 0.967. If the emitter current is 10 mA, what is the value of base current? **[AU Nov/Dec 2015; Nov/Dec 2016]**

Given $\alpha = 0.967$ and $I_E = 10$ mA

The common-base DC current gain (α) is

$$\alpha = 0.967 = \frac{I_C}{I_E} = \frac{I_C}{10\times10^{-3}}$$

Therefore, $I_C = 0.967 \times 10 \times 10^{-3} = 9.67$ mA

The emitter current $I_E = I_B + I_C$

i.e., $10 \times 10^{-3} = I_B + 9.67 \times 10^{-3}$

Therefore, $I_B = 0.33$ mA

10. A transistor has $\beta = 150$. Find the collector and base currents, if $I_E = 10$ mA. **[AU April/May, 2016]**

Refer to Example 5.4

11. Calculate the collector and emitter current levels for a BJT with $\alpha_{\text{dc}} = 0.99$ and $I_B = 20$ µA. **[AU Nov/Dec, 2014]**

Solution:

Given $\alpha_{\text{dc}} = 0.99$ and $I_B = 20$ µA.

We know that, $$\alpha_{\text{dc}} = \frac{I_C}{I_E} = \frac{I_C}{I_B + I_C}$$

i.e., $0.99\,(20 \times 10^{-6} + I_C) = I_C$

Hence, $$I_C = \frac{19.8\times10^{-6}}{0.01} = 1.98\times10^{-3} = 1.98\,\text{mA}$$

and $$I_E = I_B + I_C = (0.02+1.98)\times10^{-3} = 2\,\text{mA}$$

12. What is the need for biasing in the transistor? **[AU April/May, 2014]**

To operate the transistor in active mode, it is required to forward bias the emitter-base junction and reverse bias the collector-base junction. By providing proper bias voltage, the transistor can be made to work as an amplifier.

13. Draw input characteristics of CB transistor. **[AU Nov/Dec, 2013]**

Refer to Figure 5.20.

14. Given the biasing arrangement for an NPN transistor to operate in the active region. **[AU April/May, 2013]**

Refer to Figure 5.15.

15. Define α_{dc} and β_{dc} of a transistor. **[AU April/May, 2014]**

The DC current gain in a common base transistor α_{dc} is defined as the ratio of the collector current I_C to the emitter current I_E. It is given by

$$\alpha_{dc} = \frac{I_C}{I_E}$$

The DC current gain in a common emitter transistor β_{dc} is defined as the ratio of the collector current I_C to the base current I_B. It is given by

$$\beta_{dc} = \frac{I_C}{I_B}$$

16. What is thermal runaway in semiconductor devices? **[AU Nov/Dec, 2011]**

When the reverse bias across collector-base junction in BJT is increased, there will be an increase in reverse leakage current. This increase in reverse leakage current will further increase the current flowing through a transistor and thus the power dissipation, causing a further increase in collector-to-emitter leakage current. This process is cumulative and is termed as thermal runaway.

17. What is JFET? Give its different modes of operation. **[AU Nov/Dec, 2016]**

The JFET is an electronic device in which the flow of current through the conducting region is controlled by an electric field. Hence, the name Junction Field Effect Transistor (JFET). As current conduction is only by majority carriers, JFET is said to be a unipolar device. JFET works under depletion mode and enhancement mode.

18. Draw the structure and symbol for an *N*-channel JFET. **[AU April/May, 2014]**

Refer to Figure 5.34 for structure and Figure 5.37 for symbol of an *N*-channel JFET.

19. In which region JFET acts as a resistor and why? **[AU April/May, 2014]**

When a FET acts as a voltage variable resistor? **[AU Nov/Dec, 2012]**

Ohmic region: When gate-source voltage, $V_{GS} = 0$, the depletion layer of the channel is very small and the JFET acts like a voltage controlled resistor since the current increases with increase in voltage.

20. Define pinch-off voltage. **[AU Nov/Dec, 2014]**

In JFET, the electrons flow through a semiconducting channel between source and drain terminals. By applying a reverse bias voltage to a gate terminal, the channel is made free of charge carriers, i.e., pinched off, so that the electric current is impeded or switched off completely.

21. Sketch the graph symbols for *N*-channel and *P*-channel MOSFETs.
[AU April/May, 2011; Nov/Dec, 2014]

Refer to Figure 5.42 (b) and (c) for Enhancement *N*-Channel and *P*-Channel MOSFET and Figure 5.44 (b) and (c) for Depletion MOSFET.

22. Mention the advantages of MOSFET over JFET. **[AU Nov/Dec, 2010]**

(i) JFET can only be operated in the *depletion mode* whereas MOSFET can be operated in either *depletion* or in *enhancement mode*. In a JFET, if the gate is forward biased, excess- carrier injunction occurs and the gate current is substantial. Thus, channel conductance is enhanced to some degree due to excess carriers but the device is never operated with gate forward biased because gate current is undesirable.

(ii) MOSFET has input impedance much higher than that of JFET. This is due to negligibly small leakage current.

(iii) JFET has characteristic curves more flat than those of MOSFET indicating a higher drain resistance.

(iv) When JFET is operated with a reverse bias on the junction, the gate current I_G is larger than it would be in a comparable MOSFET. The current caused by minority carrier extraction across a reverse-biased junction is greater, per unit area, than the leakage current that is supported by the oxide layer in a MOSFET. Thus, MOSFET devices are more useful in electrometer applications than are the JFETs.

23. Differentiate between enhancement-type and depletion-type MOSFETs. [AU April/May, 2012]

(i) Gate formation of *N*-channel E-MOSFET is similar to the *N*-channel D-MOSFET.

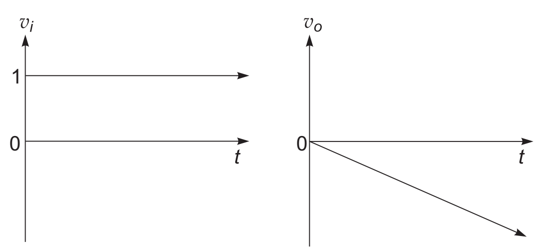
(ii) E-MOSFET can be worked only in enhancement mode and hence, this MOSFET is called enhancement MOSFET or E-MOSFET.

(iii) E-MOSFET has no conducting channel between the source and gate terminals whereas D-MOSFET has conducting channel between the source and gate terminals.

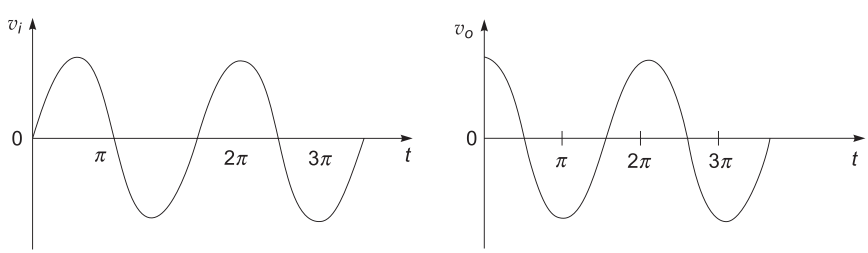
24. What do you understand by an integrator? [AU April/May, 2013]

A circuit in which the output voltage is the time integral of the input voltage is called *integrator* or *integrating amplifier*. Integrator produces a summing action over a required time interval and the circuit is based on the general parallel-inverting voltage feedback model.

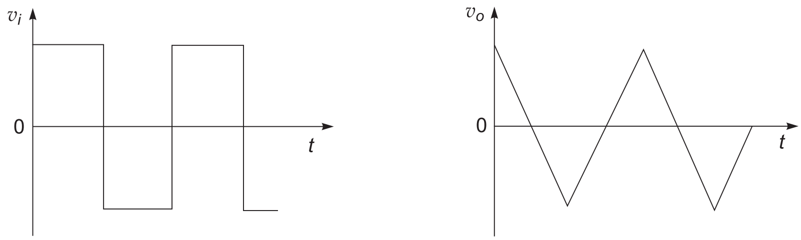
25. Draw the I/O waveforms of the integrator for (a) a step input, (b) a sine wave input and (c) a square wave input. [AU April/May, 2013]



(a) Step input and output



(b) Sinewave input and output



(c) Square wave input and output

26. List the applications of practical integrator. **[AU April/May, 2013]**

The practical integrators can be used in combination with summers and amplifiers to form analog computers and the analog computers are used to model a variety of physical systems in real time. Integrator produces a summing action over a required time interval. Further integrators are used for solving differential equations in analog to digital converters, and ramp generators. The integrator circuits are used as wave-shaping circuits and to convert square waves into triangular waves.

27. Why are integrators preferred over differentiators? **[AU Nov/Dec, 2011]**

The process of integration involves the accumulation of signal over time, and hence sudden changes in the signal are suppressed. Therefore, an effective smoothing of the signal is achieved, and integration can be viewed as *low-pass filtering.* On the other hand, the process of differentiation involves the identification of sudden changes in the input signal. Constant and slowly changing signals are suppressed by a differentiator. Therefore, the differentiator can be viewed as a form of *high-pass filtering* which will unduly enhance the surge noise signals.

28. What is the difference between normal rectifier and precision rectifier? **[AU April/May, 2015]**
How does precision rectifier differ from the conventional rectifier? **[AU Nov/Dec, 2012]**
State the difference between the conventional rectifier and precision rectifier.
[AU Nov/Dec, 2014]

The signal processing applications with very low voltage, current and power levels require rectifier circuits. The ordinary diodes cannot rectify voltages below the *cut-in* voltage of the diode. A circuit which can act as an *ideal diode* or *precision signal-processing rectifier circuit* for rectifying voltages which are further down below the level of *cut-in voltage* of the diode. They can be designed by placing the diode in the feedback loop of an op-amp.

29. For an *n*-bit Flash type A/D converter how many comparators are required? State the disadvantage of that type of converter. **[AU April/May, 2013]**

To convert an analog signal into a digital signal of n output bits, $(2^n - 1)$ number of comparators are required. For example, a 2-bit A/D converter requires 3 or $(2^2 - 1)$ comparators, while a 3-bit converter needs 7 or $(2^3 - 1)$ comparators.

Disadvantages: The simultaneous type A/D converter is not suitable for A/D conversion with more than 3 or 4 digital output bits, since $(2^n - 1)$ comparators are required for an n-bit A/D converter and the number of comparators required doubles for each added bit.

30. Which is the fastest ADC? State the reason. **[AU Nov/Dec, 2011]**

Simultaneous type A/D converter is the fastest because A/D conversion is performed simultaneously through a set of comparators. Hence, it is also called *flash type* A/D converter. Typical conversion time is 100 ns or less.

31. State the two conditions of oscillations. **[AU April/May, 2015]**

For sustained oscillations, the following two condition called Barkhausen criteria are to be satisfied:

1. the magnitude of the loop gain, $Av\beta$ must be unity and
2. the total phase-shift of the loop gain, $Av\beta$, must be equal to 0° or 360°

32. State the applications of 555 timer IC. **[AU Nov/Dec, 2013]**

The application of the IC 555 timer include oscillator, pulse generator, square and ramp wave generator, *one-shot* multivibrator, safety alarm and timer circuits, traffic light controllers etc. The 555 timer can provide time delay, ranging from microseconds to hours.

33. Define Line and Load regulations of a regulator. **[AU Nov/Dec, 2014; Nov/Dec, 2013]**

Refer to section 5.19.1

34. List the advantages of IC Voltage regulators. **[AU April/May, 2013]**

The IC voltage regulators are versatile, relatively inexpensive and are available with features such as programmable output, current / voltage boosting and floating operation for high voltage application.

35. What are the limitations of IC 723 general purpose regulators? **[AU Nov/Dec, 2012]**

The limitation of the regulator IC 723 is that it has no built-in thermal protection and short circuit protection. Therefore, the current limit protection in regulator ICs is necessary for providing protection against short circuit condition across the load.

36. Draw the functional block diagram of IC 723 regulator. **[AU April/May, 2015]**

Refer to Figure 93(a)

37. State the need for current limiting in voltage regulators. **[AU April/May, 2015]**

The limitation of the regulator IC 723 is that it has no built-in thermal protection and short circuit protection. Therefore, the current limit protection in regulator ICs is necessary for providing protection against short circuit condition across the load.

REVIEW QUESTIONS

1. What is meant by intrinsic semiconductor?
2. Explain the differences between intrinsic and extrinsic semiconductors.
3. Explain what a hole is. How do they move in an intrinsic semiconductor?
4. What is meant by doping in a semiconductor?
5. Discuss the following with respect to semiconductor: (i) doping (ii) dopant (iii) donor (iv) acceptor.
6. Explain “majority and minority carriers” in a semiconductor.
7. What is meant by *N*-type impurity in a semiconductor?
8. What is meant by *P*-type impurity in a semiconductor?
9. What is a *PN* junction? How is it formed?
10. Explain the formation of depletion region in a *PN* junction.
11. Draw the energy-band diagram of a *PN* junction and explain the working of a diode.
12. Explain how a barrier potential is developed at the *PN* junction.
13. Describe the action of *PN* junction diode under forward bias and reverse bias.

14. Show that the *PN* diode works as a rectifier.
15. Explain how unidirectional current flow is possible through a *PN* junction diode.
16. Explain *V–I* characteristics of a *PN* junction diode.
17. Explain avalanche breakdown and Zener breakdown.
18. Draw the *V–I* characteristic of Zener diode and explain its operation.
19. Show that the Zener diode can be used as a voltage regulator.
20. Draw the *V–I* characteristic of backward diode and explain its operation.
21. What is a bipolar junction transistor? How are its terminals named?
22. Explain the operations of *NPN* and *PNP* transistors.
23. What are the different configurations of BJT?
24. Explain the input and output characteristics of a transistor in CB configuration.
25. Explain the Early effect and its consequences.
26. Derive the relationship between α and β.
27. Why does the CE configuration provide large current amplification while the CB configuration does not?
28. Draw the circuit diagram of an *NPN* junction transistor CE configuration and describe the static input and output characteristics. Also, define active, saturation and cut-off regions, and saturation resistance of a CE transistor.
29. What is the relation between I_B, I_E and I_C in CB configuration?
30. Explain the laboratory setup for obtaining the CC characteristics.
31. Compare the performance of a transistor in different configurations.
32. Define α, β, and γ of a transistor. Show how they are related to each other.
33. Explain how a transistor is used as an amplifier.
34. From the characteristics of CE configuration, explain the large signal, DC, and small signal CE values of current gain.
35. If I_C is 100 times larger than I_B, find the value of β_{dc}. [*Ans.* 100]
36. Find the value of α_{dc}, if β_{dc} is equal to 100. [*Ans.* 0.99]
37. Find the voltage gain of a transistor amplifier if its output is 5 V rms and the input is 100 mV rms. [*Ans.* 50]
38. Find the value of α_{dc}, when $I_C = 8.2$ mA and $I_E = 8.7$ mA. [*Ans.* 0.943]
39. If α_{dc} is 0.96 and $I_E = 9.35$ mA, determine I_C. [*Ans.* 8.98 mA]
40. Why is a Field Effect Transistor called so?
41. Explain the construction of *N* channel JFET.
42. With the help of neat sketches and characteristic curves, explain the operation of the junction FET.
43. How does the FET behave for small and large values of $|V_{DS}|$?
44. Define the pinch-off voltage V_P. Sketch the depletion region before and after pinch-off.
45. Explain the four distinct regions of the output characteristics of a JFET.
46. Define and explain the parameters transconductance g_m, drain resistance r_d and amplification factor μ of a JFET. Establish the relation between them.
47. Explain how the transconductance of a JFET varies with drain current and gate voltage.
48. What are the relative merits of an *N*-channel and *P*-channel FET?
49. Compare JFET with BJT.
50. Explain why BJTs are called bipolar devices while FETs are called unipolar devices.
51. Explain why a low-power FET is called a square-law device.
52. Briefly describe some applications of JFET.

53. What is a MOSFET? How many types of MOSFETs are there?
54. With the help of suitable diagrams, explain the working of different types of MOSFET.
55. How does the constructional feature of a MOSFET differ from that of a JFET?
56. What are the characteristics of an ideal op-amp?
57. What are the non-ideal characteristics of an op-amp?
58. What are the two conditions necessary for generation of oscillations?
59. Explain the operation of (a) Colpitts oscillator and (b) Hartley oscillator with circuit diagrams.
60. Explain the operation of an RC phase-shift oscillator.
61. Explain the operation of a Wien Bridge oscillator using op-amp with a circuit diagram.
62. Explain the operation modes of integration.
63. What are the limitations of an ideal integrator?
64. For performing differentiation, integrator is preferred to differentiator – Explain.
65. What is the principle of a differentiator using op-amp? What are its drawbacks?
66. Explain the difference between integrator and differentiator. List one application of each.
67. Draw a half-wave rectifier circuit to rectify an ac voltage of $0.2V$. Explain the circuit diagram.
68. Draw the circuit of a full-wave rectifier circuit and explain its operation.
69. What do you mean by data converters?
70. Describe the various specifications of a D/A converter.
71. List the essential parts of a D/A converter.
72. Explain the 4-bit weighted resistor type D/A converter in detail.
73. What are the limitations of weighted resistor type D/A converter?
74. Explain a 4-bit R–2R ladder type D/A converter in detail.
75. Differentiate between current-mode and voltage-mode R–2R ladder D/A converters. Explain.
76. Explain the functional diagram and operating principle of a D/A converter IC.
77. What is the need for A/D converter?
78. What are the different types of A/D converters?
79. Describe various specifications of an A/D converter.
80. Explain a typical simultaneous type A/D converter in detail.
81. What is Flash type A/D converter? Why is it called so?
82. With a neat block diagram, explain successive approximation type A/D converter in detail.
83. Draw the circuit of an astable multivibrator using op-amp and derive the expression for its frequency of oscillations. How will you modify this circuit to have independent control of ON and OFF time durations?
84. Define an astable multivibrator using IC 555.
85. List various applications of 555 timer.
86. Explain the principles of obtaining a regulated power supply.
87. Sketch a variable voltage regulator circuit that uses the IC LM317/LM337 regulators. Explain the circuit operation. Write the expressions for the output voltage from the two regulators.
88. Draw the internal block diagram of IC 723 voltage regulator and explain the function of each block. How can the IC 723 voltage regulator be used to provide output voltages ranging from 2V to 7V?
89. Explain the circuit operation of a high voltage regulator using IC 723 with a circuit diagram. Write the equation for output voltage and state the ranges of output voltage and current that can be obtained using the circuit.

CHAPTER 6

Electrical Measurement and Instrumentation

6.1 Introduction

The process of measuring a quantity is known as measurement and the apparatus used to measure the quantities like voltage, current, power, energy, resistance and so on are called measuring instrument. The quantity to be measured using the measuring instrument is called measurand. The measurement of a measurand is the result of comparison between the unknown quantity to be measured and the standard quantity. The measuring instruments either indicate or record or display or integrate the electrical quantity to be measured using direct or comparison method during a specified period. The measuring instruments operating on many principles are discussed in this chapter. Different types of indicating instruments, its construction, working principles and torque equation are dealt in this chapter. In addition, the applications of indicating instrument like energy meter and wattmeter are explained. Also, the three-phase power measurement and instrument transformer are discussed. Further, this chapter deals with the classification of transducers, construction and operation of electrical, displacement and mechanical transducers.

6.2 Essential Requirements of Measuring Instruments **[AU Nov/Dec, 2009]**

The necessary or essential requirements for any measuring instruments are:

- When the instrument is used in the circuit, its conditions should not be altered and therefore the quantity to be measured goes unaffected.
- It should consume low power.
- It should possess very high efficiency and high sensitivity.
- The output should be linearly proportional to the input.
- It should be less affected by the noise, modifiable and properly priced.

6.3 Elements of The Measuring Instruments **[AU Nov/Dec, 2013]**

The elements present in measuring instruments are shown in Figure 6.1.

6.3.1 Primary Sensing Unit

The first unit in the measurement system, which detects the measurand, is known as the primary sensing unit. It helps in transferring the measurand to a variable-conversion unit for further processing. For example, liquid or mercury in glass thermometer acts as a primary sensing unit. Displacement or voltage is the output of the primary sensing unit.

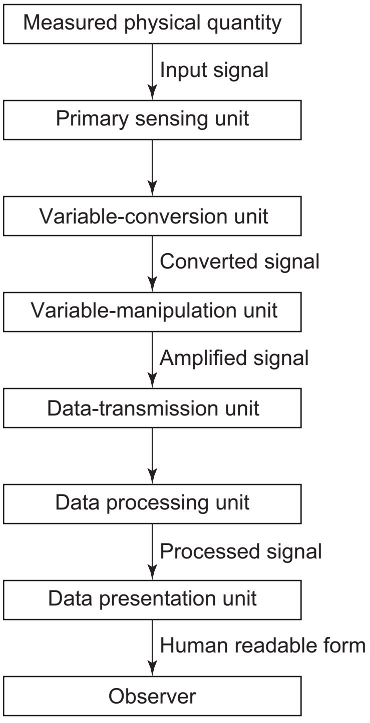


Figure 6.1 Units of the Measuring Instruments

6.3.2 Variable-conversion Unit

The conversion of primary sensing unit output to more suitable variables while preserving the information is achieved with the help of this unit. Hence, it can be called an intermediate transducer. Generally, a variable-conversion unit is not required in most of the measuring instruments. In some measuring instruments, more than one variable-conversion unit is required and in some cases, the primary sensing unit and the variable-conversion unit are combined to form a single unit called a transducer.

6.3.3 Variable-manipulation Unit

It is the intermediate stage of the measuring system, where the numerical value of the signal gets modified i.e., it manipulates the signal presented to this unit without affecting the original nature of the signal. It can be placed either before or after the variable-conversion unit. It helps in improving the output quality of the measurement system by removing the random signals like noise.

6.3.4 Data-Transmission Unit

When the functional units of the measuring system are spatially separated, the data-transmission unit acts as a communication link for transmitting the signals from one unit to another. This unit is mandatory when the system is operated remotely. Some of the common data-transmission units used are cables, wireless antennae, transducers, telemetry systems and so on.

6.3.5 Data Processing Unit

This unit is used to process the data obtained from either the variable-manipulation unit or data-transmission unit and to produce suitable output to be presented to the experimenter. In addition, it is used to compare the measured value with the standard value to produce the required output.

6.3.6 Data Presentation Unit

This unit is used for communicating the measured quantity to the experimenter, which could be used for controlling and analysing purposes.

Let us consider the thermometer to demonstrate each unit in the measuring system.

In the thermometer bulb containing mercury acts as the primary sensing unit and a variable-conversion unit. It senses the temperature, which is the input quantity. With increase in temperature, mercury in the bulb expands and its volume increases. Therefore, the temperature signal is converted into volume displacement.

As the mercury expands, it moves through the capillary tube in the thermometer stem, integrated to the bulb. With the cross-section area of the capillary being constant, the volume signal is converted into linear distance signal. The capillary, thus, has the role of signal manipulation and data transportation units. The final data presentation stage consists of the scale on the thermometer stem, which is calibrated to give the indication of the temperature signal applied to the thermometer bulb. A restriction bend is provided in the clinical thermometers at the junction of the bulb and the capillary, which does not allow the back flow of mercury to the bulb once it has expanded to the capillary. Thus, the restriction in the capillary acts as the data storage function of the instrument.

6.4 Static and Dynamic Characteristics of Instruments **[AU April/May, 2012]**

In a measuring instrument, it is necessary to have a better understanding of all the parameters involved in defining its characteristics. The performance characteristics of the measuring instrument, which decide the overall performance, can be divided into two distinct categories: static and dynamic characteristics. Under the circumstances where the quantities to be measured are either constant or vary very slowly with respect to time, a set of characteristics is defined to give a meaningful description about the measurement quality. These characteristics are called *static characteristics* of measurement system, which are to be considered when the measurement system or instrument is used under a static condition. But in practice, many quantities, to be measured, vary rapidly with respect to time. In such cases, a set of characteristics is to be defined based on the dynamic relationship existing between the input and output. These characteristics, which are normally done using differential equations, constitute the *dynamic characteristics* of the measurement system.

6.4.1 Static Characteristics of Instruments **[AU April/May, 2015]**

The characteristics, which are defined for the instruments measuring constant quantities or slowly varying with respect to time, are called **static characteristics**. These characteristics give a meaningful description about the measurement quality without interfering the dynamic descriptions, which use differential equations. The various static characteristics of instruments are:

- Scale range and scale span
- True value
- Accuracy
- Precision
- Static error and static correction
- Sensitivity
- Linearity
- Scale readability
- Reproducibility and Repeatability
- Resolution
- Threshold
- Drift
- Stability
- Tolerance
- Dead zone and dead time
- Hysteresis
- Noise
- Loading effect

Scale Range and Scale Span

The value to be measured is indicated on a scale using a pointer in an analogue instrument or using digital values in a digital instrument. Each instrument has a maximum and minimum limit within which the instrument is designed to measure, indicate or record a physical quantity. This region between the maximum and minimum limits is called the *scale range* of the instrument, given by its limits. It is the most important factor in the instrument. If the maximum and minimum values that the instrument can measure are $X_{\max}$ units and $X_{\min}$ units and the calibration is continuous between these points, then the instrument scale range is between $X_{\min}$ and $X_{\max}$. *Scale Span* represents the algebraic difference between $X_{\max}$ and $X_{\min}$ of the instrument, which is given by

$$\text{Span} = X_{\max} - X_{\min}$$

True Value

True value of the quantity is defined as the average value of an infinite number of measured values during which the average deviation of various factors tends to zero. But, in practice, it is impossible to realise due to several factors like lags, loading effect, noise and so on.

Accuracy

The uncertainty existing in the measured value is expressed in terms of accuracy, precision and error. Accuracy of a measured value of a quantity is defined as the closeness of the measured value obtained using the instrument used to measure the true value of the same quantity. It depends on the accuracy of the instrument itself, variation of the quantity, which is to be measured, observer accuracy and so on. The accuracy of an instrument may be expressed as: (a) point accuracy (b) percentage of true value or (c) percentage of scale range.

Precision

Precision is a measure of reproducibility of the measurements or a degree of agreement within a measurement group. The two characteristics of the precision term are:

- ***Conformity:*** An instrument reading consistently a particular quantity as 2.4 M instead of the true value 23456789 due to absence of proper scale.
- ***Number of significant Figures:*** It is used to obtain the precision of the instrument in which the reading is expressed and conveys the actual information about the magnitude and measurement precision of the quantity. The mathematical expression for precision is

$$P = 1 - \left| \frac{X_n - X_{an}}{X_{an}} \right|$$

where X_n is the value of n^{th} measurement and X_{an} is the average of measurement set values

Static Error and Static Correction

- ***Static error or absolute static error:*** It is the difference between the measured value and true value of the quantity, as given by

$$E_s = A_m - A_t$$

where A_m is the measured value of the quantity and A_t is the true value of the quantity. Since the static error does not indicate the accuracy of the instrument precisely, relative static error is defined.

The relative static error is defined as the ratio of the absolute static error to the true value of the quantity under measurement and is given by

$$E_r = \frac{E_s}{A_t}$$

- ***Static correction:*** It is the difference between the true value and measured value of the quantity, as given by

$$SC = A_t - A_m$$

Sensitivity

It is defined as the ratio of the change in the output of an instrument to a change in the quantity to be measured. Mathematically it is expressed as,

$$\text{Sensitivity} = \frac{\text{Change in output}}{\text{Change in input}}$$

If the input–output relation of the instrument is linear, then the slope of the curve represents the sensitivity. If the relation is not linear, then the sensitivity varies with respect to the input. Inverse sensitivity or deflection factor is given by the reciprocal of sensitivity.

Linearity

It is defined as the ability of the instrument to reproduce the true input–output characteristics symmetrically and linearly. The percentage of non-linearity existing in the instrument is given by

$$\%\ \text{non}-\text{linearity} = \frac{\text{Maximum deviation of output from true curve}}{\text{Full scale reading}}$$

The representation of linearity of the instrument is represented in Figure 6.2.

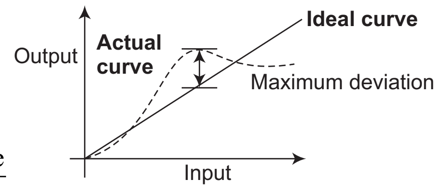


Figure 6.2 Representation of Linearity Between Input and Output

Scale readability

In analogue instruments, the closeness to which the scale can be read is known as scale readability and it depends on factors such as: (a) number of graduations (b) spacing between the graduations (c) size of the pointer and (d) discriminating power of the observer.

Reproducibility and Repeatability

- ***Reproducibility:*** It is the degree of closeness with which a given value may be repeatedly measured using the same instrument, under different conditions like changes in the method of measurement, observer, measuring instrument location, conditions of use and time of measurement. It is specified in terms of scale readings over a given period of time.
- ***Repeatability:*** It is the instrument characteristic which describes the closeness with which a given value is repeatedly measured on the same instrument, at the same location, by the same observer, under the same measurement conditions and when the same input is given to the instrument repetitively over a particular time. It is specified as a variation in scale reading.

Resolution or Discrimination of the Instrument

The smallest change in input which is required to obtain a change in output or smallest measurable input change is known as resolution i.e., when the input is slowly increased from some arbitrary non-zero value, there will be no change in the output till an increment is achieved.

Threshold

The minimum value of input quantity required to change the output reading from zero is known as threshold. It is defined as the minimum value below which there exists no output signal or smallest measurable input.

Drift

It is the measure of deviation in the instrument output for a particular period. If an instrument has no drift, then it has the capability of producing the same reading at different times when there is a variation in the measured variable. The factors which contribute towards the drift are: (i) wear and tear (ii) mechanical vibration (iii) stress developed in the instrument components (iv) variation in temperature (v) stray electric and magnetic fields and (vi) thermal emf.

Stability

It is the measure of capability of the instrument to maintain standard of performance over a prolonged period of time. The instrument will have zero stability if the instrument restores to zero when the input quantity reaches zero, while other conditions remain the same.

Tolerance

The maximum error, which can be expected in the measured value, is known as tolerance.

Dead Zone and Dead Time

- ***Dead zone:*** A region in the input where there is no output is known as dead zone or dead space or neutral zone, which is shown in Figure 6.3. It is also defined as the largest change required in the input variable to make the instrument respond.
- **Dead time:** The time taken by the instrument to respond after the change in input variable has taken place is known as dead time.

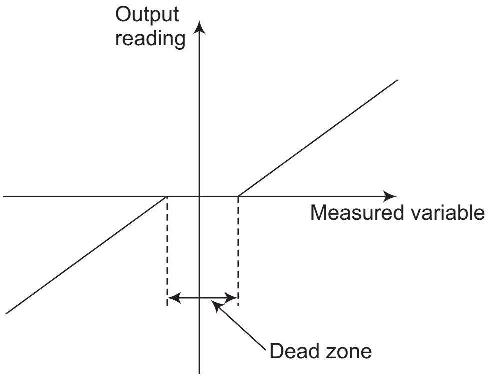


Figure 6.3 Dead Zone

Hysteresis

The phenomenon that occurs in the measuring instrument, which shows different characteristics during loading and unloading, is known as hysteresis, as shown in Figure 6.4. It occurs in the instrument due to mechanical friction, motion in bearing, magnetic and thermal effects.

Noise

Random fluctuation in a signal, which does not convey any information or an error or undesired random disturbance in the useful signal, is known as noise. The common sources of noise are stray electrical and magnetic fields, mechanical shocks and vibrations.

Loading Effect

The loading effect, which occurs due to both electrical and mechanical elements, is the alteration caused in the voltage, current etc. when the measurement is made.

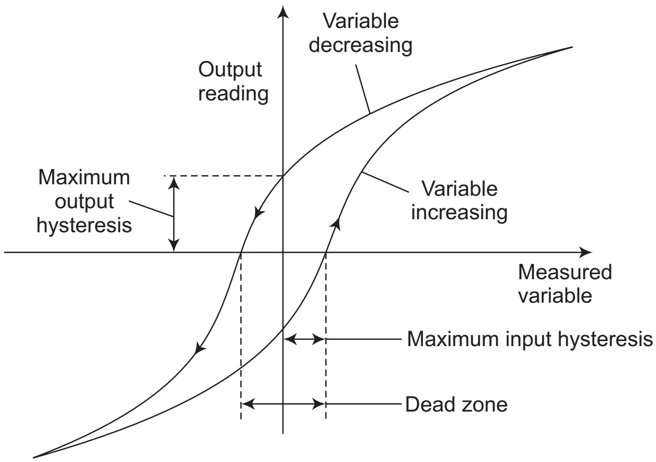


Figure 6.4 Hysteresis

6.4.2 Dynamic Characteristics of Instruments **[AU Nov/Dec, 2010]**

The static characteristics of measuring instruments described in the previous section are for the instruments subjected to non-varying inputs. However, in practice, the input varies from instant to instant and so does the output. The behaviour of the system subjected to varying inputs is known as dynamic response and its characteristics are known as dynamic characteristics. In general, the step, ramp and frequency response of the measuring instrument determine the dynamic characteristics of the measuring instrument. The different dynamic characteristics of measuring instruments are:

- Speed of response
- Response time
- Lag
- Fidelity
- Dynamic error
- Time constant

Speed of Response

It is defined as the rapidity with which an instrument responds to changes in input quantity or the quantity to be measured.

Response Time or Settling Time

It is the time required by the instrument to settle to its final steady state value after the input quantity is applied. For example, in a step-input function, the response time is the time taken by the instrument to settle at a specified percentage of the output, after the application of the input.

Lag

The delay existing in the dynamic response of the instrument when a change in input quantity is applied is known as lag. Though its value is very small, it becomes important for high-speed measurements. Therefore, in the high-speed measurement systems, the time lag should be minimum. The two different types of lag are:

Retardation type: In this type of measuring lag, the output is obtained immediately after a change in measured quantity has occurred.

Time delay: In this type of measuring lag, the output is obtained after a dead zone.

Fidelity

It is the capability of the instrument to reproduce the output in the same form as the input is known as fidelity. It is also defined as the degree to which an instrument indicates a change in the input quantity without any dynamic error.

Dynamic Error

The difference between the true value of the time varying quantity, which is changing with time and the output value indicated by the instrument, if no static error is assumed, is known as dynamic error. Generally, the total dynamic error of the instrument is given by the combination of its fidelity and the time lag between input and output of the system.

Time Constant

It is defined as the time taken to reach 63.2 % of the final output value. If the system has less time constant, it indicates that the final output value will be attained earlier.

6.5 Errors in Measurement **[AU May/June, 2012]**

The error in measurement is defined as the difference between the true or actual value and the measured value. The different sources of error in measurement are as follows:

- Gross error
- Systematic error
- Random error

The classification of errors is shown in Figure 6.5.

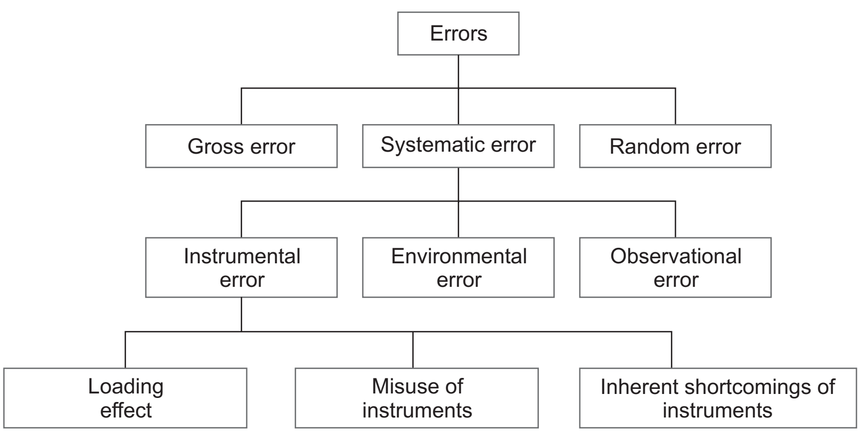


Figure 6.5 Sources of Errors

6.5.1 Gross Error

An error that is caused by an experimenter while reading, recording and calculating the measurement result is known as gross error. For example, the experimenter can possibly read the temperature as 21.5° C while the actual value is 31.5° C or the experimenter can possibly read the value as 34.5° C and instead might record it as 35.4° C. These types of errors are called gross errors. It can be avoided by taking more care, while reading and recording the data and by taking more readings of the quantity under measurement by a different experimenters.

6.5.2 Systematic Error

An error that occurs due to the presence of a fault in the measuring instrument is known as systematic error. Correcting the measurement instrument can rectify this error.

Further, these errors are classified into different categories as:

- Instrumental Error
- Environmental Error
- Observational Error

Instrumental Error

An error that arises due to faulty construction and calibration of the measuring instrument is known as instrumental error. The main reasons behind these instrumental errors are:

- Inherent shortcomings of instrument
- Misuse of instrument
- Loading effect

Inherent Shortcomings of Instrument

These types of errors exist in the instrument inherently due to its mechanical structure. These errors occur due to manufacturing, measurement, calibration or operation of the measuring instruments and causes it to read too low or too high of the measurand. For example, if the spring of a permanent magnet has become weak, that particular device will always read very high.

This type of error can be avoided by carefully planning the measurement procedure, applying correction factors after finding these errors and carefully recalibrating the instrument.

Misuse of Instruments

When a good instrument is operated in an unintelligent manner, it results in these types of errors. Examples of these types of errors are: failing to adjust the zero of instruments, poor initial setting and so on. Though this improper usage of instrument does not lead to a permanent damage, it causes errors in the measurement.

Loading Effect

The most common type of error caused by the instrument due to improper usage is known as loading effect. A well-calibrated voltmeter will give a wrong reading when it is connected across a high-resistance circuit and will give dependable reading when it is connected across the low resistance circuit. Using the meters intelligently can eliminate this error. For example, when measuring a low resistance, using ammeter–voltmeter method, a voltmeter that has a very high resistance should be used.

Environmental Error

An error that occurs due to some external condition of the measuring instrument is known as environmental error. The external conditions that affect the measuring instrument are: temperature, pressure, humidity, dust, vibration due to magnetic or electrostatic field and so on.

These errors can be eliminated or reduced by using the following methods:

- Keeping the conditions like humidity and temperature, as constant as possible using some techniques.
- Using equipment that does not get disturbed by these external conditions.
- More care should be taken to ensure that there exists no external electrostatic or magnetic fields around the measuring instrument.
- Applying computed corrections.

Observational Error

An error caused due to the wrong observation of the reading in the measuring instrument is known as an observational error. There exist different sources of this error and parallax error is the most common error. For example, as the pointer in any instrument resets slightly above the surface of the scale, this error will not occur, unless the vision line of the experimenter is exactly above the pointer. This error can be minimised by using a highly accurate meter provided with mirrored scale.

6.5.3 Random Error

An error caused by sudden change in the experimental conditions, noise, or tiredness of the experimenter is known as random error, and it can be either positive or negative. This error will still remain even after elimination of other types of errors. Hence, such type of an error is also called a residual error and some examples of this type of error are: humidity change, temperature change, voltage fluctuations and so on. Taking the average of a large number of readings can help in reducing this error.

6.6 Classification of Instruments

6.6.1 Basic Classification

The basic classifications of measuring instruments are:

- ***Mechanical instrument:*** It is a reliable instrument to measure under static and stable conditions but does not respond faster to the dynamic and transient conditions.
- ***Electrical instrument*:** In this instrument, electrical quantities are used to indicate the output of detector. It is more rapid when compared to a mechanical instrument but depends on the mechanical meter movement as an indicating device.
- ***Electronic instrument*:** A rapid response can be obtained by using this instrument.

6.6.2 Other Classifications

Other classifications of measuring instruments are:

- ***Absolute or primary instruments:*** The instruments that give the magnitude of the measurement quantity in terms of physical constants of the instrument and deflections are called absolute or primary instruments. Also, these instruments do not require any comparison with the standard instrument. It is generally not used in laboratories and are seldom used by electricians and engineers. The time required to determine the magnitude of the measuring quantity is high. Examples of such instruments are: tangent galvanometer, Rayleigh current balance, absolute electrometer and so on.

- ***Secondary instruments:*** The instruments used to measure the quantity only by the output indicated by the instruments are known as secondary instruments. These instruments are calibrated by comparing with the absolute or primary instrument. It is usually preferred to absolute instruments, as the primary instrument takes less time to compute the output. These instruments are generally used in laboratories. Some of the widely used secondary instruments are: ammeters, voltmeters, wattmeters, energy meters (watt-hour meters), ampere-hour meters and so on.

The secondary instruments are further classified based on:

- **Various effects used to measure electrical quantity**
  - ***Magnetic effect:*** Used in ammeter, voltmeter, wattmeter etc.
  - ***Thermal effect:*** Used in ammeter and voltmeter
  - ***Chemical effect:*** Used in DC ampere-hour meter
  - ***Electrostatic effect:*** Used in voltmeter
  - ***Electromagnetic induction effect:*** Used in AC ammeter, voltmeter, wattmeter etc.
- **Nature of the instrument operation**
  - ***Indicating instrument:*** Indicate the quantity to be measured using a pointer, which moves over a scale. Eg: ammeter, voltmeter and so on
  - ***Recording instruments:*** Record the quantity that is continuously varying with respect to time. It can make a permanent record of the indication.
  - ***Integrating instruments:*** Record the consumption of total quantity of electricity, energy and so on.
  - ***Displaying instruments:*** These instruments measure the electrical quantities in the form of waves on the screen. Eg: Oscilloscope
- **Nature of quantity that can be measured**
  - ***DC instruments:*** Measure only DC quantities
  - ***AC instruments:*** Measure only AC quantities
  - ***Both DC and AC instruments:*** Measure both DC and AC quantities
- **Methods used**
  - ***Direct measuring instruments:*** Convert the energy of the measured quantity directly into a form, which actuates the instrument and the value of the unknown quantity, is indicated or measured or recorded directly.
  - ***Comparison measuring instruments:*** Measure the unknown quantity with the help of comparison with the standard quantity. Example: AC Bridge.

6.7 Principles of Indicating Instruments

An indicating instrument essentially consists of a moving system and a stationary system. A pointer is attached to the moving system, which indicates the electrical quantity to be measured, on a graduated scale. In order to ensure the proper operation of the indicating instruments, the following torques are required:

- Deflecting or operating torque
- Controlling or restoring torque
- Damping torque

6.7.1 Deflecting Torque

The deflecting torque acts on the moving system of the instrument to give the required deflection and indicates the corresponding electrical quantity to be measured on a graduated scale. It exists as long as the instrument is connected to the supply. It is produced by any one of the following effects:

- ***Magnetic effect:*** When a current-carrying conductor is placed in a uniform magnetic field, it experiences a force, which causes the conductor to move. Example: moving-iron attraction and repulsion type, permanent-magnet moving-coil instrument.
- ***Thermal effect:*** When the current to be measured is allowed to flow through a small element, heat gets generated, which causes rise in temperature and it is then converted to an emf. Example: hot-wire instrument, thermocouple instrument.
- ***Electrostatic effect:*** When two charged plates are placed together, a force is exerted between them, which makes any one plate to move.
- ***Induction effect:*** When a non-magnetic conducting disc is placed in a magnetic field produced by an electromagnet, an emf gets induced in it.
- ***Hall effect:*** If a current-carrying bar of semiconducting material is placed in a uniform magnetic field, an emf is produced between the two edges of conductor.

6.7.2 Controlling Torque

The controlling torque is produced by a spring or gravity, which opposes the deflecting torque. The pointer comes to rest at a particular position corresponding to the electrical quantity to be measured, when these two torques are equal. This torque is always present in the instrument whether it is connected to the supply or not. The controlling torque increases with the deflection of the moving system. The controlling torque is also essential to bring back the moving system to its *initial* or *rest* or *zero* position, once the instrument is disconnected from the supply. The control torque can be produced using spring or gravity as explained below:

Spring Control

Two helical springs of rectangular cross-sections are connected to the spindle of the moving system, as shown in Figure 6.6. With the movement of the pointer, the springs get twisted in the opposite direction, which affects the moving system. In spring-controlled instruments, the scale is linear if the deflecting torque is proportional to the quantity being measured.

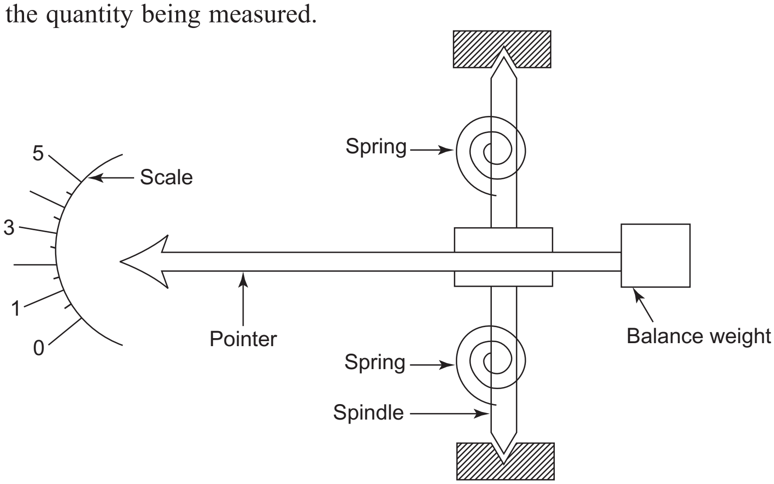


Figure 6.6 Spring Control

Gravity Control

In this method, small weights which can be adjusted are added to the moving system as shown in Figure 6.7. When the pointer deflects, this weight also takes a deflected position.

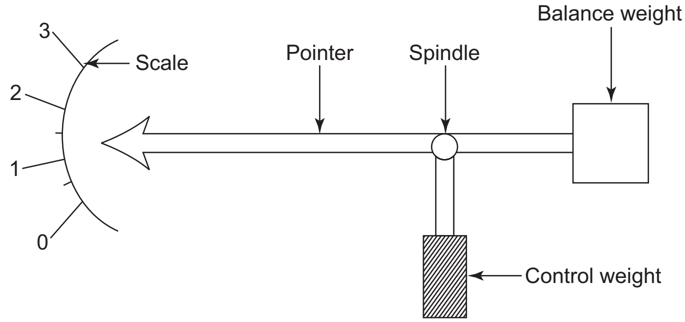


Figure 6.7 Gravity Control

The required controlling torque is produced by the gravitational force, which is acting on the moving weight. The instrument using this method of producing controlling torque has the following disadvantages:

1. Non-uniform scale will be present in the instruments i.e., the scale will be crowded near the minimum limit and uniform near the maximum limit.
2. The instrument can be used only in the vertical position.

6.7.3 Damping Torque

The torque that is used to reduce the oscillations of the pointer and to bring it to the final deflected position is known as damping torque. It acts on the pointer only when the instrument is in operation. If sufficient damping torque is not produced, the pointer makes under-damped oscillations before reaching the steady deflection. If the damping torque is more than the required value, the pointer becomes sluggish and it takes longer than the required time to reach the final deflection, as shown in Figure 6.8. Critical damping or dead beat is the condition where the magnitude of damping torque is sufficient enough to make the pointer to read the correct reading without passing or oscillating about it.

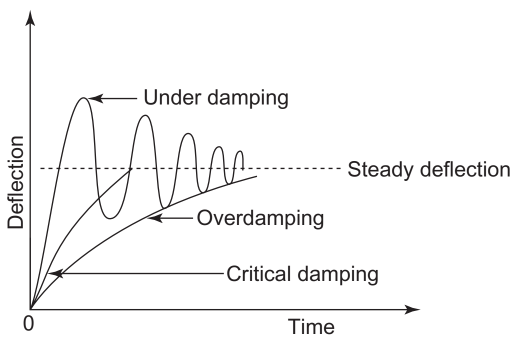


Figure 6.8 Characteristics of Damping Torque

The required damping torque can be produced by the following methods:

Air-Friction Damping

The different methods of producing damping torque using air are shown in Figures 6.9 (a) and (b). Figure 6.9 (a) shows the arrangement where a piston, attached to the spindle of the moving system, moves inside the air chamber provided with a very small clearance between the piston and the chamber. When the deflecting torque acts on the moving system, the suction and compression actions on the air inside the air chamber produce the necessary damping torque.

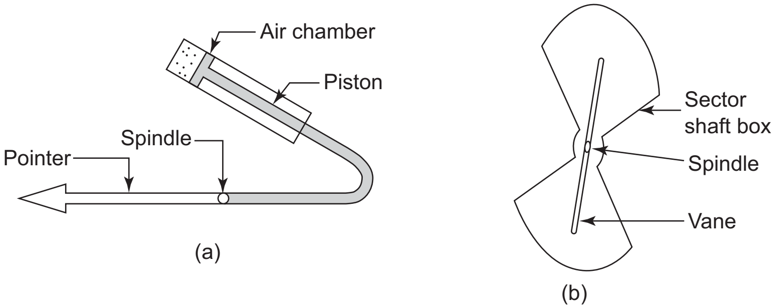


Figure 6.9 Air-Friction Damping

Another arrangement is shown in Figure 6.9(b). There is a sector-shaped box containing air. This box moves a pair of vanes attached to the spindle of the instrument. The movement of the vanes in the air produces the required damping torque. Both the box and the vanes are made of aluminium.

Eddy Current Damping

A thin disc of a conducting but non-magnetic material (like copper or aluminium) is mounted on the spindle, which carries the moving system and the pointer, as shown in Figure 6.10. The position of the disc is such that while in motion, it cuts the magnetic flux produced by the permanent magnet and eddy currents are produced in the disc. These currents flow in such a direction that the motion of the disc is opposed. Thus, the required damping torque is produced.

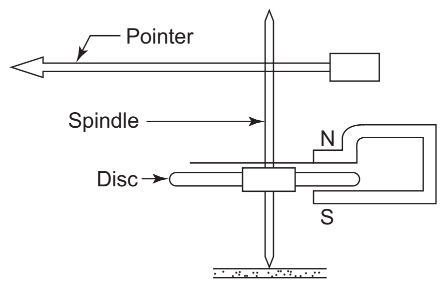


Figure 6.10 Eddy Current Damping

6.8 Types of Indicating Instruments

The indicating instruments are of different types, as listed below:
- Moving-iron instruments
- Moving-coil instruments

6.9 Moving-iron Instruments

Moving-iron instruments are generally used to measure the flow of alternating voltage and current with the help of moving iron. When compared to other AC instruments, the moving-iron instruments are precise, low-priced and rugged. The basic principle of the moving-iron instrument is that when an iron piece is brought near the magnet, it gets attracted towards the magnet. This movement of the soft-iron piece is used to measure the current or voltage, which produces the magnetic field. The strength of magnetic field, which depends on the magnitude of current passing through the magnet, decides the force of attraction of the iron piece. The two different types of moving-iron instruments are:
- Moving-iron instrument–attraction type
- Moving-iron instrument–repulsion type

6.9.1 Construction of a Moving-iron Instrument

The different components existing in the moving-iron instruments are briefly described below:

- ***Moving element:*** A soft-iron piece, in the form of a vane or a rod or a plate, is placed in such a way that it gets moved freely in the magnetic field.
- ***Stationary coil:*** Used to generate the magnetic field when it is excited by the voltage or current flowing through the coil, whose magnitude is to be measured. Also, it is used to magnetise the moving element. The strength of the magnetic field increases or decreases based on the current flowing through it.
- ***Spring or weight:*** It is used to provide the control torque.
- ***Damping device:*** It consists of an air chamber and a moving vane attached to the instrument spindle and is used to generate the damping torque, which is normally pneumatic.
- ***Aluminium pointer:*** It is used to indicate the movement produced by the deflecting torque over a graduated scale.

In addition to the above components, in repulsion type, there exists another soft-iron piece in the form of a vane or a rod or a plate, which is fixed and magnetised with the same polarity of the moving element.

6.9.2 Torque Equation in Moving-iron Instrument

At any instant of time, let I be the current flowing through the coil, which has a self-inductance L and produces a deflection θ in the needle. The deflection θ, can also be known as the angular position of the soft-iron piece. Therefore, the initial energy stored in the coil in the form of magnetic field is given by

$$E_i = \frac{1}{2}LI^2 \tag{6.1}$$

Let $d\theta$ be the increment in the deflection indicated in the instrument using the deflecting torque, T_d, when a small increment in current, dI is supplied to the coil. Therefore, the mechanical work done due to such deflection is given by

$$W_m = T_d \times d\theta \tag{6.2}$$

Also, due to this change in current dI, there will be a change in inductance dL. Therefore, the final energy stored in the coil is given by

$$E_f = \frac{1}{2}(L + dL)(I + dI)^2 \tag{6.3}$$

Therefore, the change in energy stored in the coil is given by

$$dE = E_f - E_i$$

Substituting Eqn. (6.3) and Eqn. (6.1) in the above equation, we get

$$dE = \frac{1}{2}(L + dL)(I + dI)^2 - \frac{1}{2}LI^2$$

Expanding the above equation and eliminating the higher order terms, we get

$$dE = LIdI + \frac{1}{2}I^2 dL \tag{6.4}$$

In addition, the emf induced in the coil also increases due to this change in current, as given by

$$e = \frac{d(LI)}{dt} = I\frac{dL}{dt} + L\frac{dI}{dt} \tag{6.5}$$

Therefore, the electrical energy supplied by the source is given by

$$E_s = eIdt$$

Substituting Eqn. (6.5) in the above equation, we get

$$E_s = I^2 dL + LIdI \tag{6.6}$$

According to the law of conservation of energy, the electrical energy supplied by the source is converted into stored energy in the coil and the mechanical work done for deflection of needle in the instruments. Therefore,

$$E_s = dE + W_m$$

Substituting Eqn. (6.4) and Eqn. (6.2) in the above equation and solving, we get

$$T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \tag{6.7}$$

From the above equation, it is clear that the deflecting torque depends on the rate of change of inductance with the angular position of the soft-iron piece and the square of the RMS current flowing through the coil. The controlling torque, T_C, provided by the spring arrangement in the instrument is given by

$$T_C = k_s \theta \tag{6.8}$$

where k_s is the spring constant.
In equilibrium state, the deflecting and controlling torques are equal, as given by

$$T_d = T_C$$

Substituting Eqn. (6.7) and Eqn. (6.8) in the above equation, we get the deflection of the needle or the angular position of the soft-iron piece as

$$\theta = \frac{1}{2k_s} I^2 \frac{dL}{d\theta}$$

From the above equation, it is clear that the deflection is proportional to the square of the current flowing through the coil and is independent of the direction of current.

6.9.3 Moving-Iron Instrument-Attraction Type

The basic working principle of attraction type of moving-iron instrument is that, when a soft-iron piece is brought near to the magnet, the magnet attracts it. The schematic diagram of the attraction type moving-iron instrument is shown in Figure 6.11.

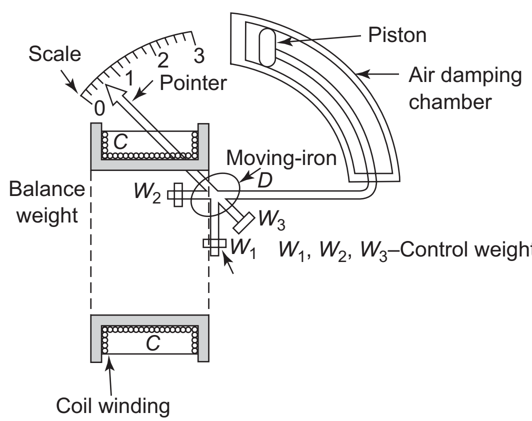


Figure 6.11 Moving-Iron Instrument-Attraction Type

Construction

This type of instrument consists of a fixed coil C, which is flat with a narrow slot-like opening and a moving-iron piece D, which is a flat disc mounted on the spindle supported by the jewel bearings. The pointer used to indicate the alternating current or voltage moves over a graduated scale and is fixed with the spindle. The range of the alternating current or voltage measured by the instrument is directly proportional to the number of turns in the fixed coil. Springs are used to provide the controlling torque. But in a vertically-mounted instrument, the gravity control can be used to provide the controlling torque. Air friction, with the help of a light aluminium piston that fixed to the moving system and moves in a fixed chamber, is used to provide the damping torque.

Working Principle

When the measuring instrument is connected to the circuit, the current starts flowing through the coil and generates a magnetic field. Now, the coil behaves like a magnet, thereby attracting the soft-iron piece towards the centre of the coil, where the flux density is maximum. As a result, the spindle and the pointer attached to the spindle move from their initial positions and give a proportional deflection due to deflecting torque. If the current flowing through the coil is reversed, then the direction of the magnetic field, and hence the polarity formed in the soft-iron piece, get reversed. Hence, there will be no change in the direction of deflecting torque. Therefore, this instrument can be used to measure both DC and AC quantities.

6.9.4 Moving-Iron Instrument-Repulsion Type

The basic working principle of a repulsion type moving-iron instrument is that, when two soft-iron pieces are magnetised to the same polarity, a force of repulsion exists between them, which cause the movement. In this instrument, there are two pieces of soft-iron inside the coil: one is fixed and the other is movable. If one of the two soft-iron pieces is made to move, the existing repulsive force makes the other soft-iron piece to move. This movement is used to measure the current or voltage, which produces the magnetic field. The two different designs of repulsion type instruments are:

- Radial vane type
- Co-axial or concentric vane type

Radial-vane Type

The schematic diagram of a radial vane type repulsive moving-iron instrument is shown in Figure 6.12. When compared to other moving-iron instruments, this is more sensitive and has a linear scale.

Construction

In this type, the radial strips of soft-iron piece are used and are placed within the coil. The fixed soft-iron piece is attached to the coil and the movable one is attached to the spindle of the instrument. Using the deflecting torque, the pointer attached to the moving-iron moves over the scale. The controlling torque is produced by spring mechanism, and the air-friction damping provides the damping torque.

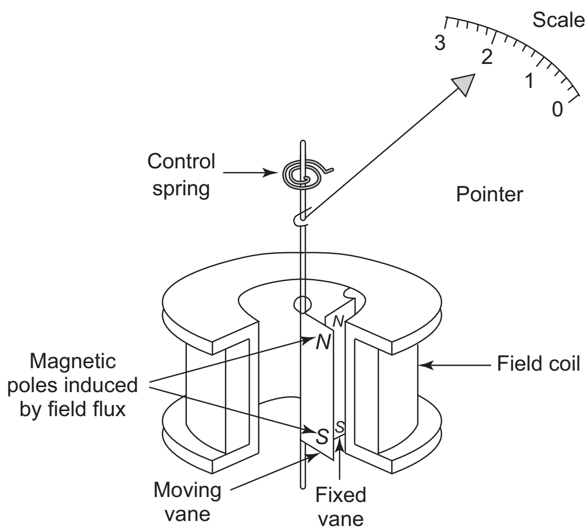


Figure 6.12 Radial Vane Type of Moving-Iron Instrument

Working Principle

The magnetic field, which magnetises both the soft-iron pieces, is produced when the current starts flowing through the operating coil. Hence, a repulsive force exists between these two soft-iron pieces. This repulsive force, when acting on the moving iron, pushes away from its initial position. Thus, the spindle attached to the moving-iron moves and hence the pointer gives a proportional deflection. Now, even when the alternating current flows through the coil, a repulsive force always exists between the two soft-iron pieces. Therefore, the deflection of the pointer is always in the same direction and is directly proportional to the actual current. Hence, this instrument can be used to measure both AC and DC quantities.

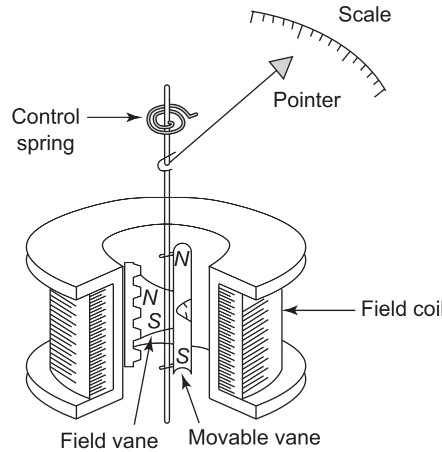


Figure 6.13 Fixed Coaxial Vane Type of Moving-Iron Instrument

Coaxial or Concentric Vane Type

The schematic diagram of a concentric vane type repulsive moving-iron instrument is shown in Figure 6.13. In this type of instrument, the fixed and moving vanes are sections of co-axial cylinders, as shown in Figure 6.13. This instrument is moderately sensitive and has low square response. Thus, the scale of the instrument is non-uniform in nature.

Construction

The instrument has two concentric vanes or soft-irons. One is fixed to the coil frame rigidly while the other rotates coaxially inside the fixed vane. The shaft, which holds the pointer, is attached to the coaxial moving vane. The controlling torque is provided by the spring or gravity arrangement, and pneumatic or air-damping arrangement provides the damping torque.

Working Principle

The soft-iron pieces or vanes are magnetised to the same polarity by the magnetic field produced due to the current flowing through the coil. A repulsive force, which exists between the two vanes, causes the movable system to rotate. The movable vane attached to the pivoted shaft causes the rotation of shaft. The pointer attached to the shaft shows deflection that is proportional to the current flowing through the coil. Similar to the other moving-iron instruments, whatever may be the direction of the current in the coil, the deflection in this instrument will be in the same direction.

6.9.5 Advantages, Disadvantages and Applications of Moving-iron Instrument

Advantages

1. As the instrument is independent of the direction of current, it can be used for both DC and AC circuits.
2. Since the components present in the instrument are simple and require less number of turns, it is cheap.
3. It is robust in nature because of its simple construction.
4. High operating-torque is available in this instrument.
5. Reasonably accurate measurements are possible with these instruments.
6. It can withstand overload momentarily.
7. It can be used in low frequency and high-power circuits.
8. Frictional error existing in the instrument is very less, as it has high torque-to-weight ratio.
9. Possible to extend the range of the instrument.

Disadvantages

1. Because of pneumatic damping, the scale of the instrument is not uniform, which results in less accurate readings.

2. The instrument is not very sensitive.
3. Errors exist due to hysteresis, frequency and stray magnetic field.
4. There exists difference in calibrating AC and DC instruments.
5. High power consumption
6. Increase in temperature increases the resistance of coil and decreases the stiffness of the spring, and permeability of soft-iron, which affects the reading.

Applications

1. Heavy-current moving-iron ammeter
2. Moving-iron voltmeter
3. Moving-iron power factor meter
4. Moving-iron synchroscope

6.10 Moving-Coil Instruments

Moving-coil instruments are mainly used in measuring DC quantities. When fed through appropriate rectifiers, this instrument can be used in measuring AC quantities. The basic principle of moving-coil instruments is that, when a current-carrying coil is placed in a magnetic field, a force or torque is exerted on it, which moves the coil away from the magnetic field. This movement of the coil helps in measuring current or voltage. The different types of moving-coil instruments are:

- Permanent magnet moving-coil instrument (PMMC): used for DC
- Dynamometer type: used for both AC and DC

6.10.1 Permanent Magnet Moving-Coil Instrument (PMMC)

If the permanent magnet is used in the instrument for creating the stationary magnetic field in which current-carrying coil moves, it is known as the permanent magnet moving-coil (PMMC) instrument. The PMMC instrument, which gives accurate results while measuring DC quantities, works on the principle that the torque is exerted on the moving-coil placed in the magnetic field generated using permanent magnet.

Construction

The different components existing in a PMMC instrument are:

- Moving-Coil
- Magnet System
- Control Spring
- Damping
- Pointer and Scale

The schematic diagram of a PMMC instrument is shown in Figure 6.14.

- ***Moving-coil:*** It is the current-carrying part in a PMMC instrument, which moves in the magnetic field produced

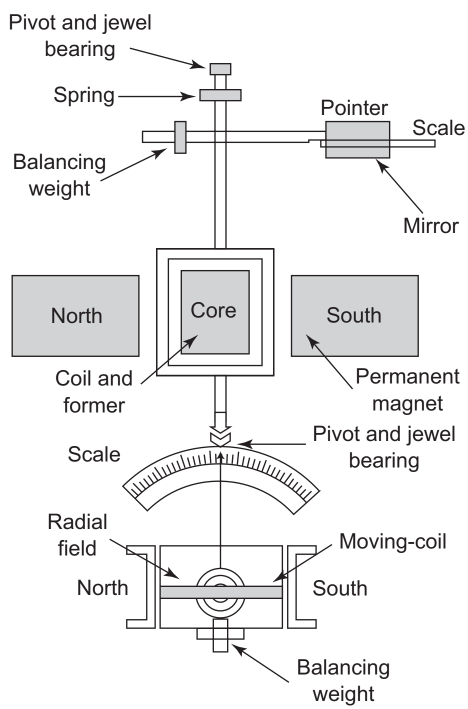


Figure 6.14 PMMC Instrument

by the permanent magnet. It is made up of many turns of copper and is mounted on an aluminium rectangular former placed between the poles of the magnet. The aluminium rectangular former, which is pivoted on jewel-bearing, is used to increase the magnetic field between the air gap of the poles. The current flowing through the coil deflects it and this deflection is used to measure the magnitude of the current or voltage. When PMMC is used as a voltmeter, the moving-coil is wound on the metallic frame, which provides the required electromagnetic damping and when PMMC is used as an ammeter, the moving-coil is wound on non-magnetic former, as the turns of the coil are effectively shorted using an ammeter shunt. Thus, the moving-coil in a PMMC instrument provides the electromagnetic damping.

- ***Magnet System:*** In a PMMC instrument, a simple U-shaped permanent magnet, made up of Alcomax or Alnico, having high coercive force and high field intensities, helps in generating the magnetic field. The magnetic field is made radial, uniform and boring a soft-iron cylinder between the poles of the permanent magnet increases its strength.
- ***Controlling Torque:*** The two-control phosphorous-bronze springs mounted on the jewel-bearing are used to provide the controlling torque in a PMMC instrument. It also helps in providing the path for the current to flow in and out of the moving-coil.
- ***Damping Torque:*** The movement of aluminium former in the permanent magnet field provides the required damping torque, which is used for keeping the coil movement in rest. This damping torque is induced due to the development of eddy current in the former due to its movement.
- ***Pointer & Scale:*** The lightweight pointer, which gets easily deflected, is carried by the spindle, moves through a graduated scale and is linked with the moving-coil. The pointer notices the deflection of the coil and its magnitude is shown on the scale. The lightweight material used in the pointer is to avoid parallax error.

Working Principle

When the current starts flowing through the moving-coil, a magnetic field gets generated, which is proportional to the current. Based on the electromagnetic action between the current-carrying coil and the permanent magnetic field, a deflecting torque is developed. When the controlling torque provided by the two springs matches with the deflecting torque or at balanced condition, the moving-coil gets stopped. The pointer attached to the moving-coil measures the amount of electrical quantity passing through the coil, by determining the angular displacement of the coil against a fixed reference, called a scale. The damping torque prevents further oscillation of the coil i.e., after the balanced condition.

6.10.2 Torque in PMMC Instrument

The deflecting torque equation for a PMMC instrument is given by

$$T_d = NBLId = GI \tag{6.9}$$

where N is the number of turns in the moving-coil, B is the magnetic flux density existing between the permanent magnet poles, L is the length, d is the breadth of the moving-coil, I is the current flowing through the coil and $G = NBLd$ is the constant.

The magnitude of the controlling torque provided by the spring is given by

$$T_C = k_s\theta \tag{6.10}$$

where k_s is the spring constant and θ is the angular movement made by the moving-coil in radians.

At steady-state condition, deflecting and controlling torques shall be equal as given by

$$T_d = T_C$$

Substituting Eqn. (6.9) and Eqn. (6.10) in the above equation, we get

$$GI = k_s \theta$$

Therefore, the angular displacement made by the moving-coil is given by

$$\theta = \frac{G}{k_s} I \tag{6.11}$$

It is clear from the above equation that, the deflection in a PMMC instrument is directly proportional to the current flowing in the moving-coil, due to which the meter scale used for the measurement of current/voltage is linear in this instrument.

6.11 Electro Dynamometer-Type

An electrodynamometer-type instrument is used for the measurement of AC and DC quantities, unlike a PMMC instrument, which can only be used for the measurement of DC quantities. The electrodynamometer-type instrument is a transfer instrument, which is calibrated with a DC source; and without using any modifications, the same instrument can be used for AC measurements with same accuracy as that of DC measurements. This type of instrument is often used in AC voltmeters and ammeters with high accuracy and with small modifications, it can be used as wattmeter for measuring the power.

The electrodynamometer-type instrument, which is similar to a PMMC type instrument except for the permanent magnet used in PMMC-type instrument, is replaced with another fixed coil that generates the necessary magnetic field. The reason for which the PMMC cannot be used for AC quantities is used as a working principle in electrodynamometer-type instrument. In order to read the AC quantities using a moving-coil instrument, the magnetic field existing in the instrument must change along with the change in AC quantities, which is not possible in a PMMC instrument.

6.11.1 Construction

The schematic diagram of an electrodynamometer-type instrument is shown in Figure 6.15.

The various parts of the electrodynamometer-type instrument are:

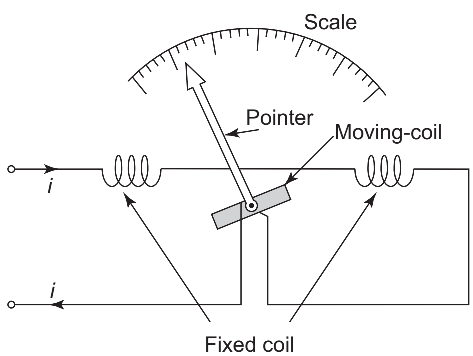


Figure 6.15 Electrodynamometer-type Instrument

Fixed Coil

This air core varnished coil is used to generate the necessary magnetic field required for the operation of the instrument and a uniform magnetic field is obtained near the centre of the coil, as the fixed coil is divided into two sections. If the instrument is used as an ammeter or a voltmeter, the fixed coils are wound with thin wire and heavy wire respectively. They are clamped in place against the coil support, which is made up of ceramic.

Moving-Coil

The moving air-cored light and rigid coil is wound as a self-sustaining coil or on a non-metallic former.

Controlling Torque

The controlling torque by the spring acts as leads to the moving-coil.

Moving System

The moving-coil is mounted on an aluminium spindle, which carries the counter weights and pointer. In some cases, a suspension is used if high accuracy is desired.

Damping Torque

A pair of aluminium vanes, which are attached to the spindle at the bottom, provides the damping torque using air friction.

Shielding

To prevent the effect of earth's magnetic field on the reading, the instrument is shielded by enclosing it in a casing made with a high-permeability alloy.

Cases and Scales

These instruments used in laboratory are usually contained in a rigid, polished wooden or metal case and are supported by adjustable levelling screws. A proper levelling of these components can be provided by the spirit level.

6.11.2 Working Principle

When the electrodynamometer instrument is used as an ammeter, both the fixed and moving-coils are connected in series to carry the same current. To limit the current that is flowing through these coils, a suitable shunt-resistance is connected to these coils. But, when the electrodynamometer instrument is used as a voltmeter, the fixed and moving-coils are connected in series with high non-inductive resistance. When it is used to measure the power, the fixed coil and moving-coil act as the current and voltage coils, connected in series with the load and across the supply terminals respectively.

When the current starts flowing through both the coils, a magnetic field is produced. The magnetic field produced by the fixed coil is proportional to the load current and the magnetic field produced by the moving-coil is proportional to the voltage. Now, the deflecting torque is produced due to the interaction of these two fields, and the deflection indicated by the pointer is proportional to the power supplied to the load. The connections of an electrodynamometer instrument as an ammeter, a voltmeter and a wattmeter are shown in Figures 6.16 (a), (b) and (c) respectively.

6.11.3 Torque Equation

Let i_1 and i_2 be the instantaneous currents flowing through the fixed and moving-coils respectively; L_1 and L_2 be the self-inductances of the fixed and moving-coils respectively and M be the mutual inductance existing between the fixed and moving-coils. The equivalent circuit of an electrodynamometer instrument is shown in Figure 6.17.

The flux linkages of coil 1 and coil 2 are given by

$$\phi_1 = L_1 i_1 + M i_2 \text{ and } \phi_2 = L_2 i_2 + M i_1 \tag{6.12}$$

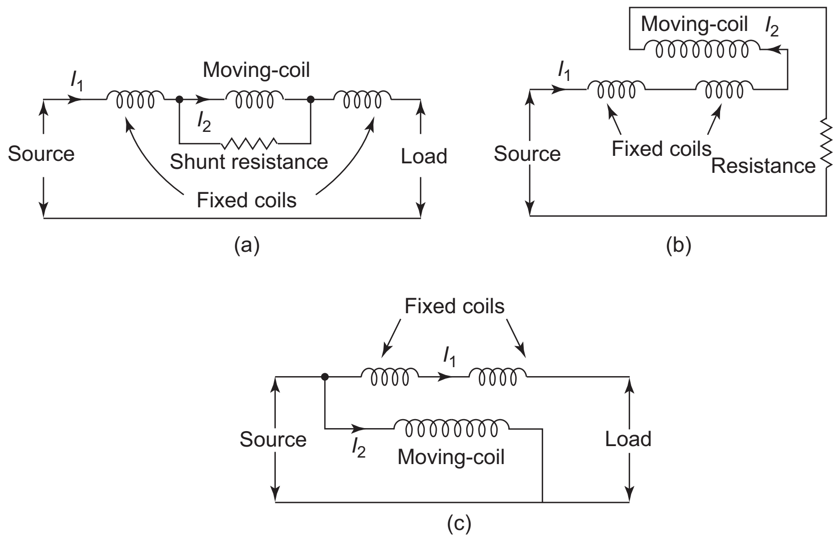


Figure 6.16 Electrodynamometer as (a) Ammeter (b) Voltmeter and (c) Wattmeter

Now, the induced emfs in the fixed and moving-coils are given by

$$e_1 = \frac{d\phi_1}{dt} \text{ and } e_2 = \frac{d\phi_2}{dt} \tag{6.13}$$

The electrical input energy is given by

$$e_i = e_1 i_1 dt + e_2 i_2 dt$$

Using Eqn. (6.13) in the above equation, we get

$$e_i = i_1 d\phi_1 + i_2 d\phi_2$$

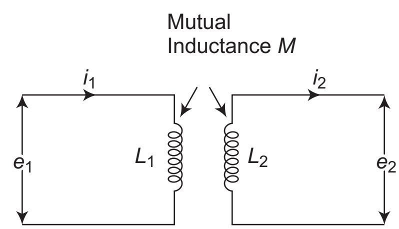


Figure 6.17 Equivalent Circuit of an Electrodynamometer Instrument

Substituting Eqn. (6.12) in the above equation, we get

$$e_i = i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1) \tag{6.14}$$

$$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1$$

The energy stored in the magnetic field due to L_1, L_2 and M is given by

$$e_s = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M$$

The change in stored energy is given by

$$de_s = d\left[\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + i_1 i_2 M\right]$$

$$= i_1 L_1 di_1 + \frac{1}{2} i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \tag{6.15}$$

According to the principle of conservation of energy, we get

Input energy = Energy stored + Mechanical energy

Therefore, Mechanical energy = Input energy – Energy stored
Substituting Eqn. (6.14) and Eqn. (6.15) in the above equation, we get

$$\text{Mechanical energy} = \frac{1}{2}i_1^2 dL_1 + \frac{1}{2}i_2^2 dL_2 + i_1 i_2 dM$$

Since the self-inductances L_1 and L_2 are constant, $dL_1 = dL_2 = 0$. Therefore, the mechanical energy = $i_1 i_2 dM$.

If T_d is the instantaneous deflecting torque and $d\theta$ is the change in deflection, then the mechanical work done is given by $T_d d\theta$. It is known that the mechanical energy is equal to the mechanical work done, and is given by

$$i_1 i_2 dM = T_d d\theta$$

Therefore, the instantaneous deflecting torque of the instrument becomes

$$T_d = i_1 i_2 \frac{dM}{d\theta}$$

DC Operation

In DC operation, $i_1 = I_1$ and $i_2 = I_2$. Therefore, the deflecting torque is given by

$$T_d = I_1 I_2 \frac{dM}{d\theta}$$

If the controlling torque, $T_C = k_s\theta$ is provided by the springs, then at the balanced condition, we get

$$T_d = T_C$$

Therefore, $k_s\theta = I_1 I_2 \frac{dM}{d\theta}$. Hence, the deflection of the pointer or the moving-coil is

$$\theta = \frac{1}{k_s} I_1 I_2 \frac{dM}{d\theta}$$

From the above equation, it is clear that in DC operation, the deflection of the moving-coil is proportional to the fixed and moving-coil currents and the rate of change of mutual inductance.

AC Operation

In AC operation, the total deflecting torque over a cycle is obtained as

$$T_d = \frac{1}{T}\int_0^T i_1 i_2 \frac{dM}{d\theta} dt \tag{6.16}$$

where T is the time period of AC quantity.

If the fixed and moving-coil currents are sinusoidal and is displaced by a phase angle, then

$$i_1 = I_{m1} \sin \omega t \text{ and } i_2 = I_{m2} \sin(\omega t - \phi) \tag{6.17}$$

where I_{m1} and I_{m2} are the maximum values of respective currents.

Substituting Eqn. (6.16) in Eqn. (6.17), we get

$$T_d = \frac{1}{T}\frac{dM}{d\theta}\int_0^T I_{m1} I_{m2} \sin(\omega t)\sin(\omega t - \phi)\, d\omega t$$

Solving the above equation, we get

$$T_d = \frac{I_{m1} I_{m2}}{2}\frac{dM}{d\theta}\cos\phi$$

Let I_1 and I_2 be the RMS values of the currents and they are given by $I_1 = \frac{I_{m1}}{\sqrt{2}}$ and $I_2 = \frac{I_{m1}}{\sqrt{2}}$. Substituting this in the above equation, we get

$$T_d = I_1 I_2 \frac{dM}{d\theta}\cos\phi$$

If the controlling torque, $T_C = k_s\theta$ is provided by the springs, then at the balanced condition, we get

$$T_d = T_C$$

Therefore, $k_s\theta = I_1 I_2 \frac{dM}{d\theta}\cos\phi$. Hence, the deflection of the pointer or the moving-coil is

$$\theta = \frac{1}{k_s} I_1 I_2 \cos\phi \frac{dM}{d\theta}$$

From the above equation, it is clear that in AC operation, the deflection of the moving-coil is proportional to the fixed and moving-coil currents, cosine of the phase angle and the rate of change of mutual inductance.

6.11.4 Advantages, Disadvantages and Applications of Electrodynamometer Instrument

Advantages

1. Free from hysteresis and eddy current losses as it has air-cored coils.
2. Has precision-grade security.
3. Can be used on both AC and DC systems.
4. Free from hysteresis errors.
5. Low power-consumption.
6. Light in weight.

Disadvantages

1. Has frictional errors, which reduce sensitivity of the instrument.
2. Non-uniformity exists in the scale.
3. Requirement of screening process to avoid the stray-field effect.
4. More expensive when compared to PMMC or MI type instruments.
5. Low torque-to-weight ratio.
6. Sensitive to overloads and mechanical impacts.

Applications

The electrodynamometer instrument can be used as:

1. Ammeter: Same current will pass through the coils.
2. Voltmeter: Coils are connected in series with high resistance.
3. Wattmeter

6.11.5 Comparison between Moving-Coil and Moving-Iron Instruments

Table 6.1 lists the comparison between the moving-coil and moving-iron instruments.

Table 6.1 *Comparison between the moving-coil and moving-iron instruments*

Moving-Coil Instrument	Moving-Iron Instrument
More accurate	Less accurate
Costly	Cheap
Uniformly distributed reading scale	Absence of uniformity in the scale
Very sensitive	Robust in construction
Power consumption is low	Power consumption is high
Uses eddy current damping	Uses air-friction damping
Can be used only for DC	Can be used on AC and DC
Spring arrangement provides the controlling torque	Gravity or spring arrangement provides the controlling torque
Deflection is directly proportional to current, $\theta \alpha I$	Deflection is directly proportional to square of the current, $\theta \alpha I^2$
Errors are set due to ageing of control springs, permanent magnet	Errors are set due to hysteresis and stray fields

6.12 Energy Meter

An instrument that helps in measuring the electrical energy is called energy meter or watt-hour meter. Motor meter and clock meter are the most basic types of energy meter. The motor meter is a single-phase induction type energy meter, used in domestic and industrial applications to measure energy in kilowatt hours. Its working principle is similar to the induction type instruments. Hence, due to interaction between two fluxes, a torque is produced that helps in rotating the disc. Since the torque produced in the instrument is based on the induction, the energy meter operates only in AC.

6.12.1 Construction

The schematic diagram of a single-phase induction type energy meter is shown in Figure 6.18.

The essential components of single-phase induction type energy meter shown in Figure 6.18 can be grouped as: (i) Driving system, (ii) Moving system, (iii) Braking system and (iv) Registering or counting system.

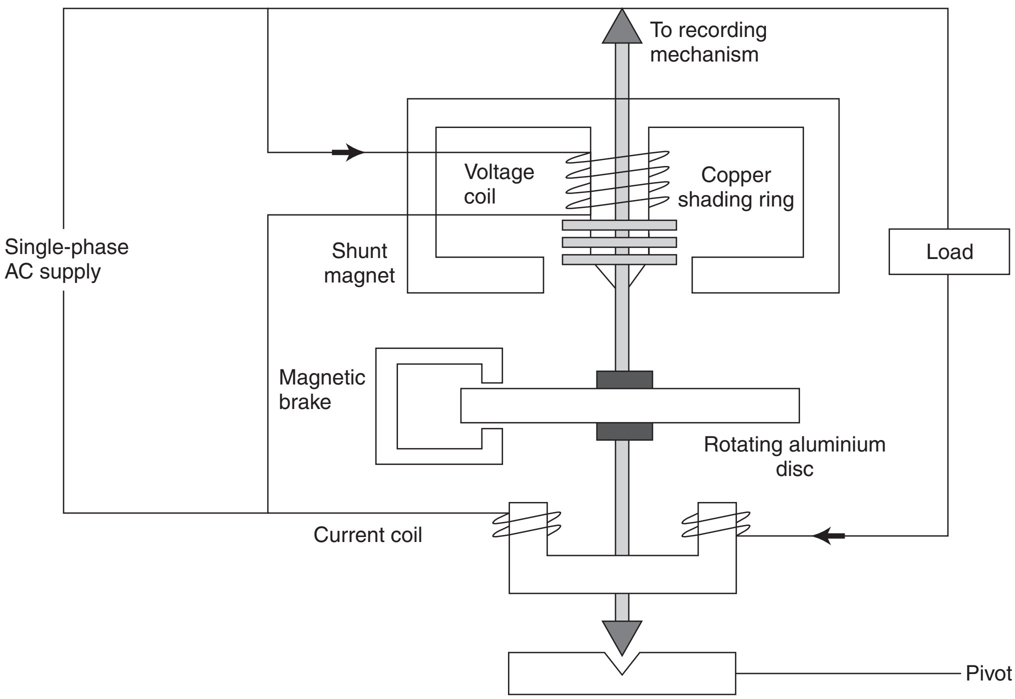


Figure 6.18 Single-Phase Induction Type Energy Meter

(i) Driving System

The driving system of the energy meter consists of two electromagnets, called shunt and series magnets. Silicon steel laminations are used for constructing these magnets. These laminations are arranged in such a way to form a strong mechanical structure which helps in distribution of magnetic flux. The M-shaped steel laminations are used to form the shunt magnet where the highly inductive coil with large number of turns of smaller cross sectional area is wounded on the centre limb of the shunt magnet. This pressure or voltage coil is connected across the AC supply. When the coil is energised by the AC supply, a magnetic field is produced and it lags the supply voltage by 90°.

The *Y*-shaped steel laminations are used to form the series magnet where each limb is wounded with less turns of coil with larger cross sectional area. This current coil is connected to one of the lines and in series with the load. When the load current flows through this coil in the series magnet, it gets energised and a magnetic field is produced that is in phase with the load current. Copper bands are provided on the shunt magnet to produce a phase difference of 90° between the magnetic field set by the shunt magnet and the supply voltage. This copper band is also called power factor compensator or compensating loop.

(ii) Moving System

The moving system of the energy meter consists of an aluminum disc mounted on an alloy shaft and is freely suspended in the air gap between the shunt and series electromagnets such that there is an interaction between the flux produced by these electromagnets and disc. The upper bearing of the moving system is a steel pin fixed to the bearing cap located at the top side of the shaft. The aluminum disc rotates on a steel pivot supported by the bearing and screwed at the bottom of the shaft. The registering or counting mechanism is connected to the shaft using a pinion. Due to variation in the magnetic field, an eddy current gets induced in the disc. This eddy current when interacts with the magnetic field, a deflecting torque is produced in the disc.

(iii) Braking System

A permanent *C*-shaped magnet called brake magnet is present in the braking system. This magnet is made up of steel alloy. It is provided with a small gap between the poles through which the disc rotates. Brake magnet helps in applying the braking or controlling torque on the aluminum disc to reduce its rotation. When the eddy current induced in the disc disturbs the magnetic field of the brake magnet, a braking torque is produced which opposes the disc movement and hence the disc speed reduces. The braking torque applied on the disc can be varied by moving the brake magnet to any other position.

(iv) Registering or Counting System

The number of rotations made by the aluminum disc is recorded by this registering or counting system. This system consists of gear driven by the pinion fixed on the shaft and helps in moving the pointer on the dial indicating the number of rotation made by the disc. The two different registering systems are: (i) pointer type and (ii) cyclometer type. In the pointer type, there are a series of five or six pointers rotating on dials with equal divisions as shown in Figure 6.19.

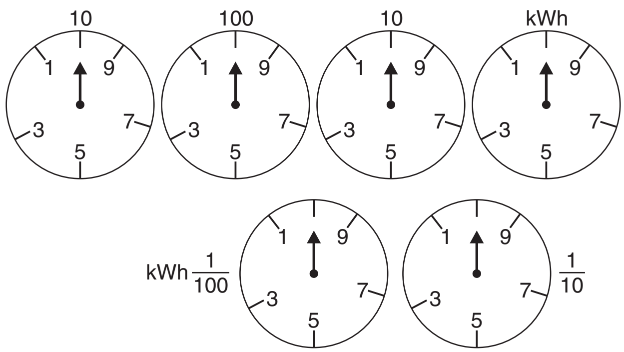


Figure 6.19 Pointer Type Registering Mechanism

6.12.2 Working

When the arrangement shown in Figure 6.19 is connected in the circuit, the pressure coil carries a current proportional to the supply voltage and the series coil carries the load current. The flux produced by the series magnet, ϕ_{se} is in phase with load current, *I* and the flux produced by the shunt magnet, ϕ_{sh} lags the supply voltage by 90°. These fluxes induce emfs e_{se} and e_{sh} in the disc and hence eddy currents i_{se} and i_{sh} are produced in phase with their respective induced emfs. A driving force is produced due to the interaction of these eddy currents and the fluxes and it helps in rotating the disc. The controlling torque is developed by the brake magnet the desired speed per unit power consumption can be obtained. When the load connected to the meter consumes power, due to the interaction between eddy currents and magnetic fluxes, the aluminum disc starts rotating. After some number of rotations, the pointer fixed to the disc displays the total power consumed by the load.

6.12.3 Errors in Single-Phase Induction Type Energy Meter

The different errors in single-phase induction type energy meter and their adjustments are given below:

(i) *Phase angle error*: The correct reading for the energy consumed is possible only if the phase difference between ϕ_{sh} and supply voltage is 90°. But due to the presence of winding resistances and iron losses, the phase angle error exists in the system. It can be rectified by adjusting the position of copper band placed in the shunt magnet.

(ii) *Speed error*: The speed error exists if the disc rotates either slower or faster which results in wrong reading of energy consumption. It can be eliminated by adjusting the position of the brake magnet.

(iii) *Friction error*: The error caused in the frictional forces existing at the bearings of moving system and in registering system at light loads is called friction error. This can be reduced by making the ratio of the ϕ_{sh} and ϕ_{se} large by using two shading rings in the outer limbs of the shunt magnet.

(iv) *Creeping*: It is the process in which the disc rotates slowly but continuously when no load is connected to the system and when the pressure coil is energised by the supply. It is due to excessive supply voltage, vibration, stray magnetic field, etc. It is prevented by drilling two holes or slots in the disc. These holes lie on the opposite side of the spindle causing the required distortion in the field and helps in making the disc stationary.

(v) *Temperature error*: Error due to temperature variation is very small as this variation neutralises each other on using power factor. Though this error is very small, great care should be taken while designing the energy meter to eliminate these errors.

(vi) *Frequency error*: If the frequency of the supply voltage varies from its nominal value, the coil reactance changes and it results in a small error called frequency error. But it is not practically possible since the supply frequencies are held within its limits.

6.12.4 Advantages and Disadvantages of Single-Phase Induction Type Energy Meter

Advantages

(i) Absence of moving iron in the instrument.
(ii) High torque to weight ratio high and good damping.
(iii) No electrical contact exists between the moving element and the circuit.
(iv) Stray magnetic field has less effect on the instrument.
(v) High accuracy for a wide range of loads.

Disadvantages

(i) When proper compensation is not provided, errors exist in the instrument due to temperature, waveform and frequency.
(ii) Used only for AC measurements.
(iii) Power consumption is high.
(iv) Non-linear scales in the instrument.
(v) High cost.

6.13 Wattmeter

The most commonly used moving-coil instrument for measuring power in both AC and DC circuits is the electrodynamometer type wattmeter. It is one of the applications of electrodynamometer type instrument.

Therefore, the construction of electrodynamometer wattmeter is same as that of the electrodynamometer instrument. The different parts present in the wattmeter are fixed coil, moving-coil, moving system, controlling torque, damping torque, shielding, cases and scales. The explanation of each part has already been discussed in Section 6.11.

6.13.1 Working

The schematic diagram of electrodynamometer type wattmeter is shown in Figure 6.20.

When the power flowing through a circuit is to be measured, the fixed and moving-coils in electrodynamometer should be connected as shown in Figure 6.20. The fixed coil that carries the current is connected in series with load. The moving-coil that carries voltage in proportional to the current is connected across the load through a high resistance *R*. When the current flows through the coils, a mechanical force exists between them and it results in the movement of moving-coil over the scale. The pointer comes to a final position when deflecting torque is equal to the controlling torque. When the direction of the current changes, the direction of current that flowing through the fixed and moving-coils changes. Hence, there is no change in the deflecting torque. Therefore, this wattmeter can be used to measure AC and DC power.

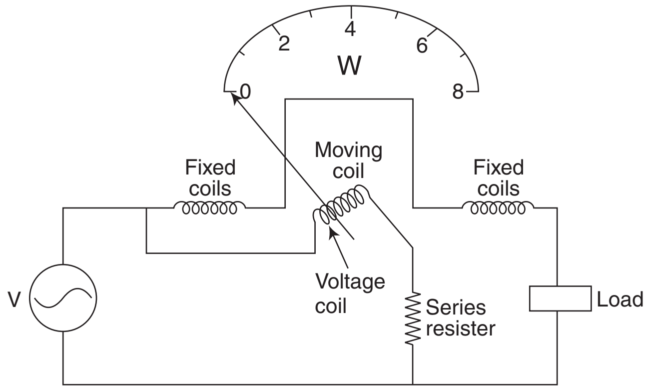


Figure 6.20 Electrodynamometer Type Wattmeter

6.14 Digital Multimeter

[AU April/May, 2015]

An instrument used to measure voltage, current and resistance is known as a multimeter. There are two types of multimeters, namely: analogue and digital. Of these two types, the digital multimeter is commonly used in laboratories and workshops because of its high input-resistance, greater accuracy, better resolution and easy readability. The DMM combines in one case the instruments for the measurements of voltage, current and resistance. The block diagram of a digital multimeter is shown in Figure 6.21.

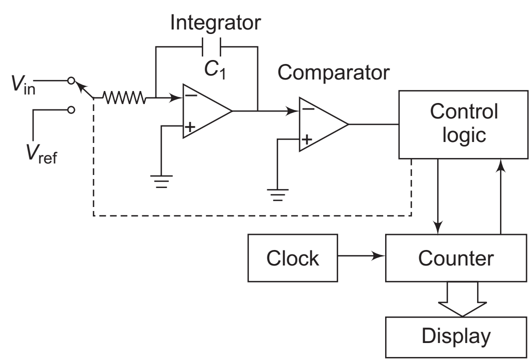


Figure 6.21 Block Diagram of a Digital Multimeter

6.14.1 Measurement of Voltage

The principle used in a digital voltmeter is used in a DMM for the measurement of voltage.

6.14.2 Measurement of Current

A series of current-sensing resistors are used to measure either DC or AC current. The current to be measured is passed through one of the sensing resistors and the DMM digitises the voltage developed across the resistor.

For example, referring to Figure 6.22, the output voltage of a current-to-voltage converter is given by

$$V_0 = -I_s R_f$$

where R_f is the known resistance. The output voltage V_0, which is proportional to the unknown source current I_s is applied to DVM section of DMM and the value of current I_s is displayed.

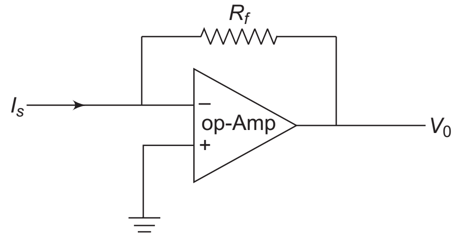


Figure 6.22 Current-to-Voltage Converter

6.14.3 Measurement of Resistance

The DMM measures the resistance by applying a known current from an internal current source to the unknown resistance and then digitising the resulting voltage developed.

For example, referring to Figure 6.23, the output voltage of a scale changer is given by

$$V_0 = \frac{-R_f}{R_i} V_i$$

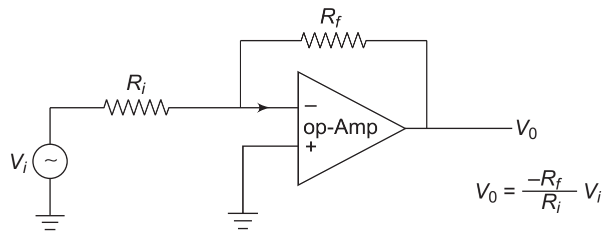


Figure 6.23 Scale Changer

where V_i and R_i are the known parameters.

The output voltage, which is proportional to the unknown resistance R_f, is applied to DVM section of DMM and the value of unknown resistance R_f is displayed.

Most of the DMMs are similar in terms of voltage, current and resistance measurements. They differ only in terms of accuracy, selection of ranges, and AC bandwidth. There are some DMMs that have built-in capacitance measuring circuitry. Most DMMs have protection from input overload by using circuit breakers, fuses, auto-ranging and diode clipper circuit. The display used can be either Liquid Crystal Display (LCD) or Light Emitting Diode (LED) display.

6.14.4 Applications

A DMM is typically used for measurement of voltage, current and resistance. It is also used to test whether the diode, transistor or SCR is good or faulty and to check circuit continuity. For example, to check a diode, the resistance is measured in one direction and then in the other direction. In the forward-biased direction, a low resistance is indicated and in the reverse-biased direction, a high resistance is indicated.

6.15 Cathode Ray Oscilloscope

The CRO is a versatile electronic testing and measuring instrument that allows the amplitude of the signal, which may be voltage, current, power, etc., to be displayed primarily as a function of time. It is used for voltage, frequency, and phase-angle measurement and also for examining the waveforms, from DC or very-low frequency to very-high frequencies.

Figure 6.24 shows the basic block diagram of a CRO. It comprises of the following main sections: (i) horizontal and vertical voltage amplifiers, (ii) power-supply circuits, and (iii) cathode-ray type (CRT).

6.15.1 Vertical and Horizontal Voltage Amplifiers

These amplifiers are connected between the input terminals and the deflection plates. The function of the amplifiers is to increase the deflection sensitivity for weak input voltages. The input signal is fed through a

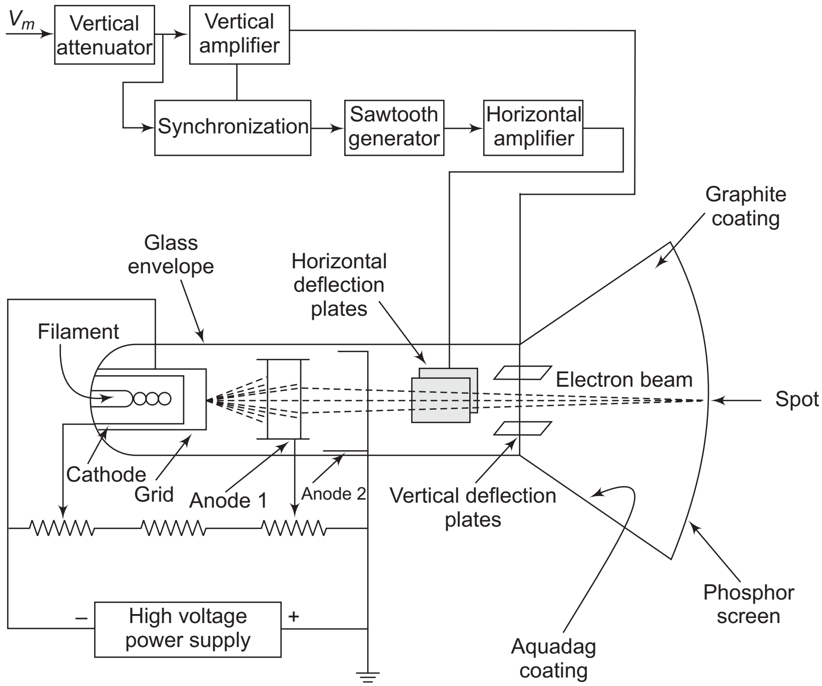


Figure 6.24 Schematic Diagram of a CRO

calibrated attenuator and a wideband high-gain vertical amplifier to the vertical deflection plates of the CRT. The horizontal amplifier, which is connected to the horizontal plates of the CRT, is fed from an internally generated time base, usually a saw-tooth waveform generator, or alternatively the horizontal amplifier can also be fed from an externally connected X input. A portion of the input signal applied to the vertical plates triggers the horizontal sweep (saw-tooth) signal. A finite amount of time (in the range of seconds) is elapsed before the saw-tooth waveform is applied to the horizontal plates. Hence, to observe the starting edge of the input signal fully, it should be delayed by the same amount of time in the delay line.

6.15.2 Power-supply Circuits

The power-supply unit provides high voltages required by the CRT to generate and accelerate the electron beam, in addition to supplying the required operating voltage for the other circuits of the oscilloscope. The CRT requires high voltages, of the order of a few thousand volts, for acceleration and a low voltage for the heater of the electron gun, which emits electrons. The CRO has various control switches on the panel. The respective control knobs can adjust the intensity of the spot and focus.

6.15.3 Cathode-Ray Tube (CRT)

The CRT is the heart of the oscilloscope. It is a vacuum tube of special geometrical shape and converts an electrical signal into a visual one. A heated cathode emits electrons, which are accelerated to a high velocity and are brought to focus on a fluorescent screen. When the electron beam strikes the screen of the CRT, a spot light is produced. The electron beam, on its journey, is deflected in response to the electrical signal under study. As a result, the waveform of the electrical signal is displayed. As shown in Figure 6.40, the CRT has various parts, which are described below.

Glass Envelope and Screen

It houses the electron gun, vertical and horizontal plates, and a screen on the conical front-end. The inner walls of the CRT between neck and screen are usually coated with a conducting material (graphite) called acquadag. This conductive coating is electrically connected to the accelerating anode so that the electrons, which accidentally strike the wall, are returned to the anode. It prevents the wall of the tube from charging to a high negative potential.

The screen is coated with a suitable fluorescent material, depending on the required colour of the spot. Some of the substances, which give characteristic fluorescent colours, are:

- Zinc orthosilicate: Green (used in CRT for general purpose)
- Calcium tungstate: Blue (used in CRT for fast photography)
- Zinc sulphide or
- Zinc cadmium sulphate: White (used in television receiver tubes).

Electron Gun

It produces a focused beam of electrons. It consists of an indirectly heated cathode, a control grid, a focusing anode and an accelerating anode. The control grid is at a negative potential with respect to the cathode, whereas the two anodes are maintained at a high positive potential with respect to the cathode. These two anodes act as electrostatic lens to converge the electron beam at a point on the screen. The cathode consists of a nickel cylinder, coated with an oxide coating that provides plenty of electrons. The control grid encloses the cathode and consists of a metal cylinder with a tiny circular opening to keep the electron beam small in size. The focusing anode focuses the electron beam to a sharp point by controlling the positive potential on it. The positive potential (about 10,000 V) on the accelerating anode is much higher than that on the focusing anode so that, this anode accelerates the narrow beam to high velocity. Therefore, the electron gun generates a narrow, accelerated beam of electrons, which produces a spot of light when it strikes the screen.

Deflection Plates

The electron beam comes under the influence of vertical and horizontal deflection plates before it strikes the screen.

When no voltage is applied to the vertical deflection plates, the electron beam produces a spot of light at the centre of the screen. If the upper plate is positive with respect to the lower plate, the electron beam is deflected upwards and strikes the screen above its centre. If the upper plate is negative with respect to the lower plate, the electron beam is deflected downwards and strikes the screen below its centre. Thus, the electron beam is made to move up and down vertically, by controlling the voltage on the vertical plates, thereby producing spots of light on the screen.

When a sinusoidal voltage is applied to the vertical deflection plates, the upper plate is positive during the positive half cycle and negative during the negative half cycle, thereby producing a continuous vertical line on the screen.

The electron beam is made to move horizontally from side-to-side, at a uniform rate by applying a saw-tooth wave, which varies linearly with time across the horizontal deflection plates.

Thus, the spot of light can be moved all over the surface of the screen by the simultaneous action of both vertical and horizontal deflection plates. In order to get the exact pattern of the signal on the screen, the signal voltage is given to the vertical deflection plates and saw-tooth wave to the horizontal deflection plates.

Types of CRTs

Conventionally, CRTs form the basis of cathode-ray oscilloscopes (CROs), TVs and consoles/monitors. They are useful in displaying numeric, alphanumeric and graphic displays with high resolution. These are of two types:

- Electrostatic (used in CROs)
- Electromagnetic (used in TVs)

There are also storage CRTs using digital storage, mesh storage, phosphor storage, and transfer storage. Flat CRTs are also available.

6.15.4 Special Oscilloscopes

Some special oscilloscopes, which are discussed briefly in the following section, are designed for specific applications.

Dual-beam Oscilloscope

A dual-beam oscilloscope is useful to observe two signals simultaneously and compare their waveforms. Figure 6.25 shows the block diagram of a dual-beam CRO. It has two completely separate electron guns, two sets of vertical deflecting plates (Y-plates) and a single set of horizontal deflecting plates (X-plates). Both the channels have a common time-base but have completely independent pre-amplifiers, delay lines and vertical amplifiers. Only one beam can be synchronised at one time because the horizontal sweep is common for both signals. In order to lock the two signals on the CRT screen, the two signals must have the same frequency and phase or must be related harmonically. The dual-beam CRO is used to observe both the input and output signals of the amplifier under test. Signal A may be the input signal and signal B may be the output signal from the amplifier.

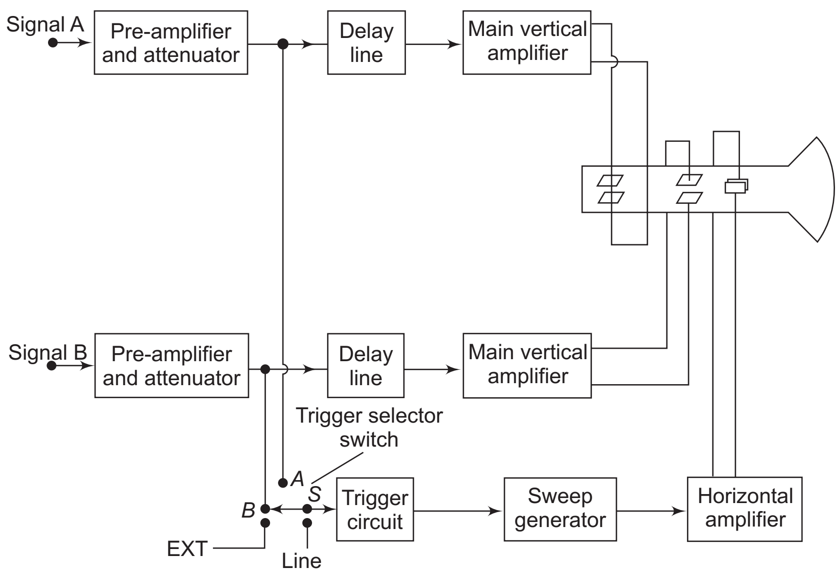


Figure 6.25 Dual-beam Oscilloscope

Dual-trace CRO

The function of a dual-trace CRO is similar to that of a dual-beam CRO but this CRO has a single electron-gun. A single electron beam is split into two beams by means of an electronic switch. The two signals are displayed simultaneously. Figure 6.26 shows a block diagram of the two vertical input channels of a dual-trace CRO. Each channel has its own calibrated input attenuator and positioning control. Therefore, the amplitude of each signal can be independently adjusted. The electronic switch alternately connects the two input channels to the vertical amplifier. The signals pass through the common vertical channel or vertical amplifier. Two channels share the horizontal channels on a time basis.

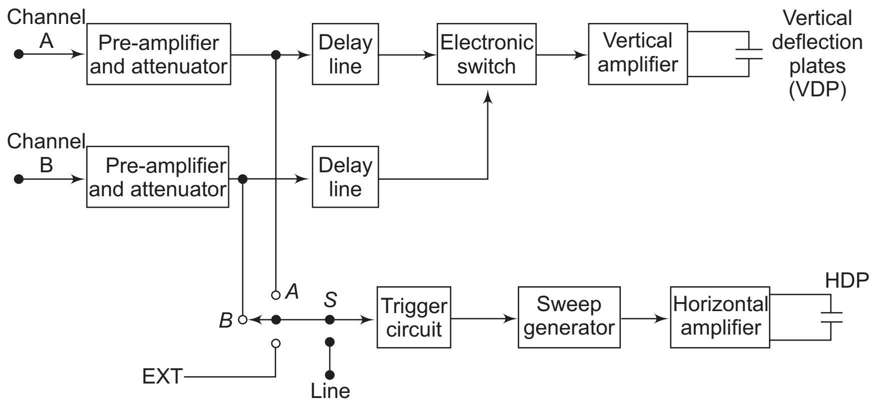


Figure 6.26 Block Diagram of the Input Channels of a Dual-trace Oscilloscope

The dual-trace oscilloscopes have four modes of operation, namely: A, B, alternate and chopped. In the A or B modes, only the input at that channel is displayed. In the alternate mode, the inputs are displayed on alternate traces. The switching rate of the electronic switch is synchronised to the sweep rate, so that the CRT spot traces the channel A signal on one sweep and the channel B signal on the succeeding sweep. This mode of operation is generally preferred when displaying relatively high-frequency signals. In the chopped mode, the electronic switch is free-running at the rate of 100–500 kHz, entirely independent of the frequency of the sweep generator. The switch successively connects small segments of A and B waveforms alternately to the main vertical amplifier at a relatively fast chopping rate of 500 kHz. If the sweep rate is low, the chopped mode is normally used as the alternate mode and would provide a display with considerable flicker.

Storage Oscilloscopes **[AU Nov/Dec, 2014]**

A limitation in conventional CROs is an event that occurs only once, will disappear from the screen after a relatively short interval of time, as the persistence of the phosphor on the screen ranges only from a few milliseconds to several seconds. In a storage CRO, the display is retained for a much longer time, sometimes even for some hours, after the image was first traced on the screen. This retention feature is, therefore, useful in the study of waveforms, which have very low frequency. In a conventional CRO, the start of such a display will fade out before the end is reached. The analogue storage oscilloscopes use the phenomenon of secondary electron emission to build up and store electrostatic charges on the surface of an insulated target.

The block diagram of a basic Digital Storage Oscilloscope (DSO) is illustrated in Figure 6.27. The input signal is digitised and stored in memory in digital form. In this digital form, it is capable of being analysed to produce a variety of information about the input signal. The digital data is reconstructed in analogue form to view the display on the CRT. In order to ensure that no information is lost, the sampling rate must be at least twice the highest frequency of the input signal. The digital oscilloscope is primarily limited in speed by the digitising capacity of the analogue-to-digital converter. Digital oscilloscope is capable of an infinite storage time using its digital memory. A crystal clock generates a time-base in a digital oscilloscope.

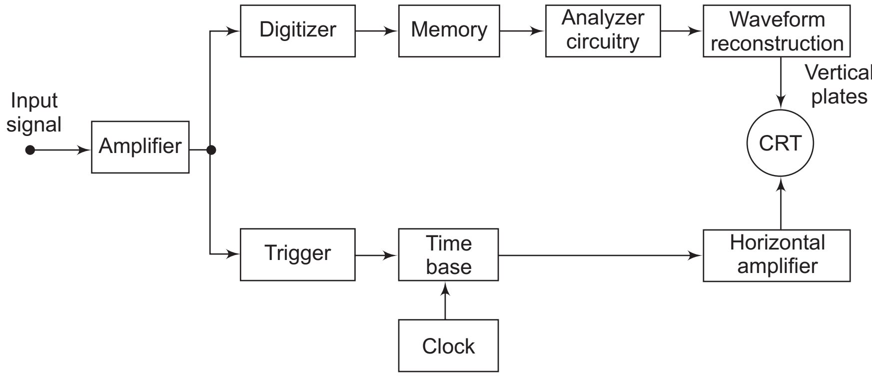


Figure 6.27 Block Diagram of a Digital Storage Oscilloscope

In addition, digital storage oscilloscopes are available in processing and non-processing types. The processing-types have built-in computing facilities and take advantage of the fact that all the data is already in digital form. DSO is also capable of operating in a look-back mode like waveform recorder. If it is triggered, it prints out the stored result on to a hardcopy recorder or disk storage.

Sampling Oscilloscope

A sampling CRO is used to examine very fast signals using instruments having bandwidth of several orders lower. The gain–bandwidth relationship of the vertical amplifier limits the frequency range of signals, which can be displayed on a CRO. As shown in Figure 6.28, samples of the input waveform are taken at different portions of the waveform over successive cycles, with one sample taken per cycle, and each sample slightly delayed with respect to the preceding sample. Then the total picture is stretched, amplified by relatively low bandwidth amplifiers and displayed as a continuous waveform on the screen. The disadvantage of a sampling CRO is that it can only make measurements on repetitive waveform signals.

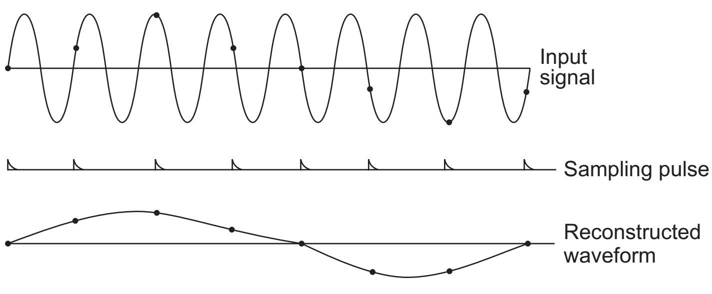


Figure 6.28 Principle of Sampling Oscilloscope

The sampling technique transforms the high-frequency input signal into lower frequency domain where conventional low-frequency circuit is capable of producing a highly effective display. The sample frequency used in sampling CROs can be as low as $\frac{1}{100}$ of the signal frequency and hence a signal frequency of 1 GHz needs an amplifier bandwidth of 10 MHz.

6.15.5 Applications of CRO

The modulation index of Amplitude Modulation (AM) waves can be measured using a CRO. The voltage–current characteristics of a *PN* junction diode and transistor, and characteristics of a transformer core can be displayed on CRO. Some more applications are discussed below.

Measurement of Voltage

If the signal is applied to the vertical deflection plates only, a vertical line appears on the screen. The height of the line is proportional to peak voltage of the applied signal. The amplitude of the signal can be measured by applying the signal to the vertical plates and the sweep is applied to the horizontal plates using internal sweep circuitry. The vertical scale on the CRT screen is marked in centimetres. Each centimetre is further subdivided into 5 parts so that each part represents 0.2 cm. If the peak amplitude of the waveform is 1.7 cm and the scale selected by the dial setting is 1 V/cm, then the amplitude of the signal is 1.4 cm × 1 V/cm = 1.4V.

Measurement of Current

When a current is to be measured, it is passed through a known resistance and the voltage across it is measured.

Measurement of Frequency

- ***Using signal waveform:*** The signal for which the frequency (f) is to be measured is given to the vertical input. The number of divisions occupied by one complete cycle of the waveform is measured. The number of divisions multiplied by the time-base setting in seconds is equal to the time period (T) of one cycle. The frequency (f) of the waveform is inverse of the time period T, i.e., $f = 1/T$.
- ***Using Lissajous Figure:*** If sinusoidal voltages are applied to both vertical and horizontal inputs of CRO, some interesting Figures are displayed, which are known as *Lissajous Figures*. Two sine waves of the same frequency produce a Lissajous Figure, which may be a straight line, an ellipse, or a circle, depending on the phase and amplitude of the two signals. Two sine waves of equal amplitudes but different frequencies will produce a Figure from which the relationship between the two frequencies can be understood. For example, Figure 6.29(a) shows that the vertical input signal has twice the frequency of the horizontal input signal. Similarly, Figure 6.29 (b) indicates that horizontal input signal has twice the frequency of the vertical one. Figure 6.29 (c) shows three loops indicating that vertical input signal has thrice the frequency of the horizontal one.

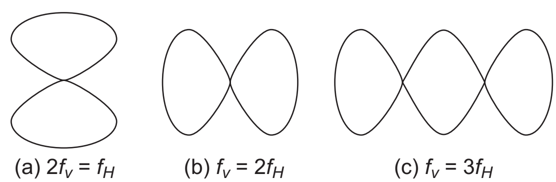


Figure 6.29 Frequency Measurement (Lissajous Figures) (a) $2f_V = f_H$ (b) $f_V = 2f_H$ & (c) $f_V = 3f_H$

A known frequency (f_H) is applied to horizontal input and unknown frequency (f_V) to the vertical input. Then a Lissajous pattern with loops is obtained. The unknown frequency (f_V) can be measured by the following relationship.

$$\frac{f_V}{f_H} = \frac{\text{No. of loops cut by horizontal line}}{\text{No. of loops cut by vertical line}}$$

Measurement of Phase Difference

The phase difference between two sinusoidal signals of same frequency can be calculated from the amplitudes A and B of the Lissajous pattern (an ellipse) shown in Figure 6.30. The phase difference (deg), $q = \sin^{-1}(A/B)$. Lissajous Figures are formed when two sine waves are applied simultaneously to the vertical and horizontal deflecting plates of a CRO. In general, the shape of the Lissajous Figures depends on amplitude, phase difference and ratio of frequency of the two waves. Two sine waves of the same frequency and amplitude may produce a straight line, an ellipse or a circle, depending on their phase difference, as shown in Figure 6.31.

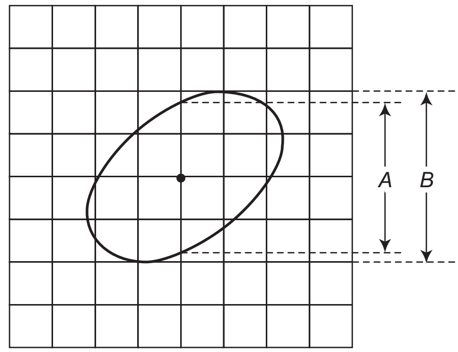


Figure 6.30 Phase Difference Measurement

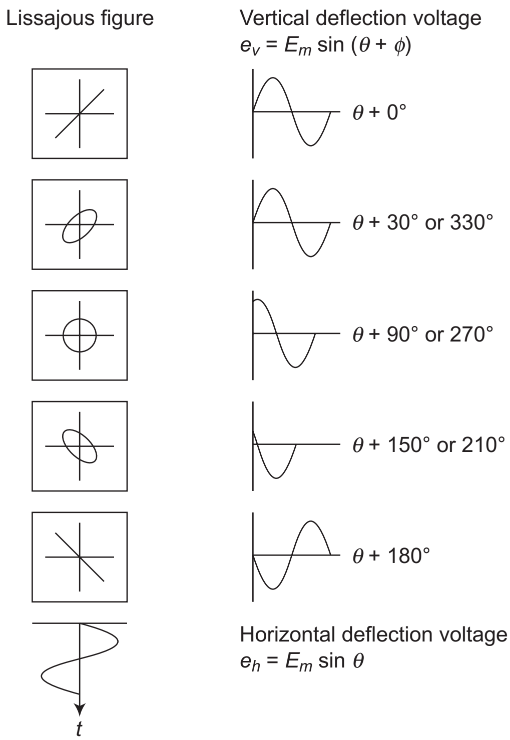


Figure 6.31 Lissajous Figure Depending on the Phase Difference of the Two Waves

Test of Distortion on Amplifier

CRO is useful to measure the distortion using Lissajous Figures. Figure 6.32 shows the connections for testing the frequency distortion of an amplifier network. The audio oscillator is adjusted to a known frequency and is connected to the deflecting plates $x - x'$. The input signal obtained at the output of the amplifier is connected to the deflecting plates $y - y'$. If the amplifier produces higher harmonics of the input frequency due to the non-linearity of the active device used, the CRO screen shows the loops in the Lissajous Figure, which indicates the presence of distortion. A straight-line display indicates the absence of distortion in the amplifier.

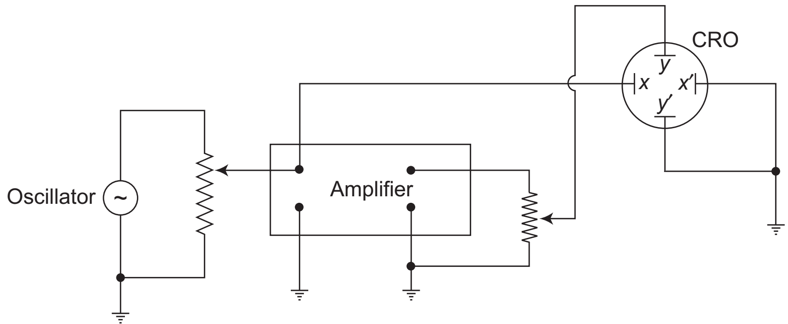


Figure 6.32 Distortion Test on Amplifier

6.16 Three-phase Power Measurement

The power consumed in a three-phase system is measured using wattmeters. Generally, one wattmeter is required for measuring power in one phase and hence three wattmeters would be required to measure power in a three-phase system. But, universally it is proved that only two wattmeters are enough to measure power in three-phase systems of any type i.e., balanced or unbalanced and star or delta-connected systems. In addition, if the system is balanced, the circuit power factor can also be determined using the readings of two wattmeters. Also, in a balanced system, the total power in all the three-phases can be obtained by multiplying the power obtained in a single-phase.

6.16.1 Methods of Power Measurement

The three methods that are used for the measurement of three-phase power in three-phase circuits are:

1. Three-Wattmeter Method
2. Two-Wattmeter Method
3. Single-Wattmeter Method

Blondel's Theorem

When power is supplied by an '*n*' wire AC system, then the number of wattmeters required to measure power is '$n - 1$', i.e., one less than the number of wires in the AC system, irrespective of a balanced or an unbalanced load. Therefore, for a three-phase four-wire system, the number of wattmeters required to measure the power is three and for a three-phase three-wire system, the number of wattmeters required to measure the power is two.

6.16.2 Three-Wattmeter Method

According to Blondel's theorem, the three-wattmeter method can be used only to measure the power in three-phase, four-wire star-connected balanced and unbalanced load, whose circuit diagram along with wattmeters W_1, W_2 and W_3 is shown in Figures 6.33, respectively.

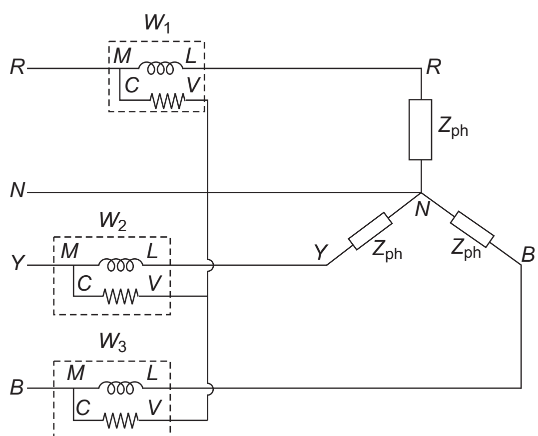


Figure 6.33 Power Measurement Using Three-Wattmeter Method

Balanced Load

Usually, the wattmeter connected across the balanced load measures the actual power measured by the load and is given by the product of root mean square (RMS) values of voltage and current. The power consumed by the three-phase load, given in Figure 6.33 measured by W_1, W_2 and W_3 are given by:

$$W_1 = |\overline{V}_{RN}||\overline{I}_R|\cos(\overline{V}_{RN} \wedge \overline{I}_R) \tag{6.18}$$

$$W_2 = |\overline{V}_{YN}||\overline{I}_Y|\cos(\overline{V}_{YN} \wedge \overline{I}_Y) \tag{6.19}$$

$$W_3 = |\overline{V}_{BN}||\overline{I}_B|\cos(\overline{V}_{BN} \wedge \overline{I}_B) \tag{6.20}$$

where $\overline{V}_{RN} \wedge \overline{I}_R, \overline{V}_{YN} \wedge \overline{I}_Y$ and $\overline{V}_{BN} \wedge \overline{I}_B$ are the angles between $\overline{V}_{RN}$ and $\overline{I}_R, \overline{V}_{YN}$ and $\overline{I}_Y$ and $\overline{V}_{BN}$ and $\overline{I}_B$ respectively.

From the phasor diagram shown in Figure 6.33, the angles $\overline{V}_{RN} \wedge \overline{I}_R, \overline{V}_{YN} \wedge \overline{I}_Y$ and $\overline{V}_{BN} \wedge \overline{I}_B$ will be ϕ and $-\phi$ for lagging and leading power factors. But it is known that $\cos(-\phi) = \cos(\phi)$. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 + W_3$$

Substituting Eqns. (6.18) to (6.20) in the above equation, we get

$$W = |\overline{V}_{RN}||\overline{I}_R|\cos\phi + |\overline{V}_{YN}||\overline{I}_Y|\cos\phi + |\overline{V}_{BN}||\overline{I}_B|\cos\phi \tag{6.21}$$

But in a balanced star-connected load, we have

$$|\overline{V}_{RN}| = |\overline{V}_{YN}| = |\overline{V}_{BN}| = |\overline{V}_{\text{ph}}| = \frac{|\overline{V}_L|}{\sqrt{3}} \text{ and } |\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L| \tag{6.22}$$

Substituting Eqn. (6.22) in Eqn. (6.21) and solving, we get the total average power measured by the three wattmeters connected across a star-connected balanced load as

$$W = \sqrt{3}\,|\overline{V}_L||\overline{I}_L|\cos\phi \tag{6.23}$$

6.16.3 Two-Wattmeter Method **[AU May/June, 2013]**

Two-wattmeter method is used to measure the total power in a three-phase, three-wire star or delta-connected balanced or unbalanced load. In this method, the current coils of the wattmeter are connected with any two lines (i.e., R and Y) and the pressure coil of the wattmeter is connected between the above two lines and the third line (i.e., B). The different phase systems for which the two-wattmeter method can be used to measure the power consumed by the load are:

1. Star-connected balanced load
2. Star-connected unbalanced load
3. Delta-connected balanced load
4. Delta-connected unbalanced load

Star-connected Balanced Load

The circuit diagram for two-wattmeter method applied to a three-phase balanced star-connected load is shown in Figure 6.34.

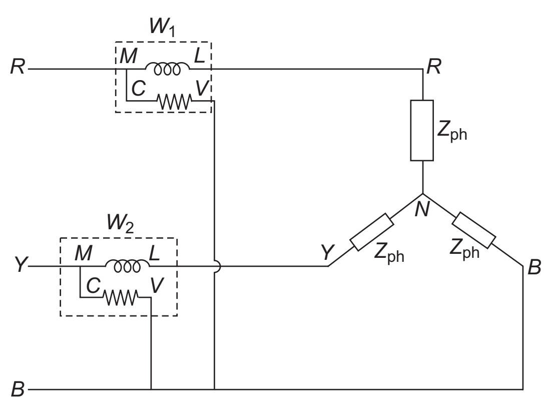


Figure 6.34 Two-Wattmeter Method for Three-phase Balanced Star-connected Load

The readings of wattmeter shown in Figure 6.34 for a balanced star-connected load is given below:

$$W_1 = |\overline{I}_R||\overline{V}_{RB}|\cos(\overline{I}_R \wedge \overline{V}_{RB}) \text{ and } W_2 = |\overline{I}_Y||\overline{V}_{YB}|\cos(\overline{I}_Y \wedge \overline{V}_{YB})$$

It is known that, $\overline{V}_{RB} = \overline{V}_{RN} - \overline{V}_{BN}$ and $\overline{V}_{YB} = \overline{V}_{YN} - \overline{V}_{BN}$. The phasor diagram for two-wattmeter method, applied to a three-phase balanced star-connected load with lagging power factor (inductive load), is shown in Figure 6.35(a).

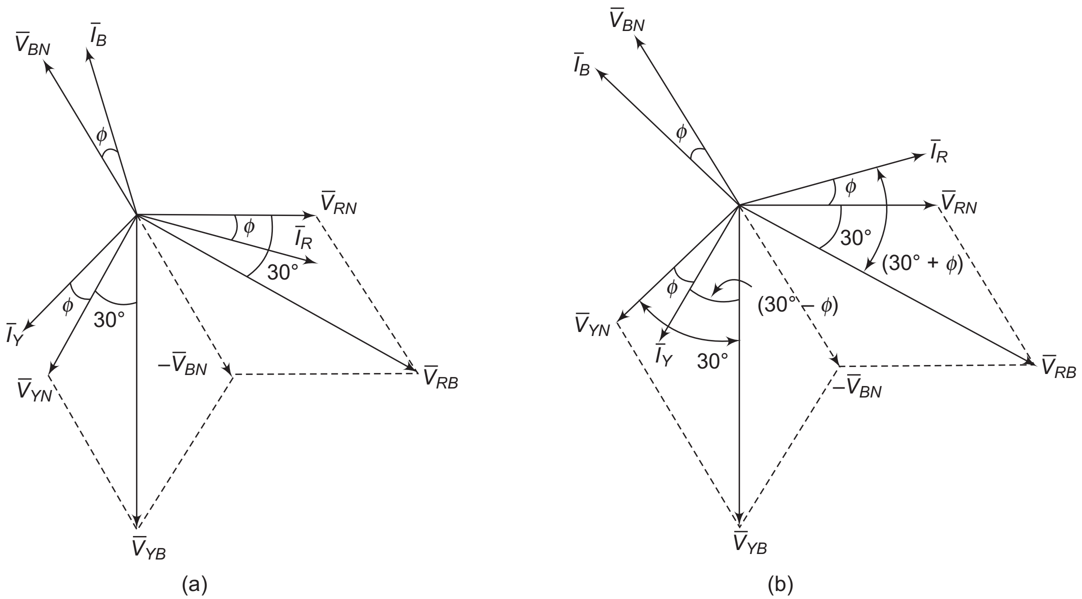


Figure 6.35 Phasor Diagram for Two-Wattmeter Method with (a) Lagging and (b) Leading Power Factor

From the phasor diagram shown in Figure 6.35 (a), the angles $\overline{V}_{RB} \wedge \overline{I}_R$ and $\overline{V}_{YB} \wedge \overline{I}_Y$ are (30° – ϕ) and (30° + ϕ) respectively. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 = |\overline{V}_{RB}||\overline{I}_R|\cos(30° - \phi) + |\overline{V}_{YB}||\overline{I}_Y|\cos(30° + \phi) \tag{6.24}$$

But, in a balanced star-connected load, we have

$$|\overline{V}_{RB}| = |\overline{V}_{YB}| = |\overline{V}_{BR}| = |\overline{V}_L| \text{ and } |\overline{I}_R| = |\overline{I}_Y| = |\overline{I}_B| = |\overline{I}_L| \tag{6.25}$$

Substituting Eqn. (6.25) in (6.24) and solving, we get

$$W = |\overline{V}_L||\overline{I}_L|[\cos(30° - \phi) + \cos(30° + \phi)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin B \sin A$ in the above equation and solving, we get the total average power consumed by the load as

$$W = \sqrt{3}\,|\overline{V}_L||\overline{I}_L|\cos\phi \tag{6.26}$$

Similarly, it is possible to obtain the total active power consumed by the load for a leading power factor. The phasor diagram for two-wattmeter method applied to a three-phase balanced star-connected load with leading power factor (capacitive load) is shown in Figure 6.35 (b).

From the phasor diagram shown in Figures 6.35 (b), it can be seen that the angles $\bar{V}_{RB} \wedge \bar{I}_R$ and $\bar{V}_{YB} \wedge \bar{I}_Y$ are (30° + ϕ) and (30° – ϕ) respectively. Therefore, the total power consumed by the load is given by

$$W = W_1 + W_2 = |\bar{V}_{RB}||\bar{I}_R|\cos(30° + \phi) + |\bar{V}_{YB}||\bar{I}_Y|\cos(30° - \phi) \tag{6.27}$$

But, in a balanced star-connected load, we have

$$|\bar{V}_{RB}| = |\bar{V}_{YB}| = |\bar{V}_{BR}| = |\bar{V}_L| \text{ and } |\bar{I}_R| = |\bar{I}_Y| = |\bar{I}_B| = |\bar{I}_L| \tag{6.28}$$

Substituting Eqn. (6.28) in (6.27) and solving, we get

$$W = |\bar{V}_L||\bar{I}_L|[\cos(30° + \phi) + \cos(30° - \phi)]$$

Using $\cos(A \pm B) = \cos A \cos B \mp \sin B \sin A$ in the above equation and solving, we get the total average power consumed by the load as

$$W = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi \tag{6.29}$$

It is noted from Eqns. (6.26) and (6.29) that, the total active power consumed by the load with lagging or leading power factor is same.

6.16.4 Power Factor Calculation by Two-Wattmeter Method

[AU May/June, 2016]

In case of a balanced star or a delta-connected load, the power factor can be calculated from readings of W_1 and W_2.

The wattmeter readings in two-wattmeter method for lagging power factor loads are:

$$W_1 = |\bar{V}_L||\bar{I}_L|\cos(30° - \phi) \text{ and } W_2 = |\bar{V}_L||\bar{I}_L|\cos(30° + \phi)$$

Let,
$$W_1 + W_2 = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi \tag{6.30}$$

$$\begin{aligned} W_1 - W_2 &= |\bar{V}_L||\bar{I}_L|[\cos(30° - \phi) - \cos(30° + \phi)] \\ &= |\bar{V}_L||\bar{I}_L|[\cos 30° \cos\phi + \sin 30° \sin\phi - \cos 30° \cos\varphi + \sin 30° \sin\varphi] \\ &= |\bar{V}_L||\bar{I}_L|[2\sin 30° \sin\phi] = |\bar{V}_L||\bar{I}_L|\left[2 \times \frac{1}{2}\sin\phi\right] \end{aligned}$$

Therefore,
$$W_1 - W_2 = |\bar{V}_L||\bar{I}_L|\sin(\phi) \tag{6.31}$$

Taking ratio of the Eqn. (6.31) to Eqn. (6.30), we get

$$\frac{W_1 - W_2}{W_1 + W_2} = \frac{|\bar{V}_L||\bar{I}_L|\sin\phi}{\sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi} = \frac{\tan\phi}{\sqrt{3}}$$

$$\tan\phi = \frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}$$

On taking the inverse of tan function on both sides, we get

$$\phi = \tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]$$

Therefore, power factor, $\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$

For leading power factor, we get negative value for tan ϕ. But, cosine of negative angle is always positive. Hence, cos ϕ is always positive but its nature must be determined by observing the sign of tan ϕ.

6.16.5 Effect of Power Factor on Wattmeter Readings

For a lagging power factor load, we have the wattmeter readings as:

$$W_1 = |\overline{V}_L||\overline{I}_L|\cos(30° - \phi) \text{ and } W_2 = |\overline{V}_L||\overline{I}_L|\cos(30° + \phi)$$

The effect of power factor on wattmeter readings, for different values of ϕ, is analysed as follows:
Case (i): Here, cos ϕ = 0, Therefore, ϕ = 90°
Therefore,

$$W_1 = |\overline{V}_L||\overline{I}_L|\cos(30° - 90°) = +\frac{1}{2}|\overline{V}_L||\overline{I}_L|$$

$$W_2 = |\overline{V}_L||\overline{I}_L|\cos(30° + 90°) = -\frac{1}{2}|\overline{V}_L||\overline{I}_L|$$

i.e., $W_1 + W_2 = 0$ (or) $W_2 = -W_1$ (or) $|W_1| = |W_2|$

It is noted that as a wattmeter has a positive scale, it cannot show negative readings. Negative reading is indicated when the pointer tries to deflect in negative direction i.e., to the left of zero. In such a case, reading can be converted to positive by interchanging either pressure coil connections i.e., ($C \leftrightarrow V$) or by interchanging current coil connections ($M \leftrightarrow L$). By interchanging connections of both the coils, there will not be any effect on wattmeter reading.

Such a reading obtained by interchanging connections of either of the two coils will be positive but it must be taken as negative for calculation purposes.

In this case, the reading on W_2 must be taken by reversing the connections as shown in Figure 6.36 and must be taken as negative for calculation purpose.

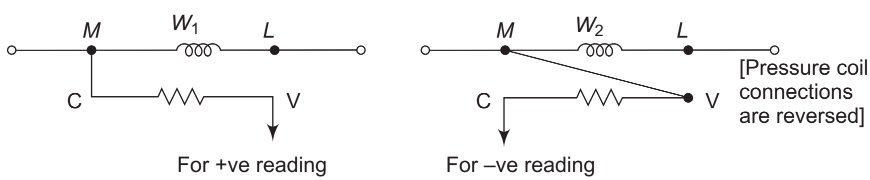


Figure 6.36 Positive and Negative Reading on Wattmeter

Case (ii): Here, cos ϕ = 0.5, Therefore, $\phi = \cos^{-1}(0.5) = 60°$

Therefore, $W_1 = |\overline{V}_L||\overline{I}_L|\cos(30° - 60°) = |\overline{V}_L||\overline{I}_L|\cos 30° = \text{positive}$

$$W_2 = |\overline{V}_L||\overline{I}_L|\cos(30° + 60°) = 0$$

Therefore, $W_1 + W_2 = W_1$ = total power

One wattmeter shows zero reading for cos ϕ = 0.5. For all power factors between 0 and 0.5, W_2 shows negative and W_1 shows positive readings, for a lagging power factor.

Case (iii): Here, $\cos \phi = 1$, Therefore, $\phi = \cos^{-1}(1) = 0°$

$$W_1 = |\overline{V}_L||\overline{I}_L|\cos(30° + 0) = |\overline{V}_L||\overline{I}_L|\cos 30° = \text{positive}$$
$$W_2 = |\overline{V}_L||\overline{I}_L|\cos(30° - 0) = |\overline{V}_L||\overline{I}_L|\cos 30° = \text{positive}$$

Therefore, both W_1 and W_2 are equal and positive. For all power factors between 0.5 and 1, both wattmeters give positive reading.

The results can be summarized as given in Table 6.2.

Table 6.2 *Variations of W_1 and W_2 for Different Power Factors*

Range of Power Factor	Range of 'ϕ'	W_1 sign	W_2 sign	Remark
$\cos \phi = 0$	$\phi = 90°$	Positive	Negative	$\|W_1\| = \|W_2\|$
$0 < \cos \phi < 0.5$	$90° < \phi < 60°$	Positive	Negative	—
$\cos \phi = 0.5$	$\phi = 60°$	Positive	0	—
$0.5 < \cos \phi < 1$	$60° < \phi < 0°$	Positive	Positive	—
$\cos \phi = 1$	$\phi = 0°$	Positive	Positive	$W_1 = W_2$

It is noted that Table 6.2 is applicable for lagging power factor loads, but the same table is applicable for leading power factor loads by interchanging the columns of W_1 and W_2. The effect of power factor on wattmeter readings i.e., curve of power factor against K, is shown in Figure 6.37, where K is given by:

$$K = \frac{\text{Smaller reading}}{\text{Larger reading}}$$

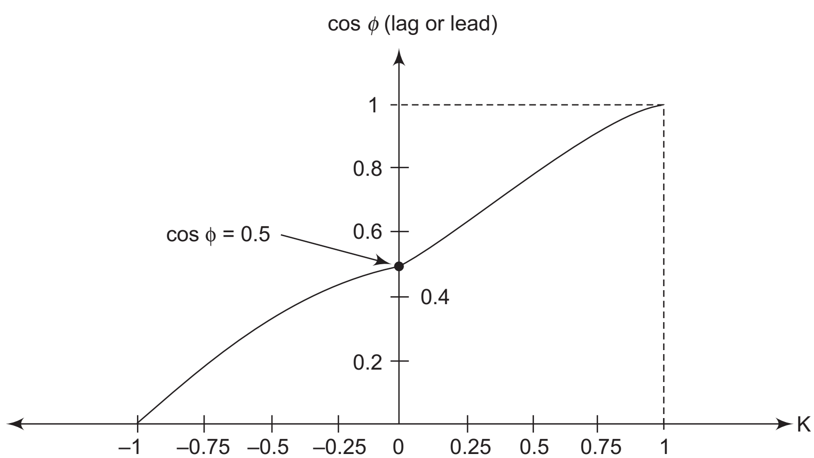


Figure 6.37 Effect of Power Factor on Wattmeter Readings

6.16.6 Reactive Volt-Amperes by Two-Wattmeter Method

The reactive Volt-Amperes can be determined as follows:

It is known that, $W_1 - W_2 = |\overline{V}_L||\overline{I}_L|\sin \phi$

The total reactive Volt-Ampere for a three-phase circuit is given by,

$$Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\sin\phi = \sqrt{3}\,(W_1 - W_2)\ \text{VAR}$$

Thus, reactive Volt-Ampere of a three-phase circuit can be obtained by multiplying $\sqrt{3}$ with the difference of the two-wattmeter readings.

Therefore, apparent power, $S = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|$ VA

Active power, $P = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\cos\phi = W_1 + W_2$ W

Reactive power, $Q = \sqrt{3}\,|\bar{V}_L||\bar{I}_L|\sin\phi = \sqrt{3}\,(W_1 - W_2)$ VAR

Example 6.1

Two wattmeters are connected to measure the power in a three-phase, three-wire balanced load. Determine the total power and power factor if the two wattmeters read: (i) 1000 W each, both positive (ii) 1000 W each, of opposite sign. **[AU April/May, 2010]**

Solution

Case (i) When $W_1 = W_2 = 1000$ W

The total power consumed by the load is given by

$$W = W_1 + W_2 = 2000\ \text{W}$$

The power factor of the given load is

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

Substituting the known values, we get

$$\cos\phi = 1$$

Therefore, the system is a unity power factor system.

Case (ii) When $W_1 = 1000$ W and $W_2 = -1000$ W

The total power consumed by the load is given by

$$W = W_1 + W_2 = 0\ \text{W}$$

The power factor of the given load is

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

Substituting the known values, we get

$$\cos\phi = 0$$

Therefore, the system is a zero power factor system.

Example 6.2

The two-wattmeters method produces wattmeter readings: $W_1 = 1560$ W and $W_2 = 2100$ W, when connected to a delta-connected load. If line voltage is 220 V, calculate (i) the per-phase average power (ii) the per-phase reactive power (iii) the power factor and (iv) the phase impedance. **[AU May/June, 2003]**

Solution

Given W_1 = 1560 W and W_2 = 2100 W

(i) The total active power is given by

$$P = W_1 + W_2 = 3660 \text{ W}$$

Therefore, active power per phase is $\dfrac{3660}{3} = 1220 \text{ W}$

(ii) The total reactive power is given by

$$Q = \sqrt{3}\,(W_1 \sim W_2) = \sqrt{3}\,(2100 - 1560)$$
$$= 935.31 \text{ VAR}$$

Therefore, reactive power per phase is $\dfrac{935.31}{3} = 311.77 \text{ VAR}$

(iii) The power factor of the system is given by

$$\cos\phi = \cos\left\{\tan^{-1}\left[\frac{\sqrt{3}\,(W_1 - W_2)}{(W_1 + W_2)}\right]\right\}$$

Substituting the known values, we get

$$\cos\phi = 0.97 \text{ and } \phi = 14.06°$$

(iv) Line voltage, V_L = 220 V

The total active power is given by

$$P = \sqrt{3} V_L I_L \cos\phi$$

Substituting the known values in the above equation, we get

$$\bar{I}_L = \frac{3660}{\sqrt{3} \times 220 \times 0.97} = 9.9024 \text{ A}$$

The phase current in the delta-connected load is obtained as

$$\bar{I}_{\text{ph}} = \frac{\bar{I}_L}{\sqrt{3}} = 5.7173 \text{ A}$$

But, the phase current in the delta-connected load is given by

$$\bar{I}_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{Z_{\text{ph}}}$$

Substituting the known values in the above equation, we get

$$Z_{\text{ph}} = \frac{\bar{V}_{\text{ph}}}{\bar{I}_{\text{ph}}} = \frac{220}{5.7173} = 38.47\ \Omega$$

Impedance angle will be 30° + ϕ = 30° + 14.06° = 44.06°

Therefore, the phase impedance existing in the delta-connected load is 38.47∠44.06° Ω.

6.17 Instrument Transformer

Instrument transformer is a special type of transformer constructed with accurate turns ratio and is used in measuring and controlling the heavy current and high voltage AC circuits. Since direct measurements require large and costly instruments, relays etc., small and cheap instrument transformers are used for measuring such heavy current and high voltage AC circuits. In addition, these instrument transformers help in increasing the range of instrument, provide isolation from heavy current and high voltage AC circuit and act as a protective device for the operator and control equipment. Therefore, the use of these transformers increases the accuracy level and safety. The instrument transformer is generally classified as:

(a) Current transformer (CT): used in measuring current
(b) Potential transformer (PT): used in measuring voltage

6.17.1 Ratio of Instrument Transformers

(a) Transformation Ratio, K

It is defined as the ratio of primary winding current (voltage) to secondary winding current (voltage) for CT (PT).

i.e., $$K = \frac{I_1}{I_2} \text{ for CT and } K = \frac{V_1}{V_2} \text{ for PT}$$

(b) Nominal Ratio, K_n

It is defined as the ratio of rated primary current (voltage) to the rated secondary current (voltage) for CT (PT).

i.e., $$K_n = \frac{I_{rt}}{I_{r2}} \text{ for CT and } K_n = \frac{V_{r1}}{V_{r2}} \text{ for PT}$$

(c) Turns Ratio, n

It is defined as the ratio of number of turns of secondary winding (primary winding) to the number of turns of primary winding (secondary winding) for CT (PT).

i.e., $$n = \frac{N_2}{N_1} \text{ for CT and } n = \frac{N_1}{N_2} \text{ for PT}$$

(d) Ratio Correction Factor (RCF)

It is defined as the ratio of transformation ratio to the nominal ratio.

i.e., $$\text{RCF} = \frac{K}{K_n}$$

6.17.2 Advantages and Disadvantages of Instrument Transformer

Advantages

(i) Normal range instrument when used along with instrument transformer helps in measuring high electrical quantities.

(ii) Irrespective of the magnitude of the electrical quantity, it helps in fixing the rating of low range meter.
(iii) Provides safety to the operator and other equipments.
(iv) Used in operating protective devices like relay.
(v) Less power loss.

Disadvantage

It is not used in measuring DC quantities.

6.17.3 Current Transformer (CT)

Current transformers are used along with normal range instruments like ammeter, wattmeter or energy meter to measure heavy AC currents. In general, CT is a two winding transformer which transfers a higher value of current to a lower value of current so that the heavy current flowing through the transmission line is safely measured using normal ammeter. The primary winding of CT has single or more turns with large cross-sectional area and the secondary winding of CT has many turns with small cross-sectional area. The primary winding is connected in series to the line carrying current and secondary winding is connected to the normal range ammeter. The CT is represented as shown in Figure 6.38.

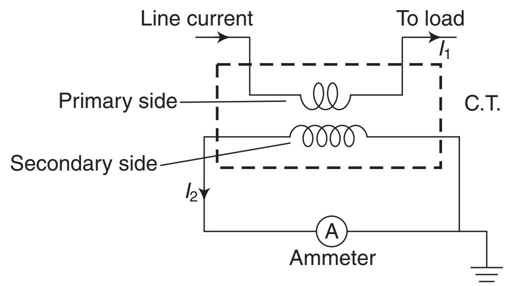


Figure 6.38 Representation of CT

Generally, CT is a step up transformer i.e., it steps up the primary voltage and step down the primary current. Hence, in current perspective, CT is a step down transformer. Normally, in a transformer,

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

Since $N_2 >> N_1$, the current transformation ratio, $\frac{I_1}{I_2}$ is also very high and this indicates the range of CT. Generally, the secondary current ratings are 5A, 1A and 0.1A while the primary current ratings vary from 10A to 3000A. If the current transformation ratio and the secondary current measured using a normal ammeter are known, the current flowing through the primary winding can be determined.

Types of CT:

Based on the construction of primary and secondary windings, CT is classified as: (i) Wound type CT and (ii) Bar type CT

Wound Type CT

In wound type CT shown in Figure 6.39, the primary winding with few turns is placed inside the transformer and is connected in series across the line through which the current is to be measured. This type of CT used to measure the current between 1A to 100 A.

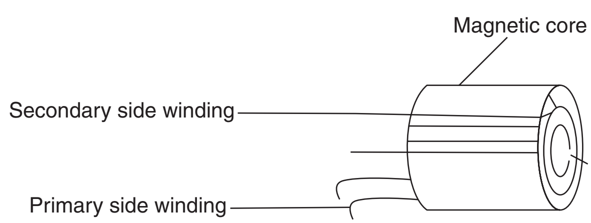


Figure 6.39 Wound Type CT

Bar Type CT

In bar type CT, only one winding, i.e., secondary winding is present. The line on which the transformer is fixed acts as a primary winding of CT as shown in Figure 6.40.

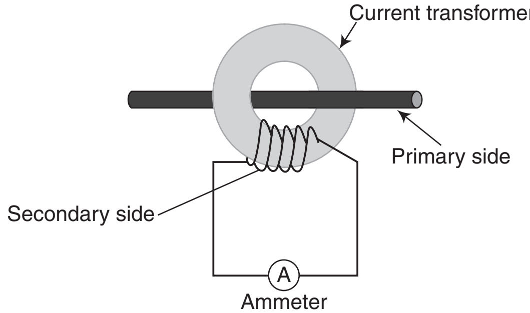


Figure 6.40 Bar type CT

Reasons for Secondary of CT to be Closed

The following are the reasons for secondary of CT to be closed:

(i) If the secondary of CT is open, the current through the secondary winding, $I_2 = 0$. Therefore, the counter mmf produced by the secondary ampere turns is zero. Due to this, the mmf produced in the primary winding is undisturbed and produces more flux with high magnitude in the core. Therefore, there will be more core losses and heat produced in the core will cross its limits.

(ii) Also, the emf induced in the primary and secondary windings will be more and hence the insulations gets damaged.

Therefore, the secondary of CT should be either grounded or connected to a low resistance coil like ammeter and wattmeter while energising the primary winding.

6.17.4 Potential Transformer (PT)

The potential transformer is similar to the current transformer and is used for transforming the higher value of voltage to lower value of voltage. In general, the potential transformer is a step down transformer and helps in measuring the high voltage value using a normal range voltmeter, watt hour meter, etc. The primary winding of PT consists of large number of turns while secondary winding consists of less number of turns. The high voltage line across which the voltage to be measured is connected to the primary winding and a low range voltmeter is connected to the secondary winding. For safety purpose, one end of the secondary winding is connected to the ground. The general representation of PT is shown in Figure 6.41. Normally, in a transformer,

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

Therefore, using the turns ratio and the voltmeter reading, the high voltage is measured. The primary winding of PT is rated for 400V to several thousand volts while the secondary winding of PT is always rated for 400V.

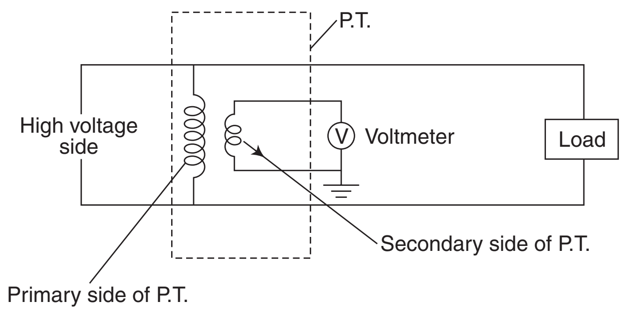


Figure 6.41 Representation of PT

Generally, the PTs are used for measuring the high level voltages, act as a protecting device for feeders, generators and help in synchronising the feeders and generators.

6.17.5 Differences between CT and PT

Table 6.3 lists the comparison between CT and PT.

Table 6.3 *Comparison between CT and PT*

Current Transformer (CT)	Potential Transformer (PT)
High current value is transformed to a low current value.	High voltage value is transformed to a low voltage value.
Primary winding of CT with small number of turns carries the current which is to be measured.	Primary winding of PT with large number of turns carries the voltage which is to be measured.
Secondary winding with large number of turns is connected to the low resistance ammeter coil.	Secondary winding with less number of turns is connected to the meter or instrument.
CT is connected in series with the instrument.	PT is connected in parallel with the instrument.
CT has high transformation ratio.	PT has low transformation ratio.
It has low impedance.	It has high impedance.

6.18 Introduction to Transducer

A transducer is a device that converts energy from one form to another form. This energy may be electrical, mechanical, chemical, optical or thermal. Transducers may be classified according to their applications, methods of energy conversion, nature of the output signal, and so on. All these classifications usually result in overlapping areas. A sharp distinction among the types of transducers is difficult. A transducer that gives electrical energy as the output is known as an *electrical transducer*. The output electrical signal could be voltage, current, or frequency, and the production of these signals is based upon resistive, capacitive, inductive effects, etc. For measuring non-electrical quantities, a detector is used, which usually converts the physical quantity into a displacement that activates the electrical transducer. The *displacement transducers*, namely, capacitive, potentiometric, photoelectric (phototube) and piezoelectric, use the principle of converting a mechanical force into displacement and then into electrical parameters. Here, the mechanical elements used for converting this applied force into displacement are called force-summing devices.

6.19 Classification of Transducers

The transducers are classified:

- **On the basis of transduction form:**
  Based on how the input quantity or measurand is converted, it is further classified as
  - Resistive transducer
  - Capacitive transducer
  - Inductive transducer

  Example: piezoelectric, thermoelectric, magneto restrictive and so on.

- **Primary and secondary transducers**
  Element which makes direct contact to the measurand or the physical quantity is called a primary transducer and the transducers which convert the output of primary transducer to electrical output are called secondary transducers.
  Example: In Bourdon-tube pressure-gauge device, the primary transducer is the Bourdon tube and secondary transducer is an LVDT (Linear variable differential transformer)
- **Passive and active transducers**
  Active transducers, also known as self-generating type, develop their own voltage or current as the output signal. The energy required for production of this output signal is obtained from the physical phenomenon being measured. *Passive transducers*, also known as externally powered transducers, derive the power required for energy conversion from an external power source. However, they may also absorb some energy from the physical phenomenon under study. A few examples of active and passive transducers are given in Table 6.4.

Table 6.4 *Active and Passive transducers*

Active Transducers	Passive Transducers
Thermocouple	**Resistance**
Piezoelectric transducer	Potentiometric device
Photovoltaic (Photojunction) cell	Resistance strain gauge
Moving-coil generator	Resistance thermometer
Photoelectric (Photoemission) cell	Thermistor
	Photoconductive cell
	Inductance
	Linear Variable Differential Transformer (LVDT)
	Capacitance
	Voltage and current
	Devices using Hall effect
	Photoemissive cell
	Photomultiplier tube

Opto-electronic transducers, such as photoconductive cells, photovoltaic cells, solar cells, phototubes, and photomultiplier tubes use the principle of converting light energy into electrical energy.

(a) **Analogue and digital transducer**
Transducers that convert the input quantity into an analogue output are known as analogue transducers. Example: Strain gauge, LVDT, thermocouple, thermistor.
Transducers that convert the input quantity into an electrical output in the form of pulses are known as digital transducers.

(b) **Transducer and inverse transducer**
A device that converts a non-electrical quantity into an electrical quantity is known as a transducer and the device that converts an electrical quantity to a non-electrical quantity is known as an inverse transducer.

Some of the basic requirements of a transducer are given as follows: **[AU Nov/Dec, 2011]**

- ***Linearity:*** The input–output characteristics of the transducer should be linear.

- ***Ruggedness:*** The transducer should withstand overloads, with measures for overload protection.
- ***Repeatability:*** The transducer should produce identical output signals when the same input signal is applied at different times under the same environmental conditions.
- ***High stability and reliability:*** The output from the transducer should not be affected by temperature, vibration, and other environmental variations, and there should be minimum errors in measurements.
- ***Good dynamic response:*** In industrial, aerospace, and biological applications, the input to the transducer will not be static but dynamic in nature, i.e., the input will vary with time. The transducer should respond to the changes in input as quickly as possible.
- ***Convenient instrumentation:*** The transducer should produce a sufficiently high analogue output signal with high signal-to-noise ratio, so that the output can be measured either directly or after suitable amplification.
- ***Good mechanical characteristics:*** The transducer, under working conditions, will be subjected to various mechanical strains. Such external forces should not introduce any deformity and affect the performance of the transducer.

Of the many effects that are used in transducers, the principal effects used are variation of resistance, inductance, capacitance, piezoelectric effect and thermal effects which are described in the following sections.

6.20 Resistive Transducer **[AU Nov/Dec, 2011]**

A transducer that converts the change in resistance of the material into an electrical signal with respect to environmental conditions is known as a resistive transducer. This transducer can be used to change resistance in both AC and DC devices. The resistive transducer is used for measuring physical quantities like temperature, displacement, vibration and so on. In general, the resistance of the material is given by

$$R = \frac{\rho l}{A}$$

where R is the resistance in ohms, A is the cross-sectional area of the conductor in metre square, L is the length of the conductor in metres and ρ is the resistivity of the conductor in materials in ohm metre.

The classification of resistive transducers is based on the variation of any one of the quantities i.e., length, area or resistivity of the material. The different types of resistance transducers are:

- Potentiometric transducer
- Strain gauges
- Resistance thermometers
- Thermistors

6.20.1 Potentiometric Transducer **[AU Nov/Dec, 2012]**

The basic circuit of a potentiometric transducer is shown in Figure 6.42. A potentiometric transducer consists of a resistance element that is contacted by a movable slider. A force-summing member is used to move the slider, thereby changing the resistance and hence the

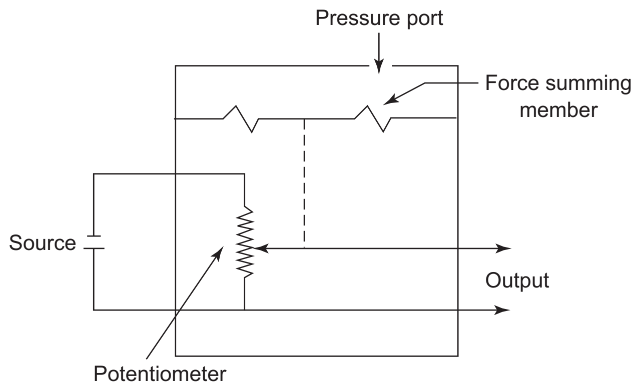


Figure 6.42 Potentiometric Transducers

output voltage changes correspondingly. The same principle can be used to vary the resistance in a bridge circuit. This transducer has high electric efficiency and provides a sufficient output to permit control operations without further amplification.

The advantages, disadvantages and applications of potentiometric transducer are discussed below:

Advantages

1. It is cheap, simple to operate and has a high resolution.
2. It is very useful in applications where there are no severe requirements.
3. It helps in measuring large amplitudes or displacement.
4. Since it has a high electrical efficiency, it is used in control applications.

Disadvantages

1. It requires a large force to move its contacts.
2. Sliding contacts can get contaminated, worn out, and there is a possibility of misalignment and generation of noise.
3. Life-time of this transducer is limited.

Applications

This transducer is used in:

1. A voltage divider to obtain an adjustable output voltage.
2. Audio-control devices for frequency attenuation, to adjust loudness and so on.
3. Televisions to control brightness, contrast and colour response.
4. Measuring the displacement

6.20.2 Electrical Strain Gauges **[AU Nov/Dec, 2014]**

If a metal conductor is stretched or compressed, its resistance changes because of dimensional changes (length and cross-sectional area) and resistivity change. If a wire is under tension and increases its length from l to $l + \Delta l$, i.e., the strain $S = \frac{\Delta l}{l}$, then its resistance increases from R to $R + \Delta R$.

The sensitivity of a strain gauge is described in terms of gauge factor G and is defined as the unit change in resistance per unit change in length, i.e.,

$$G = \frac{\Delta R/R}{\Delta l/l} = \frac{\Delta R/R}{S}$$

Unbonded Strain Gauges

The schematic diagram of a typical displacement transducer wherein the measuring forces are transmitted to the platform containing the unbonded wire structure by means of a force rod is shown in Figure 6.43 (a) and (b). The resistance wires have equal lengths.

When an external force is applied to the strain gauge, the armature moves in the direction indicated. The length of elements A and D increases, whereas, the length of elements B and C decreases. The change in resistance of the four wires is proportional to their change in length and this change can be measured using Wheatstone bridge as shown in Figure 6.43(c).

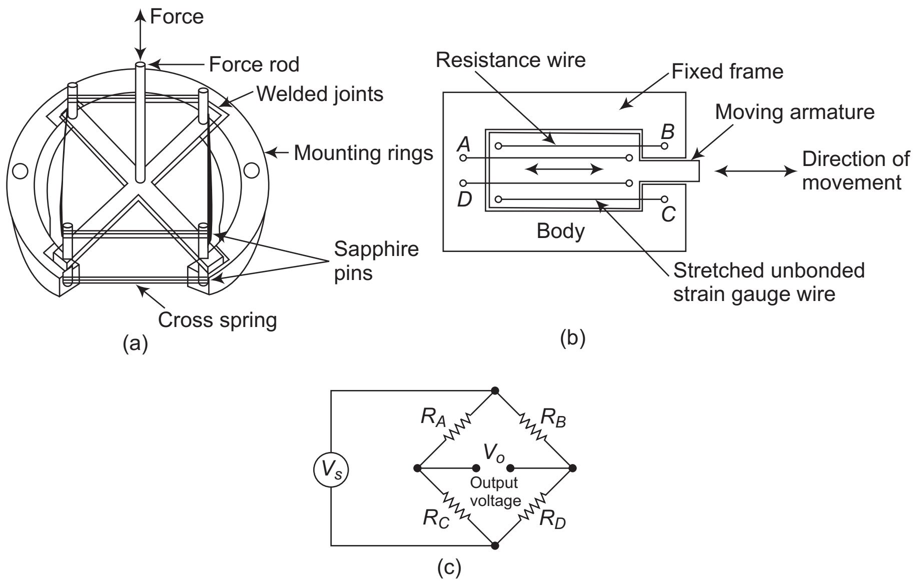


Figure 6.43 Unbonded Strain Gauge

Thus, the external force causes variation in resistance of the wires, unbalancing the bridge and causing an output voltage V_a proportional to the pressure. The bridge is balanced if

$$\frac{R_A}{R_C} = \frac{R_B}{R_D}$$

Bonded Strain Gauge

A bonded-wire strain gauge consists of a grid of fine resistance wire of a diameter of about 25 mm. The wire is cemented to a base. The base may be a thin sheet of paper or a very thin Bakelite sheet. The wire is covered with a thin sheet of material so that it is not damaged mechanically. The base is bonded to the structure under study with an adhesive material. It acts as a bonding material. It permits a good transfer of strain from base to wires. The commonly used types of bonded strain gauges are shown in Figure 6.44. The advantages, disadvantages and applications of strain gauges are given below:

Advantages

1. No moving part exists in the system.
2. Device is small and inexpensive.
3. It has faster response time.

Disadvantages

1. Non-linear characteristics exist in the transducer.
2. Transducer needs to be calibrated.
3. Very sensitive to environmental condition.
4. Has very long term-drift.

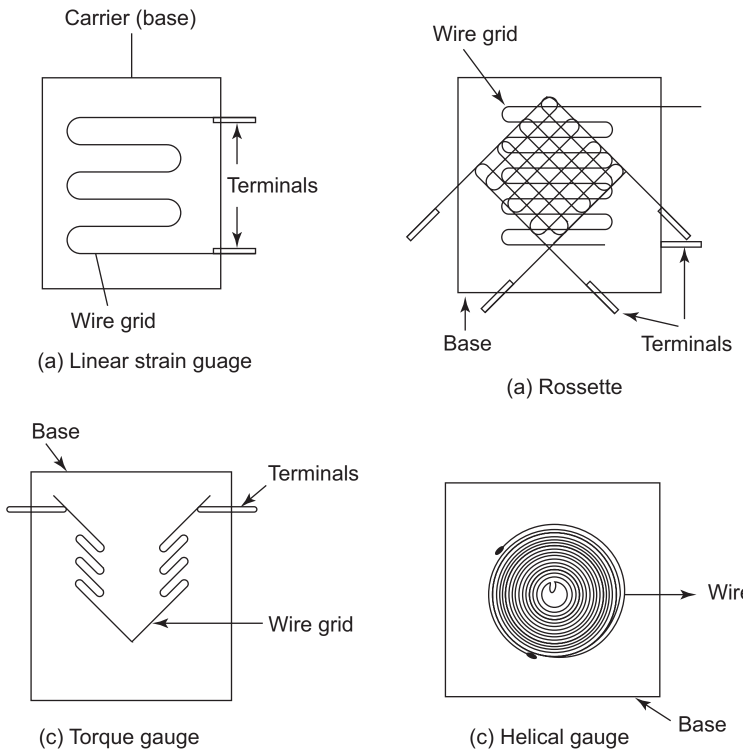


Figure 6.44 Bonded Strain Gauges

Applications

1. Used in measuring normal strains in any desired direction.
2. Can be used in measuring shear strain using some special arrangements.
3. Possible to read the reading remotely.
4. Can be used in measuring static and dynamic strains.
5. Can be used in measuring vibration, torque, bending, deflection, compression and tension.

6.20.3 Resistance Thermometer **[AU Nov/Dec, 2011]**

The resistance of most electrical conductors varies with temperature, according to the relation

$$R = R_0(1 + aT + bT^2 + \ldots)$$

where R_0 is the resistance at temperature T_0 (at 0°C), R is the resistance at T, and a and b are constants.

Over a small temperature range, depending on the material, the above equation reduces to

$$R = R_0(1 + \alpha T)$$

where α is the temperature coefficient of resistance.

Important properties of materials used for resistance thermometers are: (i) high temperature-coefficient of resistance, (ii) stable properties so that the resistance characteristic does not drift with repeated heating and cooling or mechanical strain, and (iii) a high resistivity to permit the construction of small sensors. The variation of resistivity with temperature of some of the materials used for resistance thermometers is shown in Figure 6.45. From the Figure, it can be seen that tungsten has a suitable temperature coefficient of resistance but is brittle and difficult to form. Copper has a low resistivity and is generally confined to applications where the sensor size is not restricted. Both platinum and nickel are widely used because they are relatively easy to obtain in pure state.

Platinum has an advantage over nickel, as its temperature coefficient of resistance is linear over a larger temperature range. The resistance–temperature relationship for platinum resistance elements is determined from the Callendar equations:

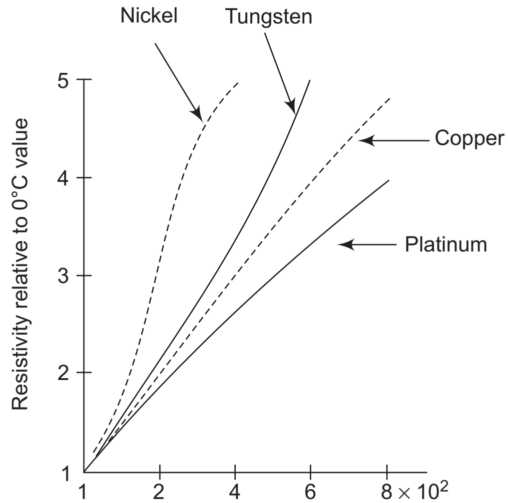


Figure 6.45 Variation of Resistivity with Temperature of Materials Used for Resistance Thermometer

$$T = \frac{100(R_T - R_0)}{R_{100} - R_0} + d\left(\frac{T}{100} - 1\right)\frac{T}{100}$$

where R_T is the resistance at temperature T, R_0 is the resistance at 0°C, R_{100} is the resistance at 100°C and d is the Callendar constant, which is approximately 1.5.

The construction of an industrial platinum resistance thermometer is shown in Figure 6.46.

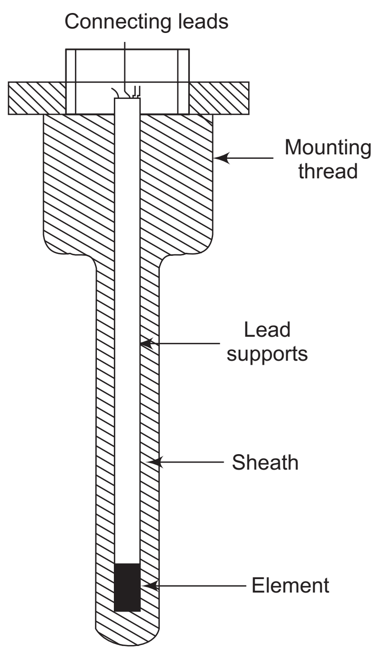


Figure 6.46 Industrial Platinum Resistance Thermometer

Advantages, Disadvantages and Applications of Resistance Thermometer

Advantages

1. Accurate measurement of quantity is possible.
2. Direct operation of indicators and recorders is possible.
3. Easily to install and replace.
4. Possible to measure differential temperature.
5. Wide range of temperature can be measured i.e., from -20° C to + 650° C.
6. Smaller in size and is suitable for remote indication.

Disadvantages

1. External power source is necessary for its operation.
2. Comparatively expensive when compared to other transducers.
3. Self-heating problem exists in the transducer.

Applications

Commonly used in aerospace, analytical equipment, food-service equipment, and semiconductor equipment.

6.20.4 Thermistor **[AU Nov/Dec, 2014]**

A thermistor, or a thermal resistor, is a two-terminal semiconductor device whose resistance is temperature sensitive. The value of such resistors decreases with increase in temperature. Materials employed in the manufacture of the thermistors include oxides of cobalt, nickel, copper, iron, uranium and manganese. The symbol for a thermistor is shown in Figure 6.47(a).

The thermistor has a very high temperature-coefficient of resistance, of the order of 3 to 5% per °C, making it an ideal temperature transducer. The temperature coefficient of resistance is normally negative. The resistance at any temperature T, is given approximately by

$$R_T = R_0 \exp \beta\left(\frac{1}{T} - \frac{1}{T_0}\right)$$

where R_T is the thermistor resistance at temperature $T(K)$, R_0 is the thermistor resistance at temperature $T_0(K)$ and β is a constant determined by calibration.

At high temperatures, this equation reduces to

$$R_T = R_0 \exp\left(\frac{\beta}{T}\right)$$

The resistance–temperature characteristic is shown in Figure 6.47 (b). The curve is non-linear and the drop in resistance from 5000 Ω to 10 Ω occurs for an increase in temperature from 20°C to 100°C. The temperature of the device can be changed internally or externally. An increase in current through the device will raise its temperature, carrying a drop in its terminal resistance. Any externally applied heat source will result in an increase in its body temperature and drop in resistance. This type of action (internal or external) lends itself well to control mechanisms.

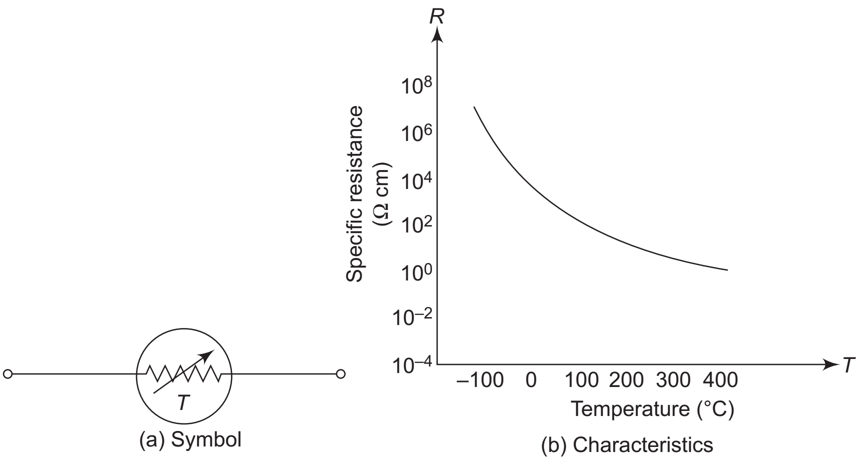


Figure 6.47 Symbol and Resistance–Temperature Characteristics of a Thermistor

Three useful parameters used in characterising thermistors are: time constant, dissipation constant, and resistance ratio. The time constant is the time taken for a thermistor to change its resistance by 63% of its initial value, for zero power dissipation. Typical values of time-constant range from 1–50 s.

The dissipation constant is the power necessary to increase the temperature of a thermistor by 1°C. Typical values of dissipation constant range from 1 mW/°C to 10 mW/°C.

Resistance ratio is the ratio of the resistance at 25°C to that at 125°C. Its range is approximately 3–60.

Advantages, Disadvantages and Applications of Thermistor

Advantages

1. Compact, low cost and longer life-time.
2. Has good stability of the system.
3. Has faster response i.e., from seconds to minutes.
4. More sensitive when compared to other temperature sensors.
5. Compatible with many devices.
6. Easy to interface with the external circuits.

Disadvantages

1. Requires shielding.
2. Requires an input power to activate.
3. Low excitation current is required to avoid self-heating.
4. Not suitable for large temperature range.
5. Non-linear resistance temperature characteristics.

Applications

Thermistor are used in:

1. Measurement of temperature.
2. Controlling of temperature.
3. Temperature compensation.
4. Measuring voltage and power at high frequencies, thermal conductivity, level, flow and pressure of liquids and composition of gases.
5. Measuring vacuum and to provide time-delay.

Comparison between RTD and Thermistor

The comparison between RTD and thermistor is given in Table 6.5.

Table 6.5 *Comparison between RTD and thermistor*

RTD	THERMISTOR
It is made up of metals.	Thermistor is made up of semiconductor materials.
Since metals have a positive temperature coefficient (PTC) of resistance, its resistance is directly proportional to temperature	Since semiconductor materials have a negative temperature coefficient (NTC) of resistance, its resistance is inversely proportional to temperature.
Has linear resistance temperature characteristics	Has non-linear resistance temperature characteristics
Less sensitive to temperature.	Highly sensitive to temperature.
Has wide operating range i.e., –200°C to 650°C	Has a narrow operating range i.e., –100°C to 300°C
Larger in size.	Smaller in size.
Costlier when compared to a thermistor.	Cheaper when compared to RTD
Has low self-resistance.	Has high self-resistance.
Provides high degree of accuracy and long-term stability.	Provides an accuracy of ± 0.01°C.
Used in laboratory and industrial applications.	Used in dynamic temperature measurement.

6.21 INDUCTIVE TRANSDUCER **[AU Nov/Dec, 2011]**

When a force is applied to a ferromagnetic armature, the air gap, as shown in Figure 6.48, gets changed, thereby varying the reluctance of the magnetic circuit i.e., the inductance of the magnetic circuit gets changed. Thus, the applied force is measured by the change of inductance in a single coil. The inductive transducer enables static and dynamic measurements.

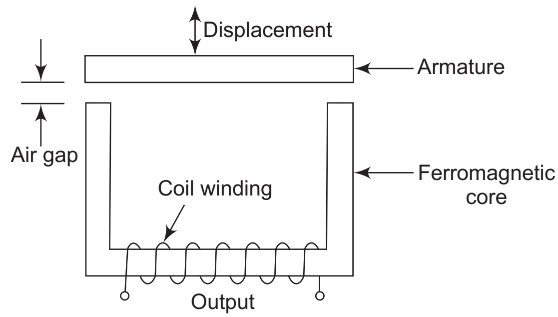


Figure 6.48 Inductive Transducers

6.21.1 Linear Variable Differential Transformer (LVDT)

[AU April/May, 2015]

The most widely used inductive transducer is the Linear Variable Differential Transformer (LVDT), as shown in Figure 6.49(a). It consists of a primary coil and two exactly similar secondary coils with a rod-shaped magnetic core positioned centrally, inside the coil. An alternating current is fed into the primary, and voltages V_{o1} and V_{o2} are induced in the secondary coils. As these coils are connected in series opposition, the output voltage is $V_o = V_{o1} \approx V_{o2}$. If the core is placed ideally in the central position (null position or reference position), $V_{o1} = V_{o2}$ and hence, the output voltage $V_o = 0$. In practice, due to incomplete balance, a residual voltage usually remains with the core in this position. As shown in Figure 6.49 (a), when the core is displaced from the null position, the induced voltage in the secondary towards which the core has moved increases, while in the other the secondary voltage decreases. This results in a differential voltage output from the transformer.

The output voltage produced by the displacement of the core is linear over a considerable range, as shown in Figure 6.49(b) but flattens out at both ends, and the voltage phase changes by 180°, as the core moves through the centre position.

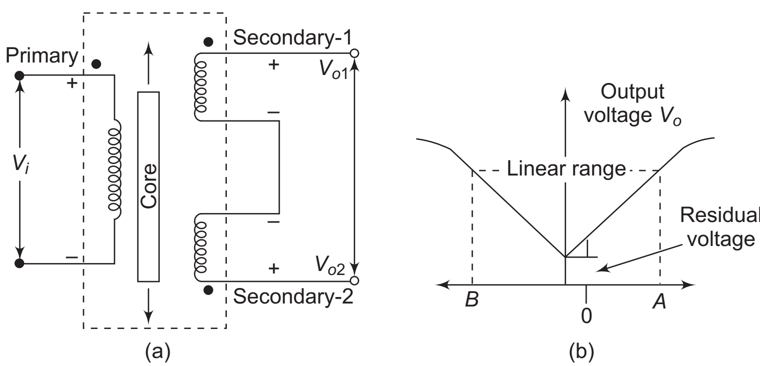


Figure 6.49 Linear Variable Differential Transformer (a) Schematic Diagram (b) Characteristics

LVDT provides continuous resolution and shows low hysteresis and hence, repeatability is excellent under all conditions. As there are no sliding contacts, there is less friction and less noise. It is sensitive to vibrations and temperature. The receiving instrument must be selected to operate on AC signals or a demodulator network must be used if a DC output is required.

Advantages, Disadvantages and Applications of LVDT

Advantages

1. It is used in wide range of applications
2. Presence of a linear relationship in the instrument
3. High sensitivity and output
4. High resolution, high sensitivity and good repeatability
5. Consumes less power
6. Produces low hysteresis
7. Low frictional losses

Disadvantages

1. Requires large displacement to get considerable differential output.
2. Very sensitive to stray magnetic field and hence requires shielding.
3. Temperature and vibrations affect output of the transducer.
4. The dynamic response is being controlled mechanically.

Applications

1. Used to measure displacement with ranging from a few mm to cm.
2. Can be used as primary and secondary transducers.
3. Used in combination with Bourdon tube to measure pressure.
4. Mostly used in servomechanisms and other industrial applications.

6.21.2 Rotary Variable Differential Transformer (RVDT) **[AU Nov/Dec, 2014]**

Rotary Variable Differential Transformer or RVDT is an inductive transducer, which senses the angular displacement of the conductor and gives a linear output proportional to it i.e., it provides a variable AC output voltage proportional to the angular displacement of the input shaft. It is similar to LVDT, except that its core is in cam shape and moves between the windings by means of a shaft. The output signal of RVDT is linear within a specified range over the angular displacement when it is energised using a fixed AC source. The schematic diagram of RVDT is shown in Figure 6.50.

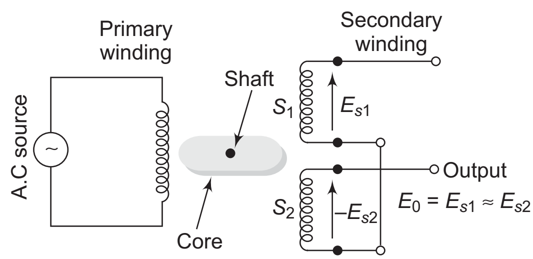


Figure 6.50 Schematic Diagram of RVDT

The RVDT consists of one primary winding and two secondary windings. The emf induced in the two secondary windings is a function of rotary displacement of the core around the shaft and both the secondary windings are placed in such a way that the emf induced is 180^0 out of phase with each other.

Working

The working of RVDT is similar to the operation of LVDT. According to the angular movement of the shaft, three differential conditions are formed.

Condition 1

When shaft is at null position, as shown in Figure 6.50, the emf induced in both the secondary windings are equal but opposite in phase. Therefore, the differential output taken from the secondary windings is zero and is explained mathematically using the following equation:

$$E_{s1} = E_{s2}$$

Where E_{s1} is the emf induced in the first secondary winding and E_{s2} is the emf induced in the other secondary winding.

Therefore, the resultant output voltage is given by $E_0 = E_{s1} - E_{s2} = 0$.

Condition 2

When the shaft starts rotating in the clockwise direction, more portion of the core comes in contact with secondary winding S_1 when compared to S_2. Hence, the emf induced across the secondary winding S_1 is more than the emf induced across the secondary winding S_2 i.e.,$E_{s1} > E_{s2}$. Therefore, the differential output drawn from these windings is positive.

i.e., $$E_0 = E_{s1} - E_{s2} = \text{positive}$$

Condition 3

When the shaft starts rotating in the anti-clockwise direction, more portion of the core comes in contact with secondary winding S_2, when compared to S_2. Hence, the emf induced across the secondary winding S_2 is more than the emf induced across the secondary winding S_1 i.e., $E_{s1} < E_{s2}$. Therefore, the differential output drawn from these windings is negative

i.e., $$E_0 = E_{s1} - E_{s2} = \text{negative}$$

The curve between the magnitude of differential output voltage and angular displacement is shown in Figure 6.51. The curve is linear for small angular displacements and beyond this range, it starts to deviate from the straight line. In practice, there will be some residual voltage in RVDT when the core is kept at null position.

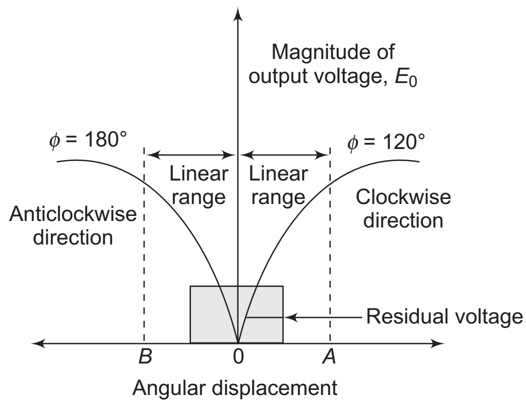


Figure 6.51 Input–Output Curve of RVDT

Advantages, Disadvantage and Applications of RVDT

The advantages, disadvantage and applications of RVDT are:

Advantages

1. Low cost due to popularity in application.
2. Solid and robust construction, which helps in operating it at different environmental conditions.
3. High accuracy and reliability can be achieved, as there is no frictional resistance.
4. Hysteresis is negligible.

Disadvantage

The RVDT provides linear output only for certain range of angular displacement.

Applications

The RVDT is used in:

1. Flight control actuation / navigation
2. Fuel-control valves
3. Cockpit controls
4. Signal conditioning as RVDT conditioner
5. Actuator feedback

6.22 Capacitive Transducer [AU Nov/Dec, 2014]

The capacitance of a parallel-plate capacitor is given by

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

where A is the area of each plate in m^2, d is the distance between parallel plates in m, ε_0 is the dielectric constant (permittivity) of free space in F/m and ε_r is the relative dielectric constant (permittivity).

The capacitance is directly proportional to the area of the plate (A) and inversely proportional to the distance between the parallel plates (d). Obviously, any variation in A or d causes a corresponding variation in the capacitance. This principle of variation in d is used in the capacitive transducer, as shown in Figure 6.52.

When a force is applied to a diaphragm, which acts as one plate of a capacitor, the distance between the diaphragm and the static plate is changed. The resulting change in capacitance can be measured with an AC bridge or an oscillator circuit in which an electric counter can measure the change in frequency and which is a measure of the magnitude of the applied force. In a capacitor microphone, the same principle is used, where the sound pressure varies the capacitance between the fixed plate and a movable diaphragm. The capacitive transducer can measure static and dynamic changes. The drawback of this transducer is its sensitivity to temperature variations.

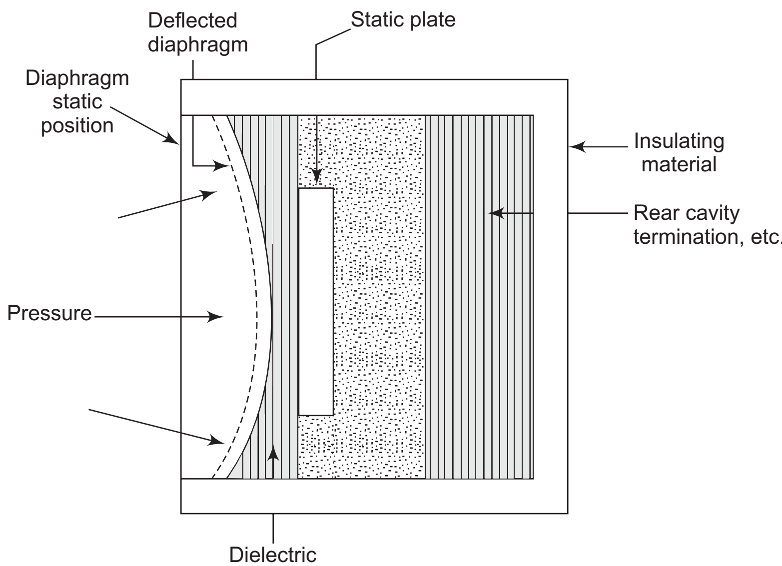


Figure 6.52 Capacitive Transducer

6.22.1 Advantages, Disadvantages and Applications of Capacitive Transducer

Advantages

1. Requires external force for operation, which makes it useful for small systems.
2. Highly sensitive and has good resolution.

3. Good frequency response.
4. Requires small power for its operation.
5. High input impedance decreases the loading effect.
6. It requires an external force for operation and hence very useful for small systems.

Disadvantages

1. Requires insulation.
2. Requires earthing to avoid stray magnetic field.
3. Sensitive to temperature changes, dust particles and moisture.
4. Presence of non-linear characteristics
5. Associates complex instrumentation circuitry.

Applications

1. Helps in measuring linear and angular displacement.
2. Used to measure force and pressure
3. Used as pressure transducer where change in dielectric constant occurs.
4. Measurement of humidity in the gases, volume, density and so on.

6.23 Thermoelectric Transducer **[AU April/May, 2011]**

A temperature transducer, which converts thermal energy into electrical energy, is known as a thermoelectric transducer. Thermocouple is the most commonly used thermoelectric transducer. Thermocouple, a type of primary transducer, is used for measuring temperature, where the change in temperature arising from two dissimilar metals is converted into an electrical energy. Thermoelectric phenomena like Seebeck effect, Peltier effect and Thompson effect are used to describe the thermocouple behaviour.

6.23.1 Seebeck Effect

This effect was introduced by Prof. Seebeck in 1821, which states that if two wires of different metals, like copper and iron, are joined together to form a closed circuit with two junctions and if those junctions are maintained at different temperatures, then an electric current will flow through the closed circuit i.e., the current will flow from copper to iron in the hot junction and from iron to copper in the cold junction. The explanation of Seebeck effect is shown in Figure 6.53 (a). In addition, it also says that an emf called Seebeck emf, which is directly proportional to the change in temperature, appears across the open circuit if the copper wire is cut at a particular point.

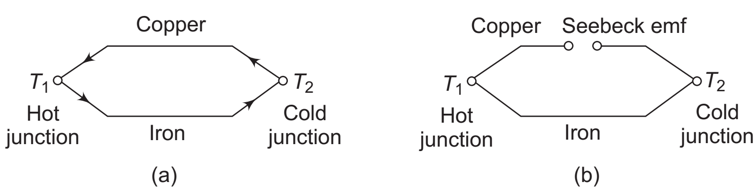


Figure 6.53 Seebeck Effect (a) Flow of Current (b) Emf

6.23.2 Peltier Effect

Professor Peltier introduced this effect, which is a reverse of Seebeck effect, in 1824. It states that if two wires of dissimilar metals form two junctions when an external voltage source is connected, as shown in Figure 6.54, then the current starts flowing through both the junctions. It also states that the heat is absorbed at a junction where the current is flowing from copper to iron, making the junction T_1 hot, and heat is liberated at a junction where the current is flowing from iron to copper, making the junction T_2 cold.

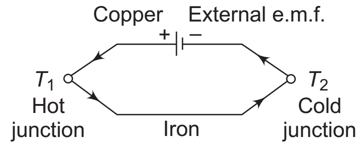


Figure 6.54 Peltier effect

6.23.3 Thompson Effect

It is a reversible heat flow effect, which was introduced by Professor Thompson. It states that when a current flows through the copper conductor with a thermal gradient along its length, then heat is released at a junction where the current and heat flow are in the same direction and heat is absorbed at a junction where different directions exist for current and heat flow.

6.23.4 Construction of a Thermocouple

Two dissimilar metals, when joined together to form two junctions T_1 and T_2, form a thermocouple, as shown in Figure 6.55 (a). Usually, T_2 is kept at constant reference temperature and is referred as cold junction or reference junction. The temperature which is to be measured is subjected to T_1 and hence it is referred as hot junction or measuring junction. When there is a temperature difference between T_1 and T_2, an emf that is proportional to the temperature gradient gets generated and can be measured using any meter or recorder, as shown Figure 6.55 (b).

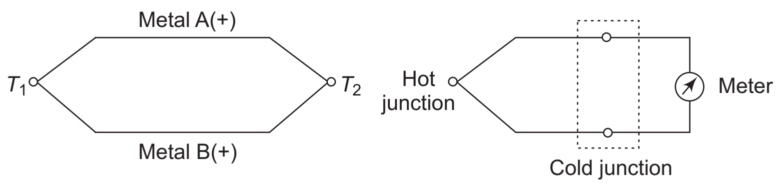


Figure 6.55 Thermocouple Circuit (a) Schematic Diagram (b) Practical Circuit

Generally, the junction in the thermocouple is formed in two ways: twisted weld and butt weld. In twisted weld, two large sized wires are twisted and welded together with several turns to give mechanical strength, while in butt weld, two comparatively small wires are fused together into a round bend. The normal sizes of metals are: 0.5 mm diameter for noble metals and 1.5 to 3 mm diameter for base metals.

6.23.5 Types of Thermocouples

Based on the materials used in the thermocouple and the range of temperature it can measure, the different types of thermocouples are listed in Table 6.6.

Table 6.6 *Types of Thermocouples*

Type of Thermocouples	Material Used	Temperature Range
T	Copper - constantan	–250°C to 400°C
J	Iron - constantan	–200°C to 850°C
K	Chromel - Alumel	–200°C to 110°C
E	Chromel - Constantan	–200°C to 850°C
S	Platinum - Platinum rhodium	0°C to 1400°C
—	Tungsten - molybdenum	0°C to 2700°C
—	Tungsten-Rhenium	0°C to 2600°C

Advantages, Disadvantages and Applications of Thermocouples

Advantages

1. Rugged construction
2. Covers wide range of temperature: –270°C to 2700°C
3. Most suitable for temperature measurement in industrial furnaces
4. Cheaper in cost
5. Easy to check the calibration
6. Offers good reproducibility
7. High response speed and good accuracy

Disadvantages

1. To have high accuracy, it is necessary to have cold junction compensation.
2. Non-linear characteristics exist between induced emf and temperature.
3. Possible to have stray voltage pickup.
4. Signal amplification is required for many applications.

Applications

1. Testing temperatures associated with different process plants e.g. chemical production, petroleum refineries, heating appliance safety, food industries, steel, iron and aluminium industries, plastics and resin industries.
2. Suitable for low temperature and cryogenic applications.
3. Temperature profiling in ovens, furnaces and kilns.
4. Temperature measurement of gas turbine and engine exhausts.

6.24 Piezoelectric Transducer **[AU May/June, 2011]**

If the dimensions of asymmetrical crystalline materials, such as quartz, Rochelle salt and barium titanite, are changed by the application of a mechanical force, the crystal produces an emf. This property is used in piezoelectric transducers. The basic circuit of a piezoelectric transducer is shown in Figure 6.56. Here, a crystal is placed between a solid base and the force-summing member. An externally applied force gives pressure to the top of the crystal. Hence, it produces an emf across the crystal, which is proportional to the magnitude of the applied pressure. As this transducer has a very good high-frequency response, it is used in high-frequency accelerometers. As it needs no external power source, it is called a self-generating transducer. The main drawbacks are that it cannot measure static conditions and the output voltage is affected by temperature variations of the crystal.

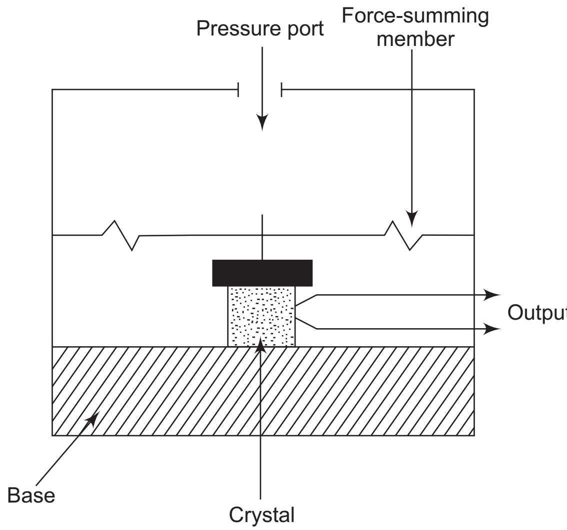


Figure 6.56 Piezoelectric Transducer

6.24.1 Advantages, Disadvantages and Applications of Piezoelectric Transducer

Advantages

1. Available in desired shape
2. Has rugged construction and it is smaller in size
3. Has good frequency response and negligible phase-shift

Disadvantages

1. Used in dynamic measurement only
2. Highly sensitive to temperature
3. Since some crystals are water-soluble, it might get dissolved in highly humid environment

Applications

1. Helps in stabilising electronic oscillators
2. Used in measuring surface roughness, accelerometer and vibration pickup
3. Used in industrial cleansing apparatus and in underwater detection systems
4. Used in spark-ignition engine, electronic watches and record players
5. Used as a sensing element e.g., piezoelectric microphones
6. Used in ultrasound imaging, chemical and biological sensors

6.25 Photoelectric Transducer

This is an optoelectronic or optical transducer, shown in Figure 6.57. It uses a phototube and a light source, separated by a small window, whose aperture is controlled by the force-summing device. The quantity of incident light on the photosensitive cathode is varied according to the externally applied force, thereby changing the anode current. This device measures both static and dynamic phenomena and it has high efficiency. It does not respond to high frequency light variation.

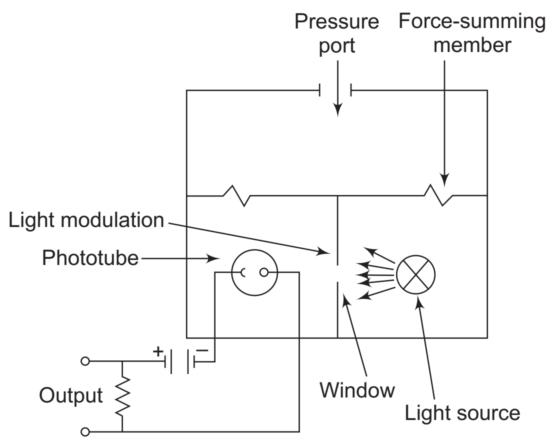


Figure 6.57 Photoelectric Transducer

6.25.1 Advantages, Disadvantages and Application of Photoelectric Transducer

Advantages

1. Capability to sense all the possible materials
2. Has long duration of life
3. Highly sensitive and reliable
4. Has very quick response time
5. Less cost

Disadvantages

1. Gets affected by atmospheric conditions.
2. Sensing range gets affected by target colour and reflexivity,

Application

Used in packaging, material handling and parts detection.

6.26 Hall Effect Transducers

When a transverse magnetic field B is applied to a specimen (thin strip of metal or semiconductor) carrying current I, an electric field E is induced in the direction perpendicular to both I and B. This phenomenon is known as the *Hall effect*.

A Hall-effect measurement experimentally confirms the validity of the concept that it is possible for two independent types of charge carriers, electrons and holes, to exist in a semiconductor.

The schematic arrangement of the semiconductor, the magnetic field and the current flow pertaining to Hall effect are shown in Figure 6.58. Under equilibrium condition, the electric field intensity, E, due to the Hall effect must exert a force on the carrier of charge, q, which just balances the magnetic force, i.e.,

$$qE = Bqv_d$$

where v_d is the drift velocity. Also, the electric field intensity due to Hall effect is given by

$$E = \frac{V_H}{d}$$

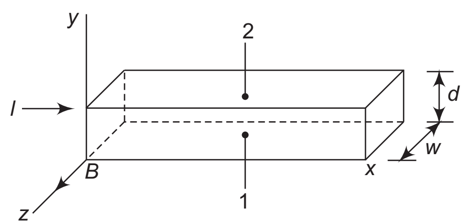


Figure 6.58 Schematic Diagram to Observe Hall Effect

where d is the distance between surfaces 1 and 2, and V_H is the Hall voltage appearing between surfaces 1 and 2. In an N-type semiconductor, electrons carry the current and these electrons will be forced downward towards side 1, which becomes negatively charged with respect to side 2.

The current density (J) is related to charge density (ρ) by

$$J = \rho v_d$$

Further, the current density (J) is related to current (I) by

$$J = \frac{I}{\text{Area}} = \frac{I}{wd}$$

where w is the width of the specimen in the direction of magnetic field (B).
Combining the above relations, we get

$$V_H = Ed = Bv_d d = \frac{BJd}{\rho} = \frac{BI}{\rho w}$$

The Hall coefficient, R_H, is defined by

$$R_H = \frac{1}{\rho}$$

so that $V_H = \frac{R_H}{w} BI$. A measurement of the Hall coefficient R_H, determines not only the sign of the charge carriers but also their concentration. The Hall coefficient for a P-type semiconductor is positive, whereas it is negative for an N-type semiconductor. This is true because the Hall voltage in a P-type semiconductor is of opposite polarity to that in an N-type semiconductor.

The advantage of Hall-effect transducers is that they are non-contact devices with high resolution and small size.

The Hall effect is used to find whether a semiconductor is N or P-type and to determine the carrier concentration. If the terminal 2 becomes charged positively with respect to terminal 1, the semiconductor must be N-type and $\rho = pq$, where n is the electron concentration. On the other hand, if the polarity of V_H is positive at terminal 1 with respect to terminal 2, the semiconductor must be P-type and $\rho = pq$, where p is the hole concentration.

The mobility (μ) can also be calculated with simultaneous measurement of the conductivity (σ). The conductivity and the mobility are related by the equation $\sigma = \rho\mu$ or $\mu = \sigma R_H$.

Therefore, the conductivity for N-type semiconductor is $\sigma = nq\mu_n$ and for P-type semiconductor, $\sigma = pq\mu_n$, where μ_n is the electron mobility and μ_p is the hole mobility.

Thus, if the conductivity of a semiconductor is also measured along with R_H, then mobility can be determined from the following relations.

For N-type semiconductor, $\mu_n = \frac{\sigma}{nq} = \sigma R_H$

and for P-type semiconductor, $\mu_p = \frac{\sigma}{pq} = \sigma R_H$

Since V_H is proportional to B for a given current I, Hall effect can be used to measure the AC power and the strength of magnetic field and sense the angular position of static magnetic fields in a magnetic field meter. It is also used in an instrument called Hall-effect multiplier, which gives the output proportional to the product of two input signals. If I is made proportional to one of the inputs and B is made proportional to the second signal, then from the equation, $V_H = \frac{BI}{\rho w}$, V_H will be proportional to the product of two inputs. Hall devices for such applications are made from a thin wafer or film of Indium Antimonide (InSb) or Indium Arsenide. As the material has very high electron mobility, it has high Hall coefficient and high sensitivity.

An electrical current can be controlled by a magnetic field because the magnetic field changes the resistances of some elements with which it comes in contact. In the magnetic bubble memory, while read-out, the Hall effect element is passed over the bubble. Hence, a change in current of the circuit will create, say, a *one*. If there is no bubble, there will be a *zero* and there will be no current change in the output circuit.

The read-in device would have an opposite effect, wherein the Hall device creates a magnetic field when supplied with a pulse of current. This, in turn, creates a little domain and then a magnetic bubble is created.

Some of the other applications are in measurement of velocity, rpm, sorting, limit sensing, and non-contact current measurements.

6.26.1 Advantages, Disadvantages and Applications of Hall Effect Transducers

Advantages

1. High-speed operation over 100 kHz is possible, whereas at high frequencies, the inductive or capacitive sensor output begins to distort.
2. As there is no wear and friction due to non-contact operation, the number of operating cycles is unlimited.
3. When packed, it is immune to dust, air and water, whereas dust triggers a capacitive sensor.
4. It can measure zero speed.
5. Highly repeatable operation.
6. Capable of measuring large current.

Disadvantages

1. Gets affected by external interfering magnetic field.
2. There exists a large temperature drift.
3. Large offset voltage.

Applications

1. Used in converting magnetic flux to electric transducer.
2. Used as current sensor.
3. Automotive fuel-level indicator.
4. Spacecraft propulsion.
5. Used in brushless DC motor to sense the position of the rotor.
6. Used in measuring power, current and displacement.

6.27 Mechanical Tranducers

A transducer that converts one form of physical quantity to another is known as a mechanical transducer. This transducer is the primary transducer, which acts as an input to the electrical or secondary transducer. It can measure the physical quantities such as pressure, force, displacement, flow rate etc. The common mechanical transducers, which convert one form of physical quantity to another form, are:

- ***Flat spiral spring***: It can produce the controlling torque in the instruments to measure electrical quantities.
- ***Torsion bar of shaft:*** The primary sensing element in torque meter, which is used to measure the torque, is known torsion bar of shaft. Its deflection or twist is directly proportional to the torque applied and hence its deformation is used to measure the torque.
- ***Proving ring:*** It is used to measure force, weight or load. It causes a deflection, which is further measured with the help of electrical transducer.

- ***Spring flexure pivot:*** It is a frictionless device used in measurements. The sensitivity of the device is almost constant for an angular displacement that is less than 15°.
- ***Bourdon tube:*** The different forms of Bourdon tubes are: (i) C type, (ii) spiral, (iii) twisted tube and (iv) helical, as shown in Figure 6.59. It is made of brass or phosphor bronze or beryllium copper or steel. It is made out of an elliptically sectioned elastic tube, which is bent to form the above-mentioned shapes. One end of the tube is closed and other end is opened for the liquid to enter. When the liquid, whose pressure is to be determined, enters the tube, a movement which can be measured is caused in the free end. It is normally used to measure gauge pressure.

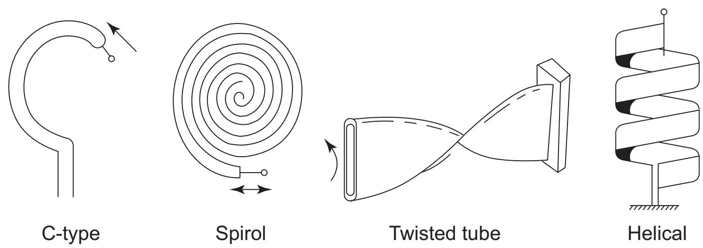


Figure 6.59 Bourdon Tubes

- ***Diaphragm:*** Flat and corrugated diaphragms, shown in Figure 6.60(a), are used to measure pressure by determining the displacement of the diaphragm. The pressure to be measured is applied at one side of the diaphragm and the other side is rigidly fixed. This type of arrangement causes deflection at the centre of the diaphragm, which is directly proportional to the pressure applied.

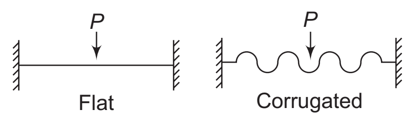


Figure 6.60(a) Diaphragms

  Deformation occurs in the flat diaphragm, when a pressure P is applied to it, as shown in Figure 6.60(b). The relation between the pressure P applied and the displacement d_m at the centre of the diaphragm is given by

$$P = \frac{256Et^3 d_m}{2(1-v^2)D^4} \text{ N/m}^2$$

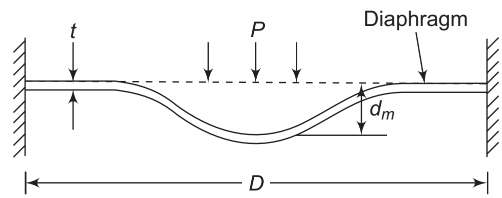


Figure 6.60(b) Flat Diaphragm with Pressure P Applied

  where E is the Young's modulus in N/m^2, t is the thickness of the diaphragm in m, D is the diameter of the diaphragm in m, d_m is the deflection at the centre of the diaphragm in m and v is the Poisson's ratio.

  The above relation between pressure P and d_m is linear for $d_m \leq 0.5t$ and non-linear in other cases.

- ***Bellow:*** A thin-walled tube, whose thickness is approximately 0.1 mm and having a corrugated shape, is known as bellow and is made up of a single piece of special brass or stainless steel. Figure 6.61 shows a simple bellow. It is also known as pressure-activated spring and its displacement for a particular pressure depends on its type and the thickness of the material used.

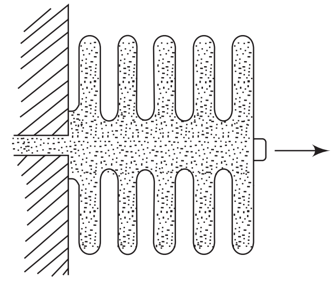


Figure 6.61 Bellow

- ***Temperature detector:*** The different principles used for detecting temperature are:
  - ***Using bimetallic strip:*** It consists of two different metals with different coefficients of thermal expansion, which are joint together. When the strip is heated due to expansion of metal, a deflection made by the bimetallic strip is converted into the movement of the pointer to indicate the temperature.
  - ***Thermocouple:*** Temperature is detected using the thermoelectric emf generated between two metals.
  - ***Resistive thermometer and thermistor:*** Temperature is detected by changing the resistance of the material used in it.
- ***Hydro-pneumatic device:*** This device is used to measure the flow and it works on the principle of simple float or a hydrometer.

6.28 Light Dependant Resistor (LDR)

The *photoconductive cell* (*PC*) or photodetector is a two-terminal device which is used as a Light Dependent Resistor (LDR). It is made of a thin layer of semiconductor material such as cadmium sulphide (CdS), lead sulphide (PbS), or cadmium selenide (CdSe) whose spectral responses are shown in Figure 6.62. The photoconducting device with the widest applications is the CdS cell, because it has high dissipation capability, with excellent sensitivity in the visible spectrum and low resistance when stimulated by light. The main drawback of CdS cell is its slower speed of response. PbS has the fastest speed of response.

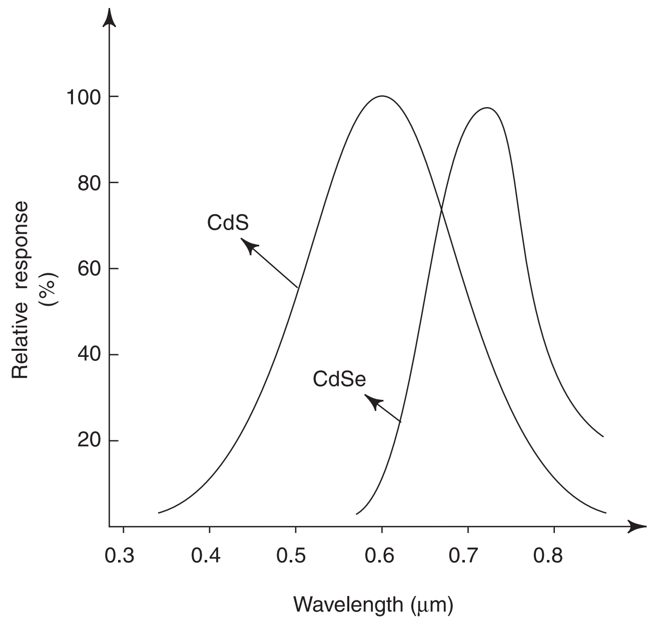


Figure 6.62 Spectral responses of CdS and CdSe

The illumination characteristics of photoconductive detectors are shown in Figure 6.63(a). It exhibits the peculiar property that its resistance decreases in the presence of light and increases in the absence of light. The cell simply acts as a conductor whose resistance changes when illuminated. In absolute darkness, the resistance is as high as 2 MΩ and in strong light, the resistance is less than 10 Ω.

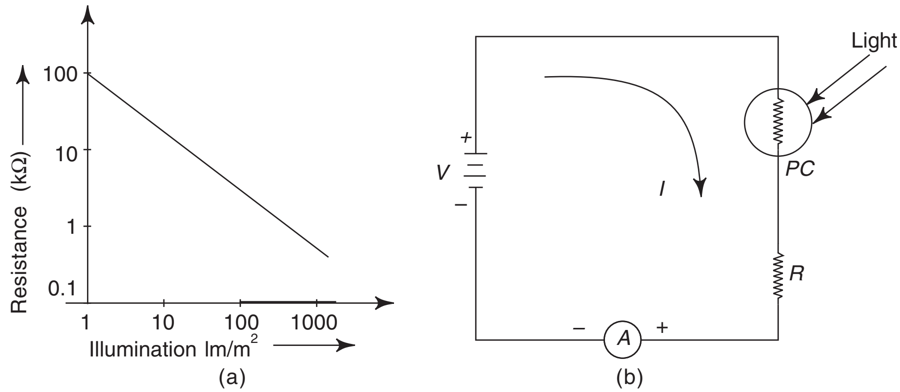


Figure 6.63 (a) Illumination characteristics of the photoconductive detector
(b) Photoconductive detector connected in a simple circuit

A simple circuit for a photoconductive detector is shown in Figure 6.63(b). The semiconductor layer is enclosed in a sealed housing. A glass window in the housing permits light to fall on the active material of the cell. Here, the resistance of the photoconductive detector, in series with *R*, limits the amount of current *I* in the circuit. The ammeter *A* is used to measure the current *I*. When no light falls on the cell, its resistance is very high and the current *I* is low. Hence, the voltage drop V_o across *R* is relatively low. When the cell is illuminated, its resistance becomes very low. Hence, current *I* increases and voltage V_o increases. Thus, this simple circuit arrangement with slight modification can be used in control circuits to control the current.

Applications

The detector is used either as an ON/OFF device to detect the presence or absence of a light source which is used for automatic street lighting or some intermediate resistance value can be used as a trigger level to control relays and motors. Further, it is used to measure a fixed amount of illumination and to record a modulating light intensity.

It is used in counting systems where the objects on a conveyor belt interrupt a light beam to produce a series of pulses which operates a counter.

It is used in twilight switching circuits. When the day light has faded to a given level, the corresponding resistance of the detector causes another circuit to switch ON the required lights.

It is widely used in cameras to control shutter opening during the flash. Twin photoconductive cells mounted in the same package have been used in optical bridge circuits for position control mechanisms and dual-channel remote volume control circuits.

Two Mark Questions and Answers

1. **What are the basic elements of a generalised measurement system?** **[AU Nov/Dec, 2013]**

   Refer to section 6.3 for the basic elements of a generalised measurement system

2. **Mention the basic requirements of measurement.** **[AU Nov/Dec, 2014]**

   The necessary or essential requirements for any measuring instrument are:

   (i) When the instrument is used in the circuit, its conditions should not be altered and therefore the quantity to be measured goes unaffected.

(ii) It should consume low power as possible.
(iii) It should possess a very high efficiency and high sensitivity.
(iv) The output should be linearly proportional to the input.
(v) It should be less affected by the noise, modifiable and properly priced.

3. What is measurement and how is it classified? **[AU April/May, 2014]**

The process of measuring the quantity is known as measurement and the apparatus used to measure the quantity like voltage, current, power, energy, resistance and so on is called *measuring instrument*.
Refer to section 6.6 for the classification of measuring instruments

4. What is primary sensing element? **[AU Nov/Dec, 2012]**

The first unit in the measurement system which detects the measurand is known as primary sensing unit. It helps in transferring the measurand to variable-conversion unit for further processing. For example, liquid or mercury in glass thermometer acts as primary sensing unit. Displacement or voltage is the output in the primary sensing unit.

5. List any four static characteristics of a measuring system. **[AU April/May, 2012]**

Refer to section 6.4.1 for the static characteristics of a measuring system

6. Define the term 'accuracy'. **[AU April/May, 2012]**

Accuracy of a measured value of a quantity is defined as the closeness of the measured value obtained using instrument to the true value of the same quantity. It depends on the accuracy of the instrument itself, variation of the quantity which is to be measured, and observer accuracy, etc.

7. Define the term 'precision'. **[AU April/May, 2012]**

Precision comes from the term precise which indicates clearly or sharply defined and it is a measure of reproducibility of the measurements or a degree of agreement within a measurement group.

8. Define 'static error' and how are static errors classified. **[AU April/May, 2010]**

Static error is the difference between the measured value and true value of the quantity as given by

$$E_s = A_m - A_t$$

where A_m is the measured value of the quantity and A_t is the true value of the quantity
Refer to section 6.5 for classification of static errors.

9. Distinguish between reproducibility and repeatability. **[AU April/May, 2012]**

The reproducibility is the degree of closeness with which a given value may be repeatedly measured using the same instrument under different conditions like changes in the method of measurement, observer, measuring instrument location, conditions of use and time of measurement is known as reproducibility and is specified in terms of scale readings over a given period of time, whereas the repeatability is the instrument characteristic which describes the closeness with which a given value is repeatedly measured on the same instrument, same location, same observer, under the same measurement conditions and when the same input is given to the instrument repetitively over a particular time. It is specified as a variation in scale reading.

10. What is meant by dynamic characteristics of instruments? **[AU April/May, 2011]**

Refer to section 6.4.2 for the dynamic characteristics of instruments

11. Define "error" in measurement. **[AU April/May, 2013; Nov/Dec, 2014]**

The "error" in measurement is defined as the difference between the true or actual value and the measured value.

12. With one example, explain "Instrumental errors". [AU Nov/Dec, 2012]

Refer to section 6.5.2 for instrumental error.

13. How are the absolute and relative errors expressed mathematically? [AU Nov/Dec, 2012]

Absolute error: $E_s = A_m - A_t$

where A_m is the measured value of the quantity and A_t is the true value of the quantity.

Relative error: $E_r = \dfrac{E_s}{A_t}$

14. What is transducer? [AU Nov/Dec, 2014; April/May, 2011]

A transducer is a device which converts energy from one form to another. This energy may be electrical, mechanical, chemical, optical or thermal.

15. What is piezoelectric effect? [AU April/May, 2015; Nov/Dec, 2011]

If the dimensions of asymmetrical crystalline materials, such as quartz, Rochelle salt and barium titanite, are changed by the application of a mechanical force, the crystal produces an emf. This property is used in piezoelectric transducers.

16. Distinguish between active and passive transducers. [AU Nov/Dec, 2013]

Refer to section 6.28 for the difference between active and passive transducer.

17. How the transducers are classified on the basis of principle of transduction? [AU April/May, 2010]

On the basis of principle of transduction, the transducers are classified as

(i) Resistive transducer
(ii) Capacitive transducer
(iii) Inductive transducer

Example: piezoelectric, thermoelectric, magneto restrictive and so on

18. List the factors to be considered for selecting a transducer. [AU April/May, 2012]

The factors which are to be considered in selecting a transducer are:

(i) Linearity
(ii) Ruggedness
(iii) Repeatability
(iv) High stability and reliability
(v) Good dynamic response
(vi) Convenient instrumentation
(vii) Good mechanical characteristics

19. Define "gauge factor" of strain gauge. [AU April/May, 2012]

The sensitivity of a strain gauge described in terms of a characteristic called the gauge factor G is defined as the unit change in resistance per unit change in length, i.e., $G = \dfrac{\Delta R/R}{\Delta l/l} = \dfrac{\Delta R/R}{S}$

20. Mention the uses of capacitive transducer. [AU Nov/Dec, 2011]

Uses of capacitive transducer are:

(i) It helps in measuring linear and angular displacement
(ii) It is used to measure force and pressure
(iii) It is used as pressure transducer where change in dielectric constant occurs and
(iv) It is used to measure the humidity in the gases, volume and density

Review Questions

1. Define measurement, measuring instrument and measurand.
2. What are the essential requirements of measuring instrument?
3. What are the basic elements of a generalised measurement system?
4. What is measurement and how is it classified?
5. What are the basic requirements of an indicating instrument? Briefly discuss them.
6. Explain the static characteristics of instruments.
7. List the dynamic characteristics of instruments and explain them.
8. Define errors in measurement.
9. Why is damping torque necessary in indicating instruments? What are the methods of producing the same? Explain with necessary sketches.
10. Justify the necessity of controlling torque in indicating instruments. Discuss the various methods of producing the same. Compare them.
11. What are the types of indicating instruments?
12. What is the working principle of a moving iron instrument with the necessary equations?
13. What to you understand by attraction type and repulsion type instruments? What are the important difference between moving-coil and moving-iron instrument?
14. What are the advantages, disadvantages and applications of moving iron instruments?
15. Describe the working principle of a moving-coil instrument with necessary equations.
16. Explain the working of a dynamometer type instrument. What are the specific requirements of the same when it is used as a wattmeter?
17. What are the advantages, disadvantages and applications of moving-coil instrument?
18. Explain the construction and working of single-phase induction type energy meter.
19. Discuss the different errors and their adjustments in single-phase induction type energy meter.
20. Explain the construction and working of wattmeter.
21. Explain the operation of a basic digital multimeter with the help of a block diagram.
22. Describe the working of a CRO with the help of a block diagram.
23. Explain how frequency and phase can be measured using a CRO.
24. What is the speciality of a dual-beam CRO? Explain its working with a block diagram.
25. Describe, with the help of a neat block diagram, the working principle of a dual-trace CRO.
26. What are the special features of storage oscilloscopes?
27. Explain the principle of operation of a digital storage oscilloscope.
28. How does the sampling CRO increase the apparent frequency response of an oscilloscope?
29. Describe the applications of CRO.
30. Describe the distortion test on amplifiers using a CRO.
31. Show that three-phase power can be measured by two wattmeters. Draw the phasor diagrams. Derive an expression for power factor in terms of wattmeter readings.
32. With a neat circuit diagram and phasor diagram, explain the three-phase power measured by two wattmeter method and also derive the expression for power factor.
33. What is instrument transformer?
34. What are different types of CTs?
35. Compare CT and PT.
36. What is a transducer? Briefly describe any one of the displacement transducers.
37. What are active and passive transducers? Why are they called so?
38. What are the basic requirements of a transducer?

39. Discuss resistive transducer with necessary diagrams
40. What is meant by gauge factor of a strain gauge?
41. Discuss, with suitable diagrams, the salient features of unbonded and bonded strain gauges.
42. Explain the principle of operation of a resistance thermometer.
43. What is a thermocouple?
44. How is a thermocouple used for temperature measurement?
45. Compare RTD and thermistor
46. Discuss the working principle of an inductive transducer.
47. Explain in detail the working of a linear variable differential transformer.
48. Describe the construction features of the linear variable differential transformer.
49. Explain the working of Rotary variable differential transfer with a neat sketch.
50. Explain the transduction principle used in the capacitor transducer.
51. Explain Hall effect. How can Hall effect be used to determine some of the properties of a semiconductor?
52. Describe the applications of Hall effect.
53. List the advantages, disadvantages and applications of different resistive transducers.
54. Write short notes on:
    (i) Thermistor
    (ii) Photoelectric transducer
    (iii) Piezoelectric transducer
    (iv) Thermoelectric transducer
55. List the advantages, disadvantages and applications of inductive, capacitive, thermoelectric, piezoelectric, and photoelectric and Hall effect transducers.
56. Explain mechanical transducers.
57. Explain LDR.